Robotics 2

Dynamic model of robots: Newton-Euler approach

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Approaches to dynamic modeling
(reprise)

energy-based approach (Euler-Lagrange) ⇌ **Newton-Euler method (balance of forces/moments)**

- multi-body robot seen as a whole
- constraint (internal) reaction forces between the links are automatically eliminated: in fact, they do not perform work
- closed-form (symbolic) equations are directly obtained
- best suited for study of dynamic properties and **analysis** of control schemes

- dynamic equations written separately for each link/body
- mainly used for inverse dynamics in real time
  - equations are evaluated in a numeric and recursive way
  - best for **synthesis** (=implementation) of model-based control schemes
- by eliminating the internal reaction forces and performing back-substitution of all expressions, we get dynamic equations in closed-form (identical to Euler-Lagrange!)

Derivative of a vector in a moving frame

… from velocity to acceleration

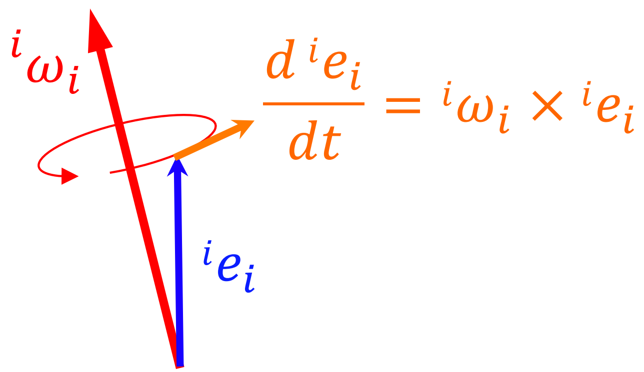
$$ {}^0v_i = {}^0R_i\, {}^iv_i \qquad\qquad {}^0\dot{R}_i = S({}^0\omega_i)\, {}^0R_i $$

$$ {}^0\dot{v}_i = {}^0a_i = {}^0R_i\, {}^ia_i = {}^0R_i\, {}^i\dot{v}_i + {}^0\dot{R}_i\, {}^iv_i $$

$$ = {}^0R_i\, {}^i\dot{v}_i + {}^0\omega_i \times {}^0R_i\, {}^iv_i = {}^0R_i \left({}^i\dot{v}_i + {}^i\omega_i \times {}^iv_i \right) $$

$$ \boxed{{}^ia_i = {}^i\dot{v}_i + {}^i\omega_i \times {}^iv_i} $$

derivative of a "unit" vector in a moving frame



Dynamics of a rigid body

- Newton dynamic equation
  - balance: sum of forces = variation of linear momentum

$$\sum f_i = \frac{d}{dt}(m v_c) = m \dot{v}_c$$

- Euler dynamic equation
  - balance: sum of moments = variation of angular momentum

$$\sum \mu_i = \frac{d}{dt}(I\omega) = I\dot{\omega} + \frac{d}{dt}\left(R\bar{I}R^T\right)\omega = I\dot{\omega} + \left(\dot{R}\bar{I}R^T + R\bar{I}\dot{R}^T\right)\omega$$

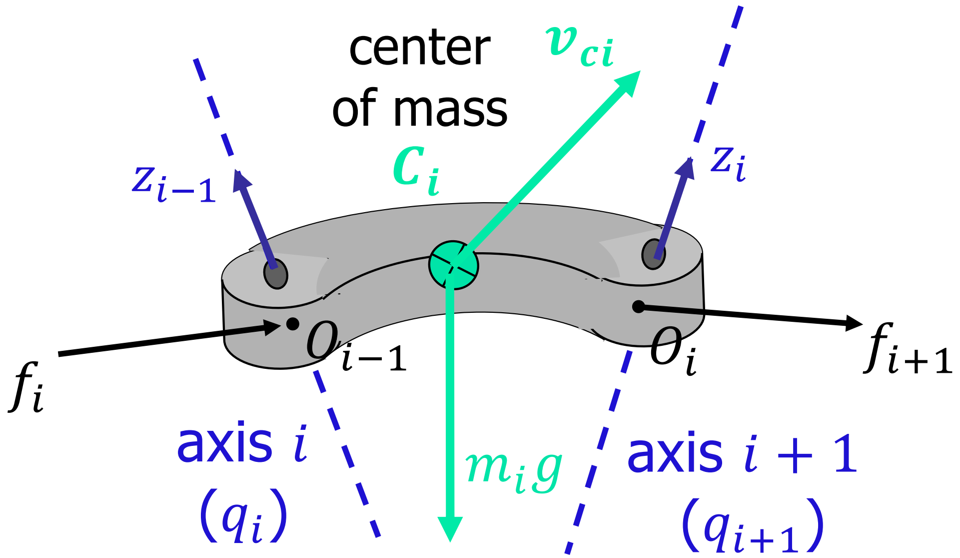
$$= I\dot{\omega} + S(\omega)R\bar{I}R^T\omega + R\bar{I}R^T S^T\cancel{(\omega)\omega} = I\dot{\omega} + \omega \times I\omega$$

- principle of action and reaction
  - forces/moments: applied by body i to body $i+1$

    = $-$ applied by body $i+1$ to body i

Newton-Euler equations - 1

link i



FORCES

f_i force applied from link $i-1$ on link i

f_{i+1} force applied from link i on link $i+1$

$m_i g$ gravity force

all vectors expressed in the same RF (better in RF$_i$ ...)

