Robotics 2

Kinematic calibration

Prof. Alessandro De Luca

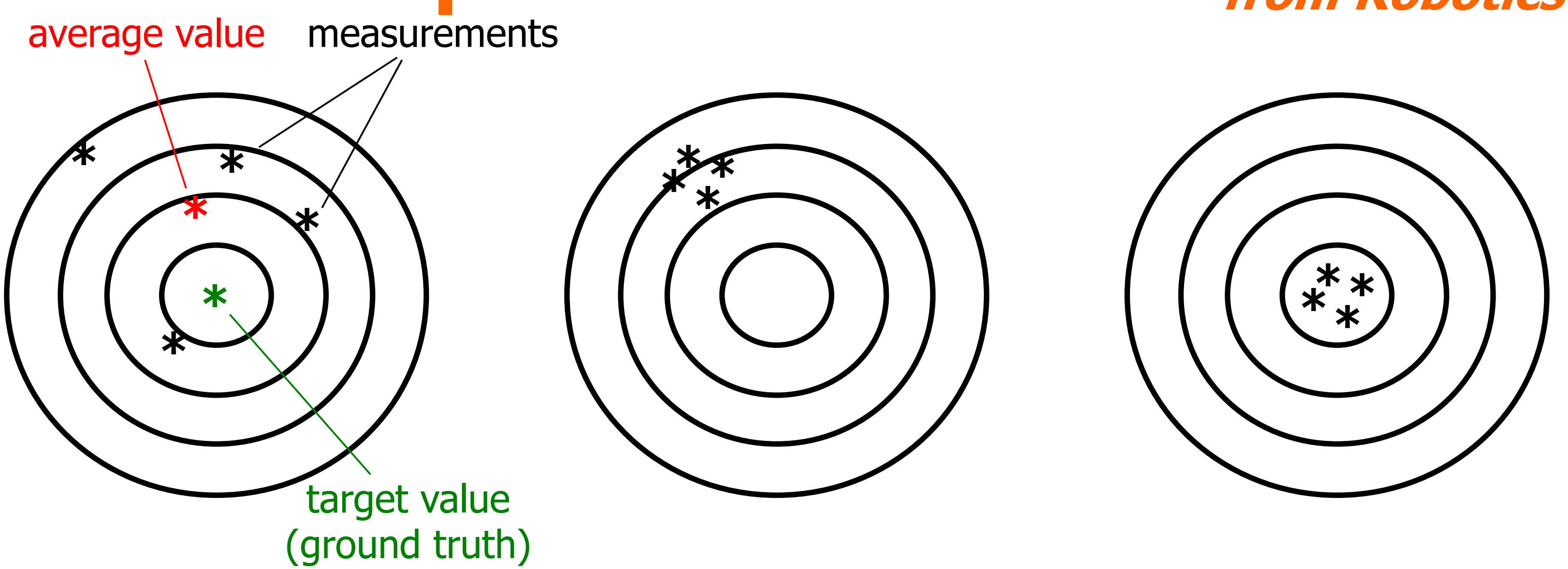
Dipartimento di Ingegneria Informatica
Automatica e Gestionale Antonio Ruberti

Sapienza
Università di Roma

Accuracy and Repeatability

robot as a measuring device

from Robotics 1



low accuracy low repeatability	low accuracy high repeatability	high accuracy high repeatability

better components!

calibration!

Direct kinematics

- nominal set of Denavit-Hartenberg (D-H) parameters

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \quad \boldsymbol{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad \boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix}$$