Newton equation

$$\boxed{f_i - f_{i+1} + m_i g = m_i a_{ci}}$$

N

linear acceleration of C_i

Newton-Euler equations - 2

link i

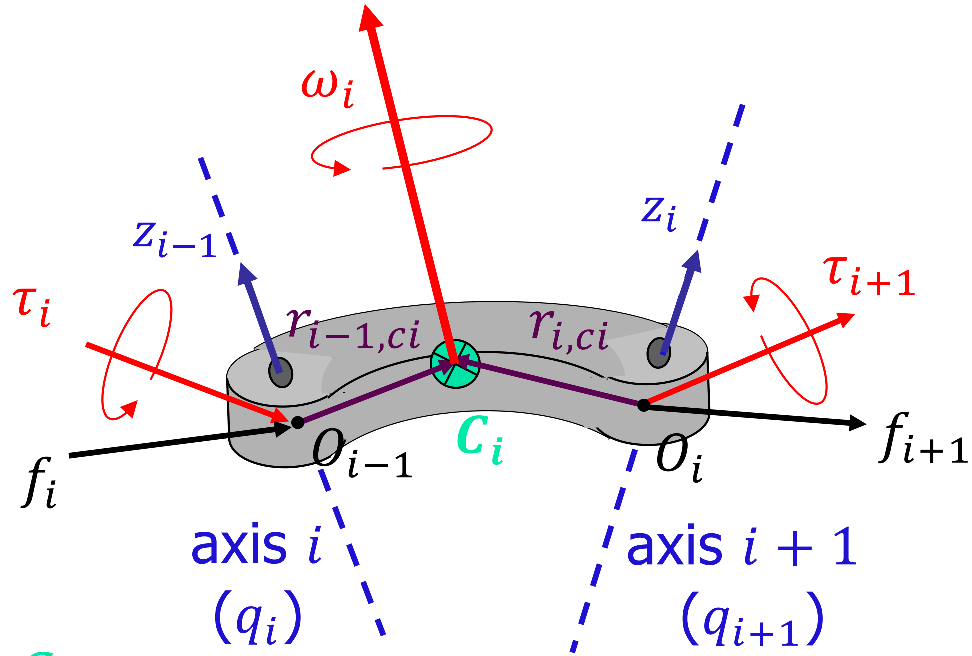
MOMENTS

τ_i moment applied from link $(i-1)$ on link i

τ_{i+1} moment applied from link i on link $(i+1)$

$f_i \times r_{i-1,ci}$ moment due to f_i w.r.t. C_i

$-f_{i+1} \times r_{i,ci}$ moment due to $-f_{i+1}$ w.r.t. C_i



gravity force gives no moment at C_i

all vectors expressed in the same RF (... RF$_i$!!)

Euler equation

$$\boxed{\tau_i - \tau_{i+1} + f_i \times r_{i-1,ci} - f_{i+1} \times r_{i,ci} = I_i \dot{\omega}_i + \omega_i \times (I_i \omega_i)}$$

E

angular acceleration of body i

Forward recursion

Computing velocities and accelerations

- "moving frames" algorithm (as for velocities in Lagrange)
- for simplicity, only revolute joints here
  (see textbook for the more general treatment)

initializations

$$ {}^{i}\omega_i = {}^{i-1}R_i^T\left[{}^{i-1}\omega_{i-1} + \dot{q}_i\,{}^{i-1}z_{i-1}\right] \quad \longleftarrow \quad {}^{0}\omega_0 $$

$$ {}^{i}\dot{\omega}_i = {}^{i-1}R_i^T\left[{}^{i-1}\dot{\omega}_{i-1} + \ddot{q}_i\,{}^{i-1}z_{i-1}\right] + {}^{i-1}\dot{R}_i^T\left[{}^{i-1}\omega_{i-1} + \dot{q}_i\,{}^{i-1}z_{i-1}\right] $$

AR

$$ = {}^{i-1}R_i^T\left[{}^{i-1}\dot{\omega}_{i-1} + \ddot{q}_i\,{}^{i-1}z_{i-1} + \dot{q}_i\,{}^{i-1}\omega_{i-1}\times{}^{i-1}z_{i-1}\right] \quad \longleftarrow \quad {}^{0}\dot{\omega}_0 $$

$$ {}^{i}a_i = {}^{i-1}R_i^T\,{}^{i-1}a_{i-1} + {}^{i}\dot{\omega}_i \times {}^{i}r_{i-1,i} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}r_{i-1,i}\right) \quad \longleftarrow \quad {}^{0}a_0 - {}^{0}g $$

$$ {}^{i}a_{ci} = {}^{i}a_i + {}^{i}\dot{\omega}_i \times {}^{i}r_{i,ci} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}r_{i,ci}\right) $$

the gravity force term can be skipped in Newton equation, if added here

Proof of recursion

for the angular acceleration ${}^{i}\dot{\omega}_i$

$$ {}^{i}\omega_i = {}^{i-1}R_i^T\left[{}^{i-1}\omega_i\right] = {}^{i-1}R_i^T\left[{}^{i-1}\omega_{i-1} + {}^{i-1}\omega_{i-1,i}\right] = {}^{i-1}R_i^T\left[{}^{i-1}\omega_{i-1} + \dot{q}_i\,{}^{i-1}z_{i-1}\right] $$

contribution due to revolute joint i

same formula used in recursive algorithm for kinetic energy

$$ {}^{i}\dot{\omega}_i = {}^{i-1}R_i^T\left[{}^{i-1}\dot{\omega}_{i-1} + \ddot{q}_i\,{}^{i-1}z_{i-1}\right] + {}^{i-1}\dot{R}_i^T\left[{}^{i-1}\omega_i\right] $$

$$ = {}^{i-1}R_i^T\left[{}^{i-1}\dot{\omega}_{i-1} + \ddot{q}_i\,{}^{i-1}z_{i-1}\right] + \left(S\left({}^{i-1}\omega_{i-1,i}\right){}^{i-1}R_i\right)^T\left[{}^{i-1}\omega_i\right] $$

$$ = {}^{i-1}R_i^T\left[{}^{i-1}\dot{\omega}_{i-1} + \ddot{q}_i\,{}^{i-1}z_{i-1}\right] - {}^{i-1}R_i^T S\left({}^{i-1}\omega_{i-1,i}\right)\left[{}^{i-1}\omega_i\right] \qquad \left(S^T = -S\right) $$

$$ = \quad \ldots \quad - {}^{i-1}R_i^T\left[{}^{i-1}\omega_{i-1,i} \times {}^{i-1}\omega_i\right] $$

$$ = \quad \ldots \quad - {}^{i-1}R_i^T\left[\dot{q}_i\,{}^{i-1}z_{i-1} \times \left({}^{i-1}\omega_{i-1} + \dot{q}_i\,{}^{i-1}z_{i-1}\right)\right] $$

$$ = \quad \ldots \quad - {}^{i-1}R_i^T\left[\dot{q}_i\,{}^{i-1}z_{i-1} \times {}^{i-1}\omega_{i-1}\right] = \quad \ldots \quad + {}^{i-1}R_i^T\left[\dot{q}_i\,{}^{i-1}\omega_{i-1} \times {}^{i-1}z_{i-1}\right] $$

$$ = {}^{i-1}R_i^T\left[{}^{i-1}\dot{\omega}_{i-1} + \ddot{q}_i\,{}^{i-1}z_{i-1} + \dot{q}_i\,{}^{i-1}\omega_{i-1} \times {}^{i-1}z_{i-1}\right] $$

Hint: by direct time differentiation of the general DH rotation matrix ${}^{i-1}R_i(\theta_i)$ of a revolute joint show that

$$ {}^{i-1}\dot{R}_i = S\left({}^{i-1}\omega_{i-1,i}\right){}^{i-1}R_i \qquad \text{with } {}^{i-1}\omega_{i-1,i} = \dot{\theta}_i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \dot{\theta}_i\,{}^{i-1}z_{i-1} $$

Proof of recursion
for the acceleration ${}^{i}a_i$

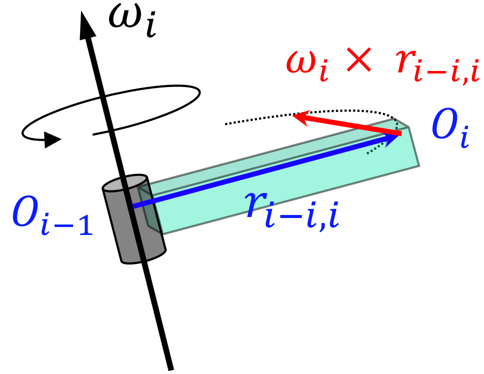
in any fixed frame

$$v_i = v_{i-1} + \omega_i \times r_{i-i,i}$$

fundamental kinematic relation of a rigid body
(relation between velocities of point O_{i-1} and O_i)

$$\dot{v}_i = \dot{v}_{i-1} + \dot{\omega}_i \times r_{i-i,i} + \omega_i \times \dot{r}_{i-i,i}$$