for simplicity, suppose an all-revolute joints manipulator

- nominal direct kinematics

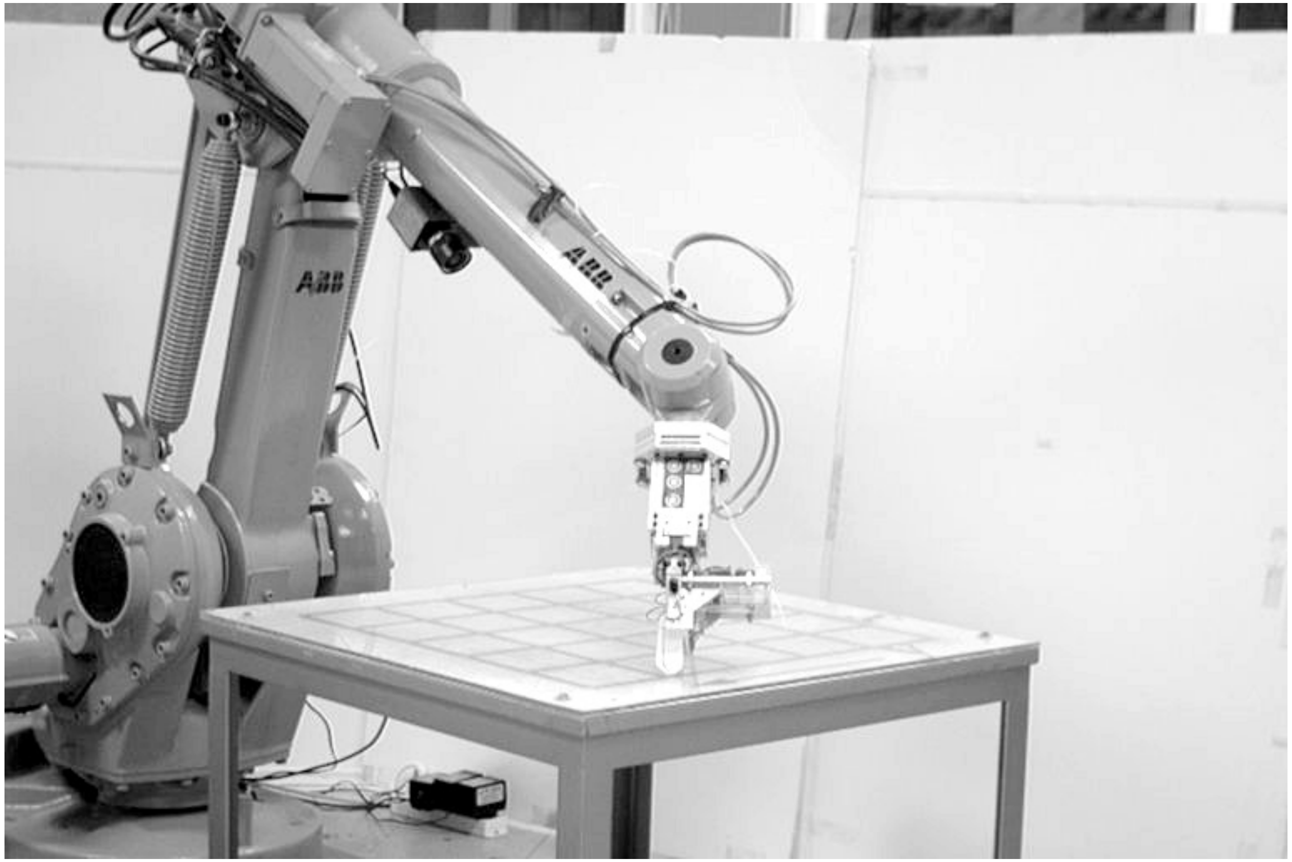
$$\boldsymbol{r}_{nom} = f(\boldsymbol{\alpha}, \boldsymbol{a}, \boldsymbol{d}, \boldsymbol{\theta}) \qquad \boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}$$