$$= \dot{v}_{i-1} + \dot{\omega}_i \times r_{i-i,i} + \omega_i \times \left(\omega_i \times r_{i-i,i}\right)$$



in moving frames

$${}^{i}a_i = {}^{i}\dot{v}_i = {}^{i-1}R_i^T\,{}^{i-1}\dot{v}_{i-1} + {}^{i}\dot{\omega}_i \times {}^{i}r_{i-1,i} + {}^{i}\omega_i \times ({}^{i}\omega_i \times {}^{i}r_{i-1,i})$$

$$= {}^{i-1}R_i^T\,{}^{i-1}a_{i-1} + {}^{i}\dot{\omega}_i \times \underbrace{{}^{i}r_{i-1,i}} + {}^{i}\omega_i \times ({}^{i}\omega_i \times \underbrace{{}^{i}r_{i-1,i}})$$

constant for a revolute joint

Backward recursion

Computing forces and moments

from N_i $\longrightarrow$ to N_{i-1}

eliminated, if inserted in forward recursion (i=0)

initializations

$$ {}^i f_i = {}^i R_{i+1} \, {}^{i+1} f_{i+1} + m_i \left({}^i a_{ci} - \cancel{{}^i g} \right) \longleftarrow f_{N+1} $$

τ_{N+1} $\downarrow$

F/MR

$$ {}^i \tau_i = {}^i R_{i+1} \, {}^{i+1} \tau_{i+1} + \left({}^i R_{i+1} \, {}^{i+1} f_{i+1} \right) \times {}^i r_{i,ci} - {}^i f_i \times \left({}^i r_{i-1,i} + {}^i r_{i,ci} \right) + {}^i I_i \, {}^i \dot{\omega}_i + {}^i \omega_i \times {}^i I_i \, {}^i \omega_i $$

from E_i $\longrightarrow$ to E_{i-1}

at each recursion step, the two vector equations ($N_i + E_i$) at joint i provide a wrench $(f_i, \tau_i) \in \mathbb{R}^6$): this contains ALSO reaction forces/moments at the joint axis $\Rightarrow$ to be "projected" along/around this axis to produce work

FP

$$ u_i = \begin{cases} {}^i f_i^T \, {}^i z_{i-1} + F_{vi} \dot{q}_i & \text{for prismatic joint} \\ {}^i \tau_i^T \, {}^i z_{i-1} + F_{vi} \dot{q}_i & \text{for revolute joint} \end{cases} \Rightarrow \boxed{N \text{ scalar equations}} \text{ at the end} $$

generalized forces (in rhs of Euler-Lagrange eqs)

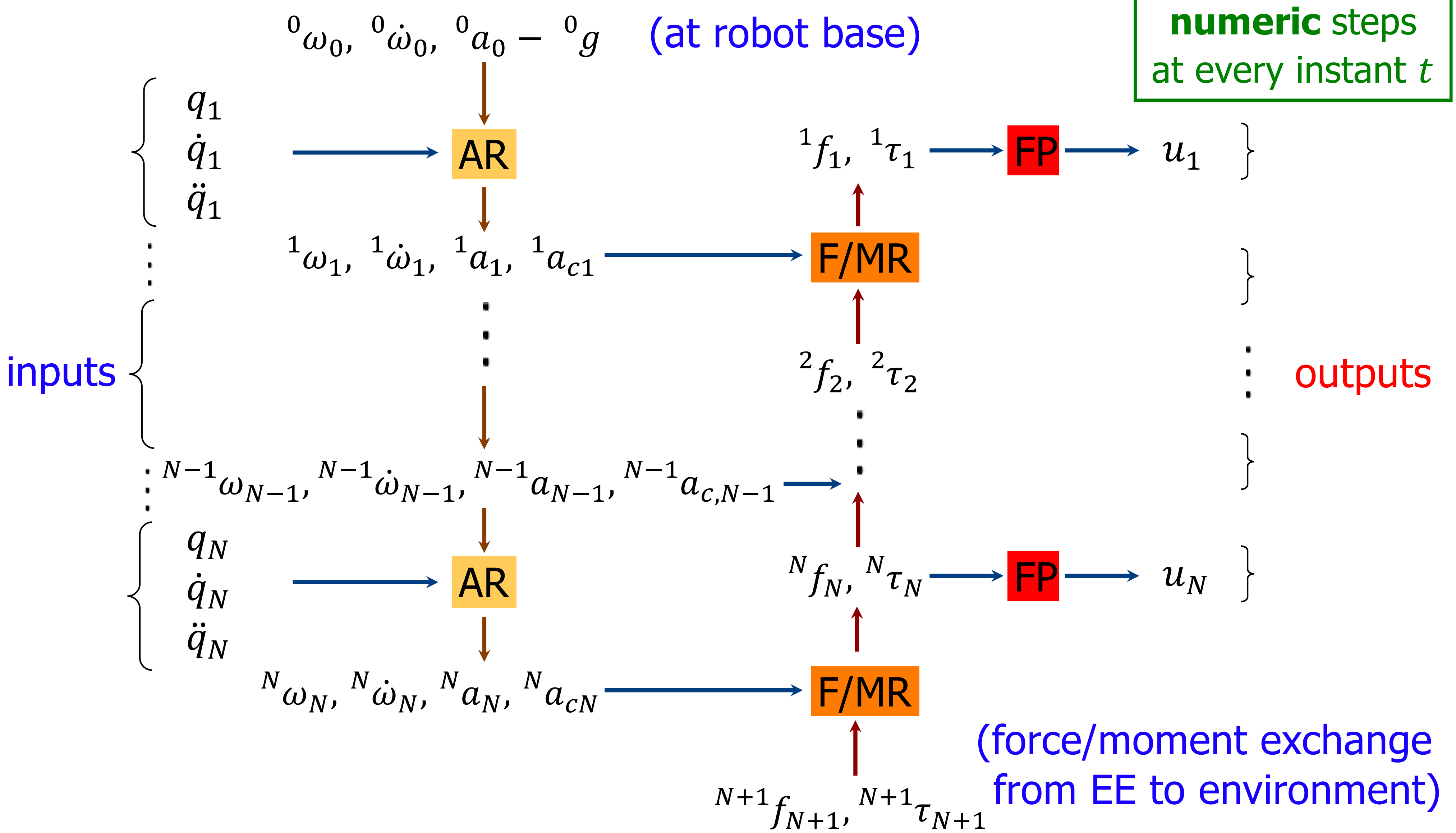
add any dissipative term (here, viscous friction only)