θ_i's typically measured by encoders

Need for calibration

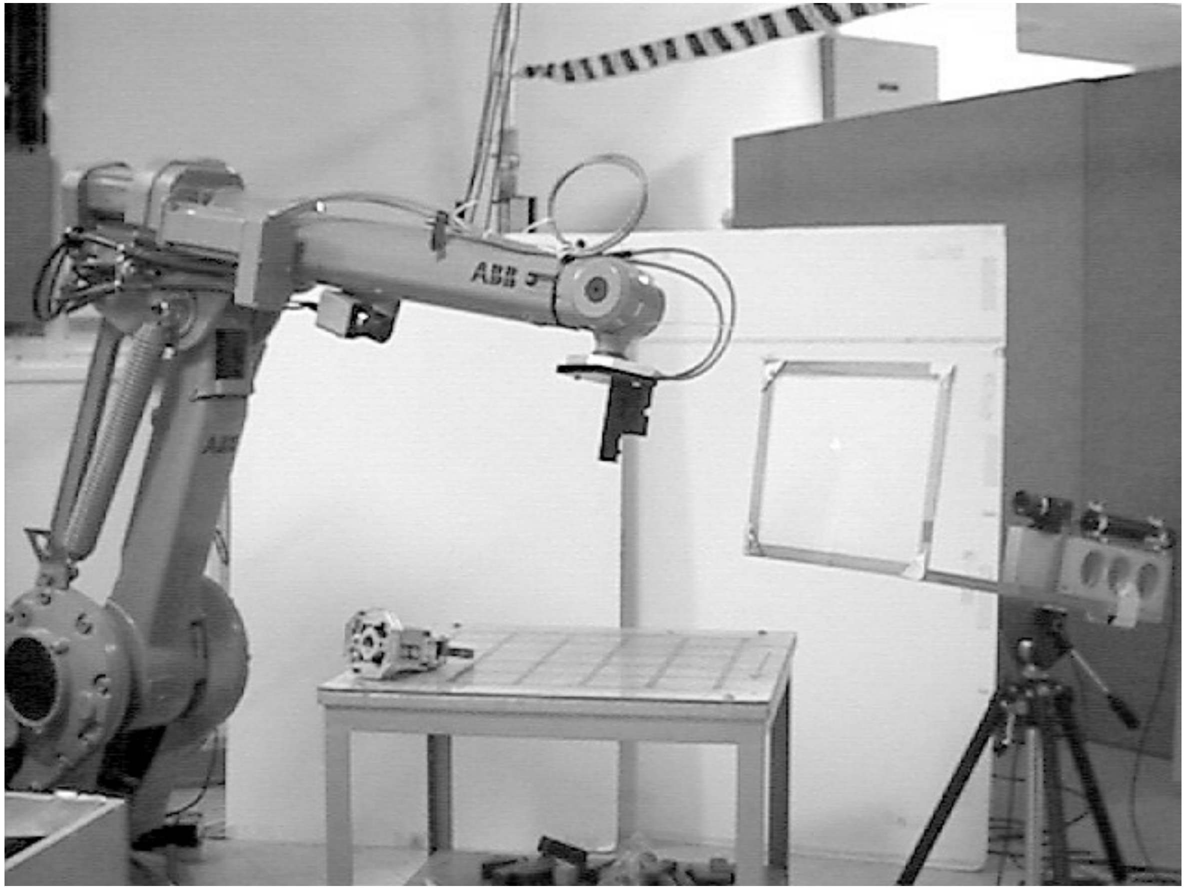
- tolerances in mechanical construction and in assembly of links/joints imply small errors in actual end-effector pose (real ≠ nominal parameters)
- encoder mounting on motor axes may not be consistent with the "zero reference" of the robot direct kinematics (joint angle measures are constantly biased)
- errors distributed "along" the arm are amplified, due to the open kinematic chain structure of most robots
- calibration goal: recover as much as possible E-E pose errors by correcting the nominal set of D-H parameters, based on independent external (accurate!) measurements
- experiments to be done once for each robot, before starting operation... (and maybe repeated from time to time)

Cartesian measurement systems - 1



calibration table

Cartesian measurement systems - 2

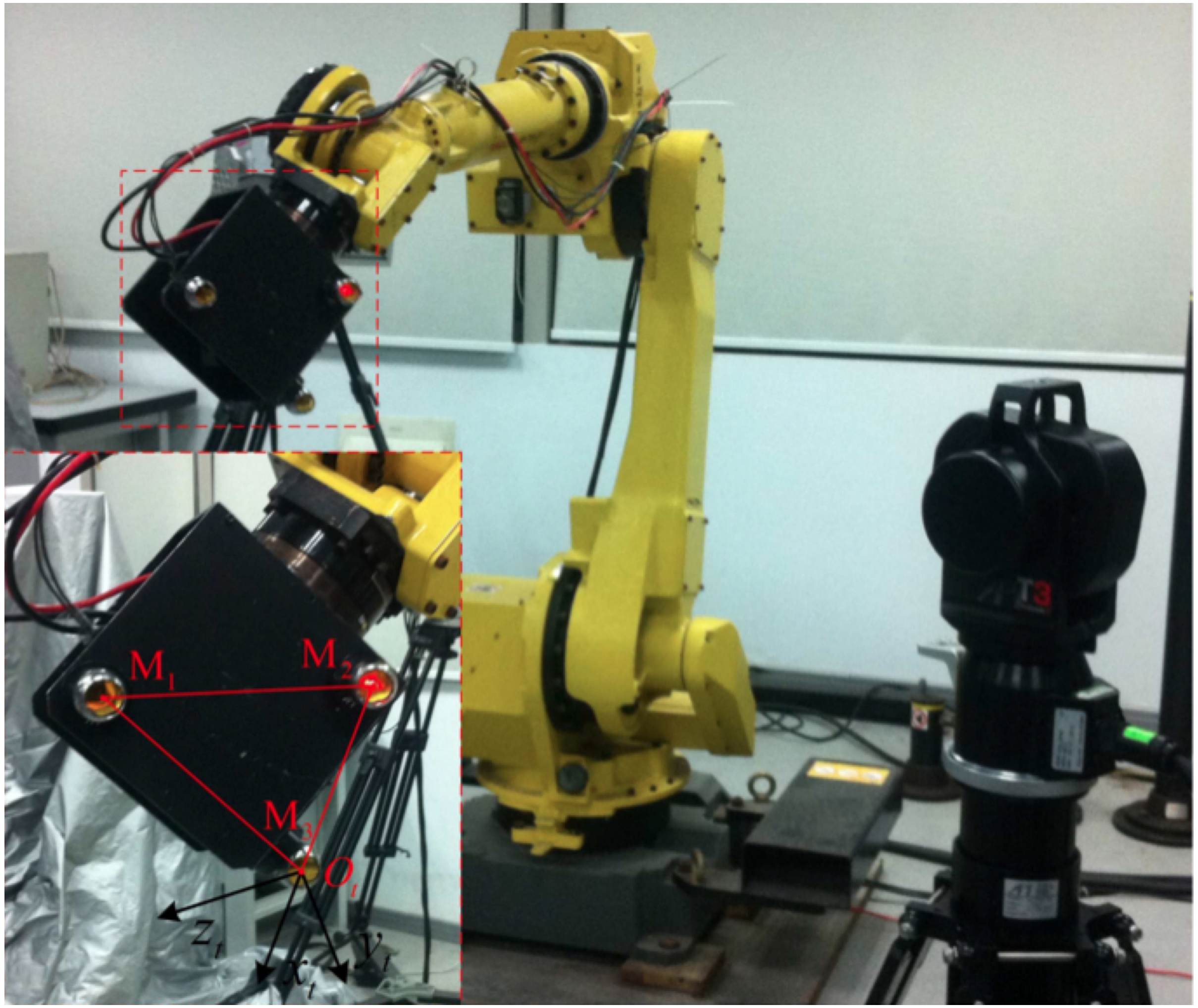


laser/camera system + triangulation

Cartesian measurement systems - 3

FANUC 6R robot
M-710iC/50

3 SMRs
(Spherically-Mounted Reflectors)