Comments on Newton-Euler method

- the previous forward/backward recursive formulas can be evaluated in symbolic or numeric form
  - symbolic
    - substituting expressions in a recursive way
    - at the end, a closed-form dynamic model is obtained, which is identical to the one obtained using Euler-Lagrange (or any other) method
    - there is no special convenience in using N-E in this way …
  - numeric
    - substituting numeric values (numbers!) at each step
    - computational complexity of each step remains constant ⇒ grows in a linear fashion with the number N of joints ($O(N)$)
    - strongly recommended for real-time use, especially when the number N of joints is large

Newton-Euler algorithm

efficient computational scheme for inverse dynamics



Matlab (or C) script

assuming no interaction with the environment ($f_{N+1} = \tau_{N+1} = 0$)

general routine $NE_\alpha(\mathrm{arg}_1, \mathrm{arg}_2, \mathrm{arg}_3)$

- data file (of a specific robot)
  - number N and types $\sigma = \{0,1\}^N$ of joints (revolute/prismatic)
  - table of DH kinematic parameters
  - list of ALL dynamic parameters of the links (and of the motors)
- input
  - vector parameter $\alpha = \{{}^0g, 0\}$ (presence or absence of gravity)
  - three ordered vector arguments
    - typically, samples of joint position, velocity, acceleration taken from a desired trajectory
- output
  - generalized force u for the complete inverse dynamics
  - … or single terms of the dynamic model

Examples of output

- complete inverse dynamics

$$u = NE_{^0g}(q_d, \dot{q}_d, \ddot{q}_d) = M(q_d)\ddot{q}_d + c(q_d, \dot{q}_d) + g(q_d) = u_d$$

- gravity term

$$u = NE_{^0g}(q, 0, 0) = g(q)$$

- centrifugal and Coriolis term

$$u = NE_0(q, \dot{q}, 0) = c(q, \dot{q})$$

- i-th column of the inertia matrix

$$u = NE_0(q, 0, e_i) = M_i(q)$$

e_i = i-th column of identity matrix

- generalized momentum

$$u = NE_0(q, 0, \dot{q}) = M(q)\dot{q} = p$$

A further example of output

- factorization of centrifugal and Coriolis term

$$u = NE_0(q,\dot{q},0) = c(q,\dot{q}) = S(q,\dot{q})\dot{q}$$

- for later use, what about a "mixed" velocity term?

$$S(q,\dot{q})\dot{q}_r \;\not\Leftrightarrow\; \begin{cases} u = NE_0(q,\dot{q}_r,0) = S(q,\dot{q}_r)\dot{q}_r \\ u = NE_0(q,e_i\dot{q}_{ri},0) = S_i(q,e_i\dot{q}_{ri})\dot{q}_{ri} \end{cases}$$ no good!