API
laser tracker III
www.apisensor.com

laser tracker + targets on end-effector

Acquiring data for calibration

FARO ION
laser tracker

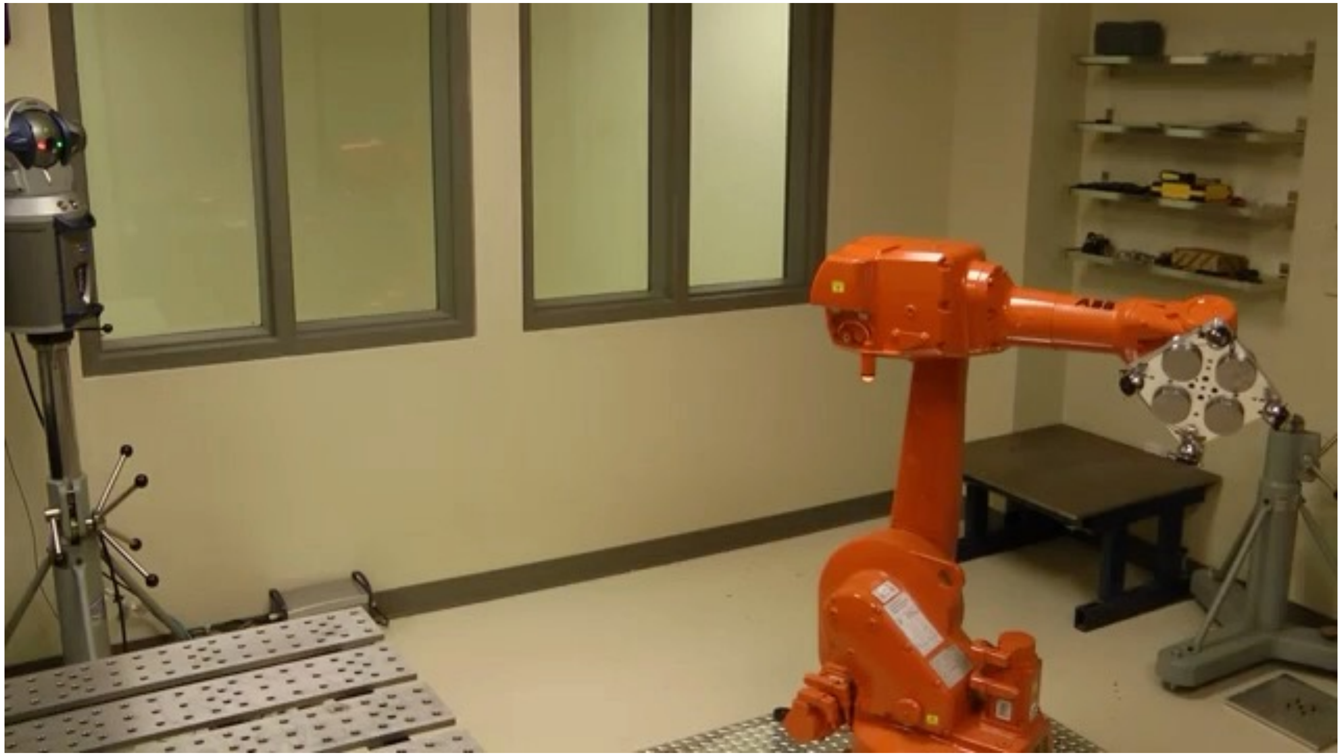
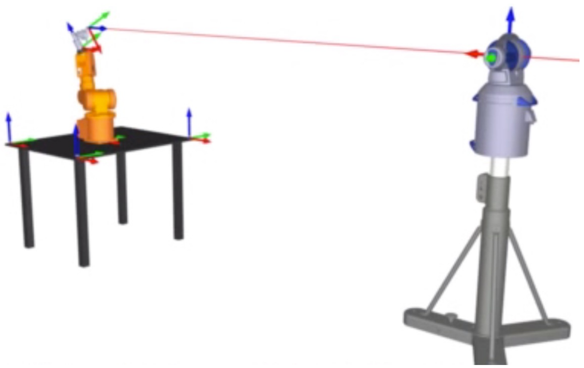
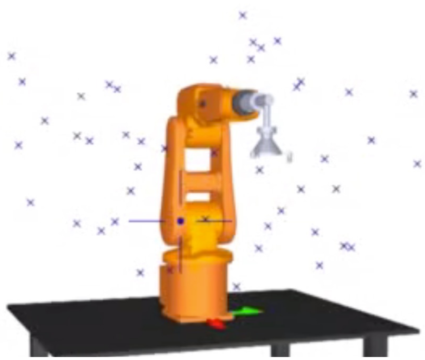


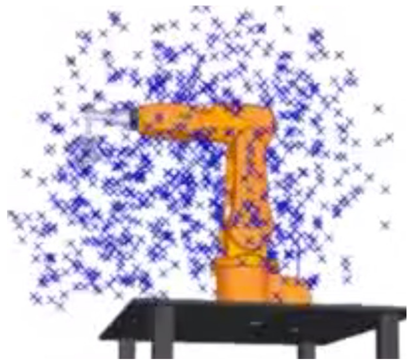
ABB
IRB 1600
robot

4 SMRs

video
@CoRo Lab
ETS Montréal







Linearization of direct kinematics

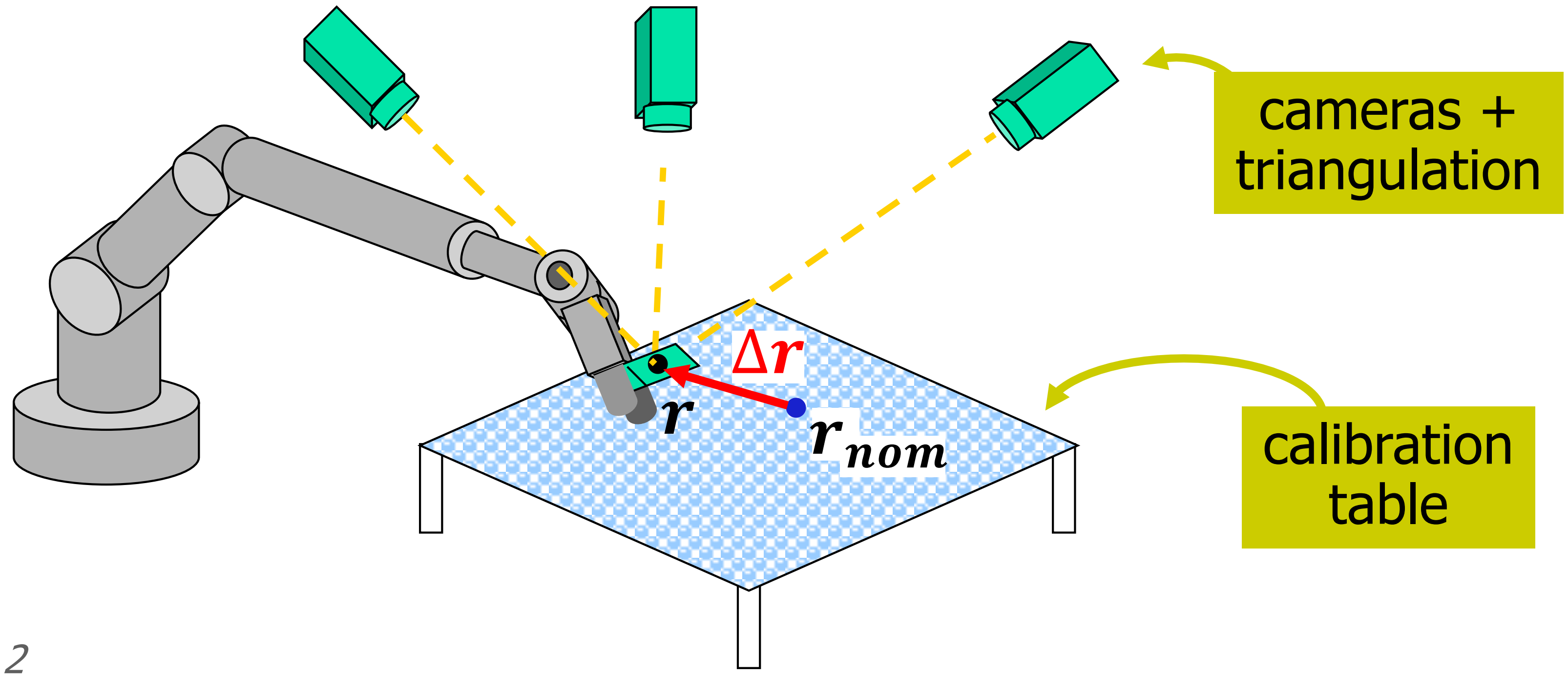
partial Jacobians evaluated in nominal conditions

$$\Delta r = r - r_{nom} = \frac{\partial f}{\partial \alpha} \cdot \Delta\alpha + \frac{\partial f}{\partial a} \cdot \Delta a + \frac{\partial f}{\partial d} \cdot \Delta d + \frac{\partial f}{\partial \theta} \cdot \Delta\theta$$

"small" errors

obtained by external measurement system

first-order variations



Calibration equation

$4n \times 1$

$$\Delta\varphi = \begin{pmatrix} \Delta\alpha \\ \Delta a \\ \Delta d \\ \Delta\theta \end{pmatrix} \qquad \Phi = \begin{pmatrix} \frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial a} & \frac{\partial f}{\partial d} & \frac{\partial f}{\partial \theta} \end{pmatrix}$$

$6 \times 4n$

$$\Delta r = \Phi\, \Delta\varphi$$

ℓ experiments ($\ell \gg$ n)

$6\ell \times 1$ $\qquad$ $6\ell \times 4n$

$$\overline{\Delta r} = \begin{pmatrix} \Delta r_1 \\ \Delta r_2 \\ \vdots \\ \Delta r_\ell \end{pmatrix} \qquad \bar{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_\ell \end{pmatrix}$$

full column rank
(for sufficiently large ℓ)

$$\overline{\Delta r} = \bar{\Phi}\, \Delta\varphi$$

measures

unknowns

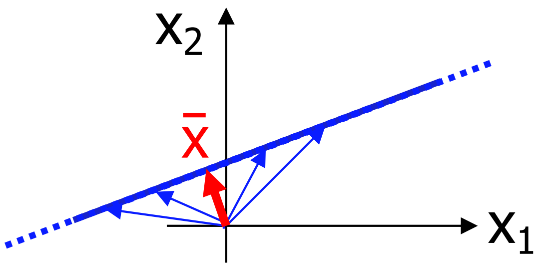
regressor matrix evaluated
at nominal parameters