a) $S(q,\dot{q})\dot{q}_r = S(q,\dot{q}_r)\dot{q}$, when using Christoffel symbols

b) $S(q,\dot{q}+\dot{q}_r)(\dot{q}+\dot{q}_r) = S(q,\dot{q})\dot{q} + S(q,\dot{q}_r)\dot{q}_r + 2S(q,\dot{q})\dot{q}_r$

$$\Rightarrow \quad u = \frac{1}{2}\left(NE_0(q,\dot{q}+\dot{q}_r,0) - NE_0(q,\dot{q},0) - NE_0(q,\dot{q}_r,0)\right)$$

$$= S(q,\dot{q})\dot{q}_r$$ (i.e., with 3 calls of standard NE algorithm)

[Kawasaki et al., IEEE T-RA 1996]

Modified NE algorithm

modified routine $\widehat{NE}_\alpha(\arg_1, \arg_2, \arg_3, \arg_4)$ with 4 arguments

[De Luca, Ferrajoli, ICRA 2009]

$$\widehat{NE}_\alpha(x, y, y, z) = NE_\alpha(x, y, z)$$ consistency property

e.g., $u = \widehat{NE}_{^0 g}(q, 0, 0, 0) = NE_{^0 g}(q, 0, 0) = g(q)$

$$u = \widehat{NE}_0(q, \dot{q}, \dot{q}, 0) = NE_0(q, \dot{q}, 0) = c(q, \dot{q}) = S(q, \dot{q})\dot{q}$$

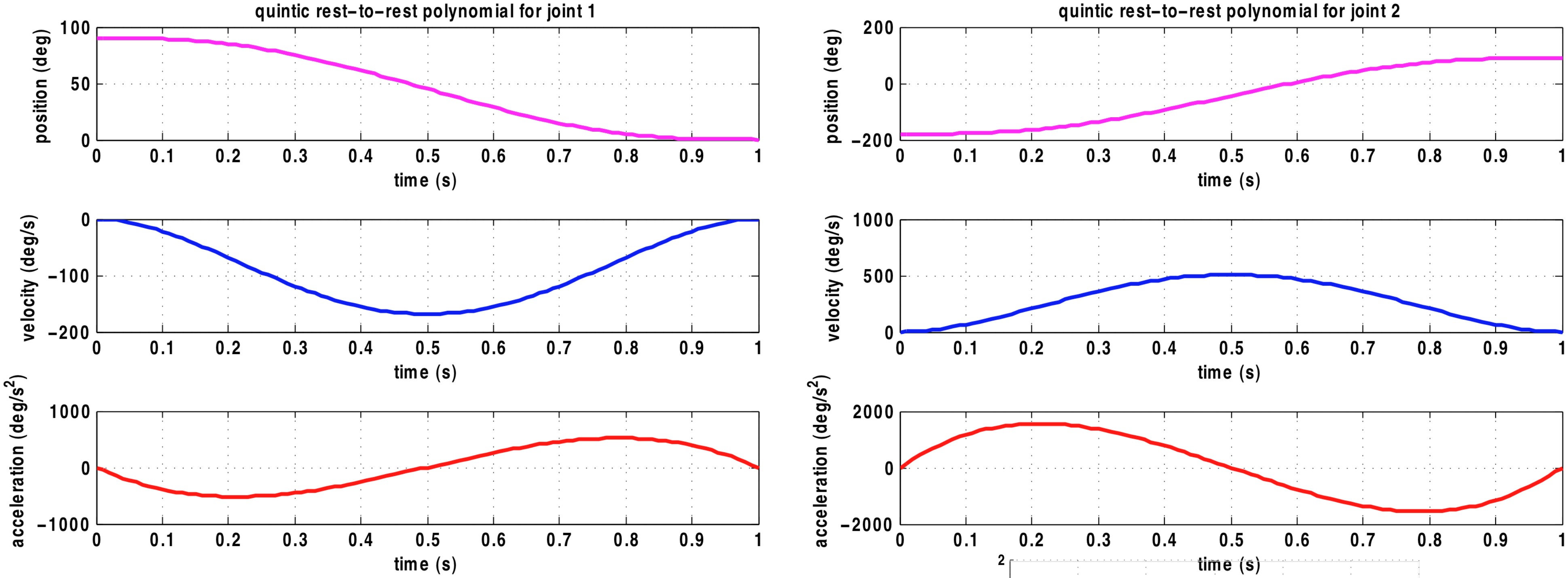
$\Rightarrow$ $u = \widehat{NE}_0(q, \dot{q}, \dot{q}_r, 0) = S(q, \dot{q})\dot{q}_r$ with $\dot{M} - 2S$ skew-symmetric

(i.e., with 1 call of modified NE algorithm)

$\Rightarrow$ $u = \widehat{NE}_0(q, \dot{q}, e_i, 0) = S_i(q, \dot{q})$

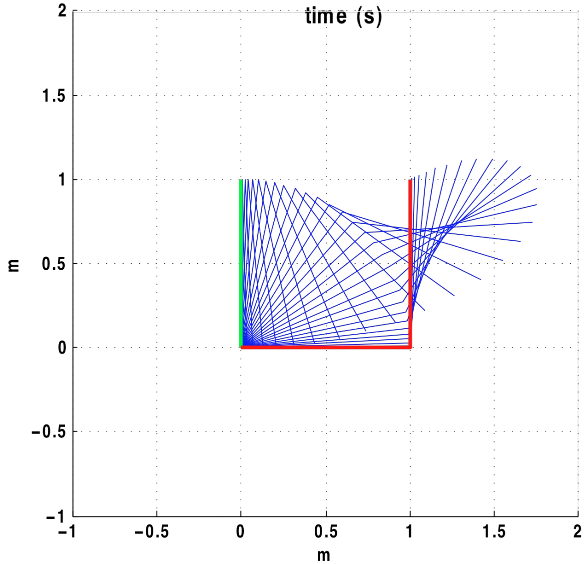
(i.e., the full matrix S satisfying the skew-symmetry of $\dot{M} - 2S$ with N calls of the modified NE algorithm)

Inverse dynamics of a 2R planar robot



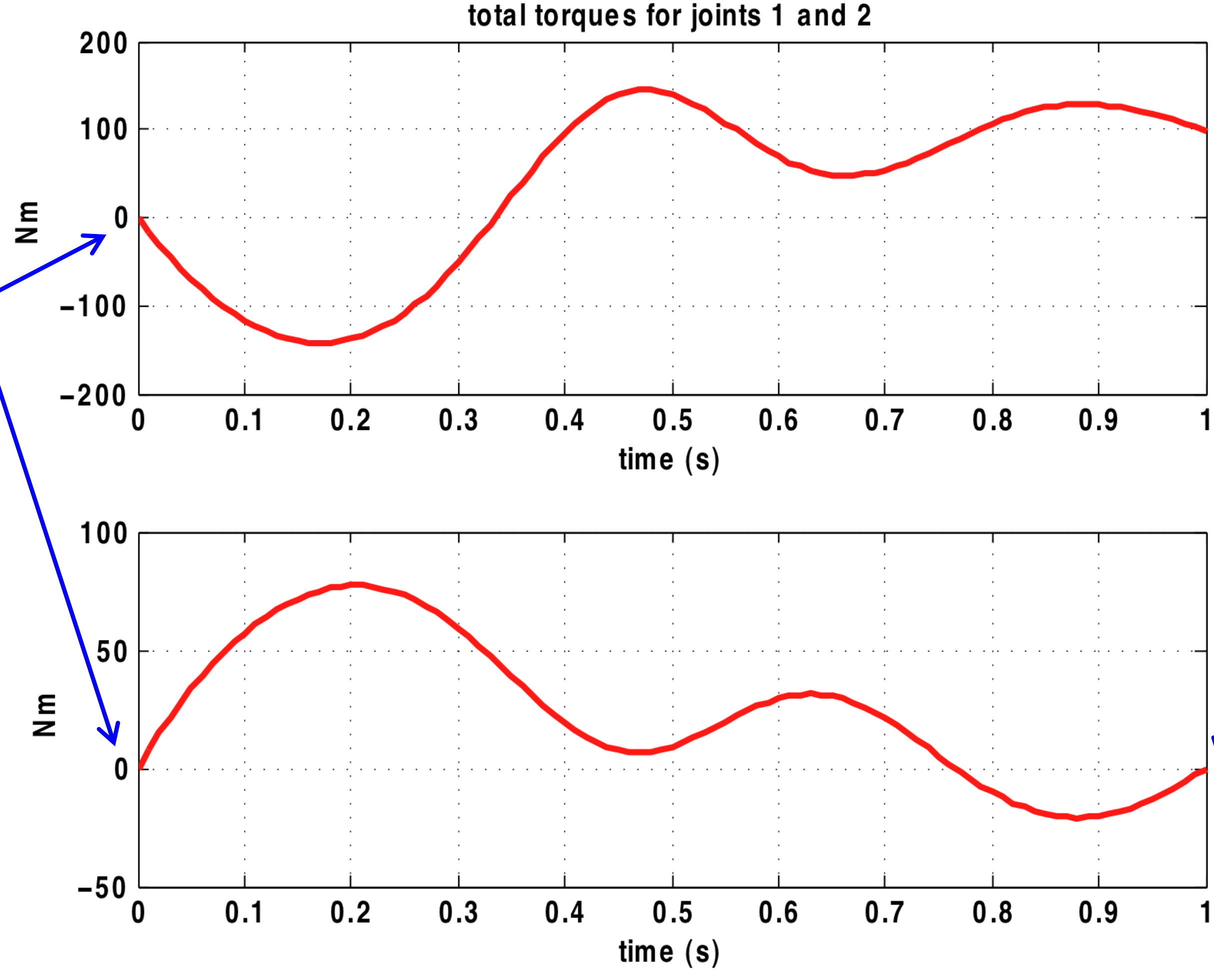
desired (smooth) joint motion:
quintic polynomials for q_1, q_2 with
zero vel/acc boundary conditions
from (90°, -180°) to (0°, 90°) in $T = 1$ s

$\Leftrightarrow$



Inverse dynamics of a 2R planar robot

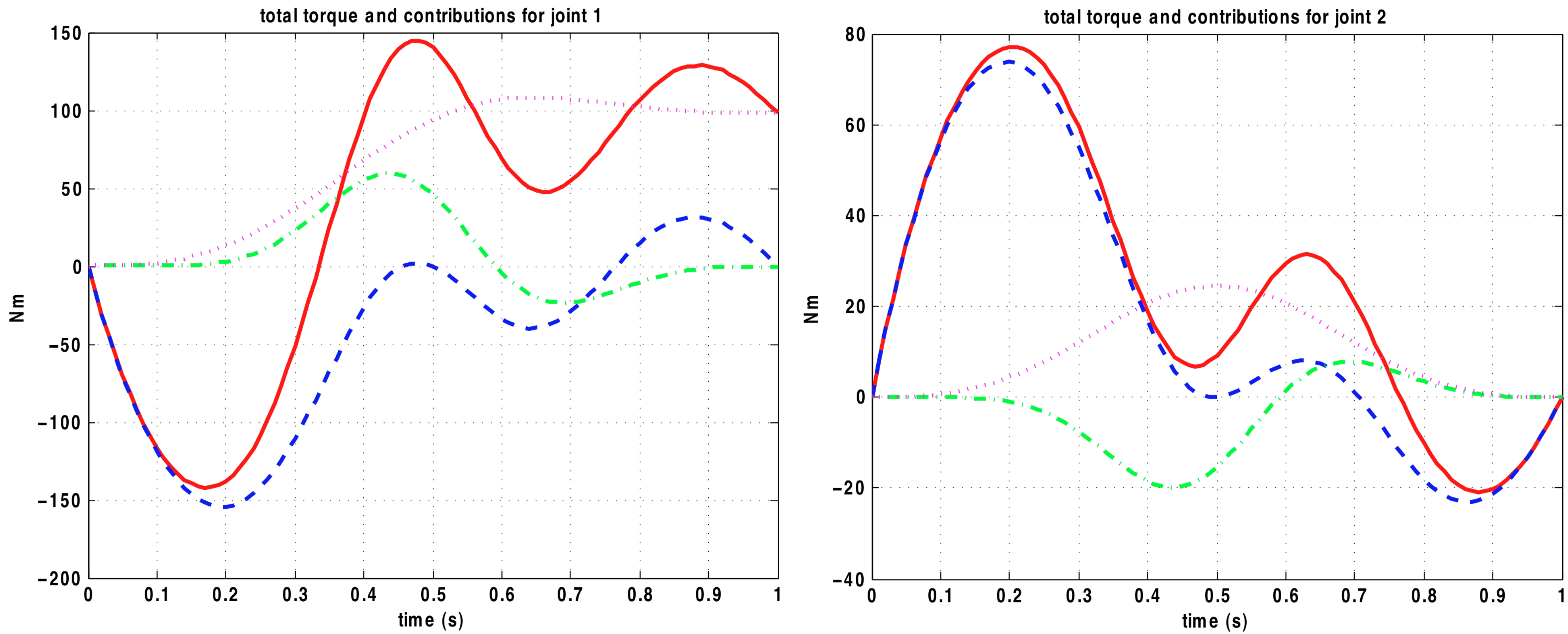
zero
initial torques =
free equilibrium
configuration
+
zero initial
accelerations



final torques
$u_1 \neq 0, u_2 = 0$
balance
link weights
in final (0°, 90°)
configuration

motion in vertical plane (under gravity)
both links are thin rods of uniform mass $m_1 = 10$ kg, $m_2 = 5$ kg

Inverse dynamics of a 2R planar robot



torque contributions at the two joints for the desired motion

—— = total, - - - - = inertial

-·-·- = Coriolis/centrifugal, ········ = gravitational

Use of NE routine for simulation
direct dynamics

- numerical integration, at current state $(q, \dot{q})$, of

$$\ddot{q} = M^{-1}(q)[u - (c(q,\dot{q}) + g(q))] = M^{-1}(q)[u - n(q,\dot{q})]$$

- Coriolis, centrifugal, and gravity terms

$$n = NE_{^0g}(q, \dot{q}, 0)$$ complexity $O(N)$

- i-th column of the inertia matrix, for $i = 1, .., N$

$$M_i = NE_0(q, 0, e_i)$$ $O(N^2)$

- numerical inversion of inertia matrix

$$InvM = \text{inv}(M)$$ $O(N^3)$ but with small coefficient

- given u, integrate acceleration computed as

$$\ddot{q} = InvM * [u - n]$$ ➡ new state $(q, \dot{q})$ and repeat over time ...