Under- and over-determined systems of linear equations

$$A\,x = b$$

full rank

A



1 equation
2 unknowns

minimum norm solution

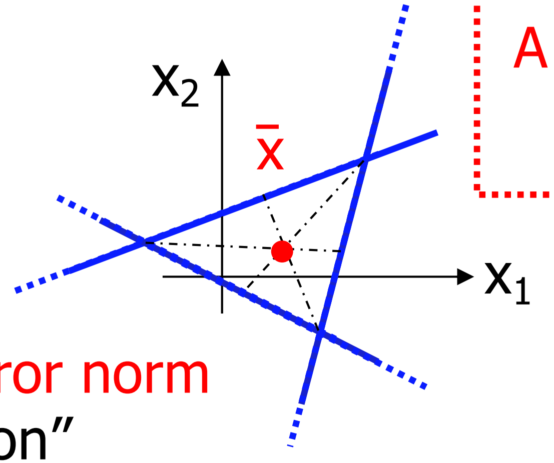
$$\min \frac{1}{2}\|x\|^2, \text{ among } x:\ A\,x = b$$

$$A^{\#} = A^T\left(A\,A^T\right)^{-1}$$

full rank

A

3 equations
2 unknowns



minimum error norm
"solution"

$$\min \frac{1}{2}\|Ax - b\|^2$$

$$A^{\#} = \left(A^T A\right)^{-1} A^T$$

$$\bar{x} = A^{\#} b$$

pseudoinverse !!

if multiple solutions x exist (having the same minimum **error** norm), the pseudoinverse provides the one having **minimum norm**

Calibration algorithm

$\epsilon_r = 10^{-4},\ i_{max} = 50$

$\varphi^{(0)} = \varphi_{nom}$

$\bar{\Phi}^{(0)} = \bar{\Phi}(\varphi^{(0)})$

$\overline{\Delta r}^{(0)} = \overline{\Delta r}(\varphi^{(0)})$

$i = 0$

$if\ \left\|\overline{\Delta r}^{(i)}\right\| \leq \epsilon_r$

$\quad \varphi^* = \varphi^{(i)}$ — final solution

$else\ if\ i \leq i_{max}$

$\quad \Delta\varphi^{(i)} = (\bar{\Phi}^{(i)})^{\#}\ \overline{\Delta r}^{(i)} = \left(\bar{\Phi}^{(i)T}\bar{\Phi}^{(i)}\right)^{-1}\bar{\Phi}^{(i)T}\ \overline{\Delta r}^{(i)}$ — minimizes $\left\|\bar{\Phi}^{(i)}\Delta\varphi - \overline{\Delta r}^{(i)}\right\|^2$

$\quad \varphi^{(i+1)} = \varphi^{(i)} + \Delta\varphi^{(i)}$ — new set of DH parameters + "bias" on measures of θ

$\quad \bar{\Phi}^{(i+1)} = \bar{\Phi}(\varphi^{(i+1)})$

$\quad \overline{\Delta r}^{(i+1)} = \overline{\Delta r}(\varphi^{(i+1)})$

evaluated with the new values

$\quad i = i + 1$

$else$

$\quad disp('no\ convergence\ in\ i_{max}\ iterations')$

$end\ if$

$$\boxed{\overline{\Delta r} = \bar{\Phi}\,\Delta\varphi}$$

linearized least squares problem needs an iterative solution

Improvement by kinematic calibration

- ABB IRB 120 6R industrial robot
- 1000 random configurations (collision-free by simulation)
- 50 arbitrary configurations used for measurement in calibration
- 950 configurations used for validation
- Cartesian position errors

	before calibration	after calibration
Average	1.746 mm	0.193 mm
Median	1.567 mm	0.180 mm
Standard Deviation	1.043 mm	0.085 mm
Min	0.050 mm	0.010 mm
Max	4.423 mm	0.516 mm

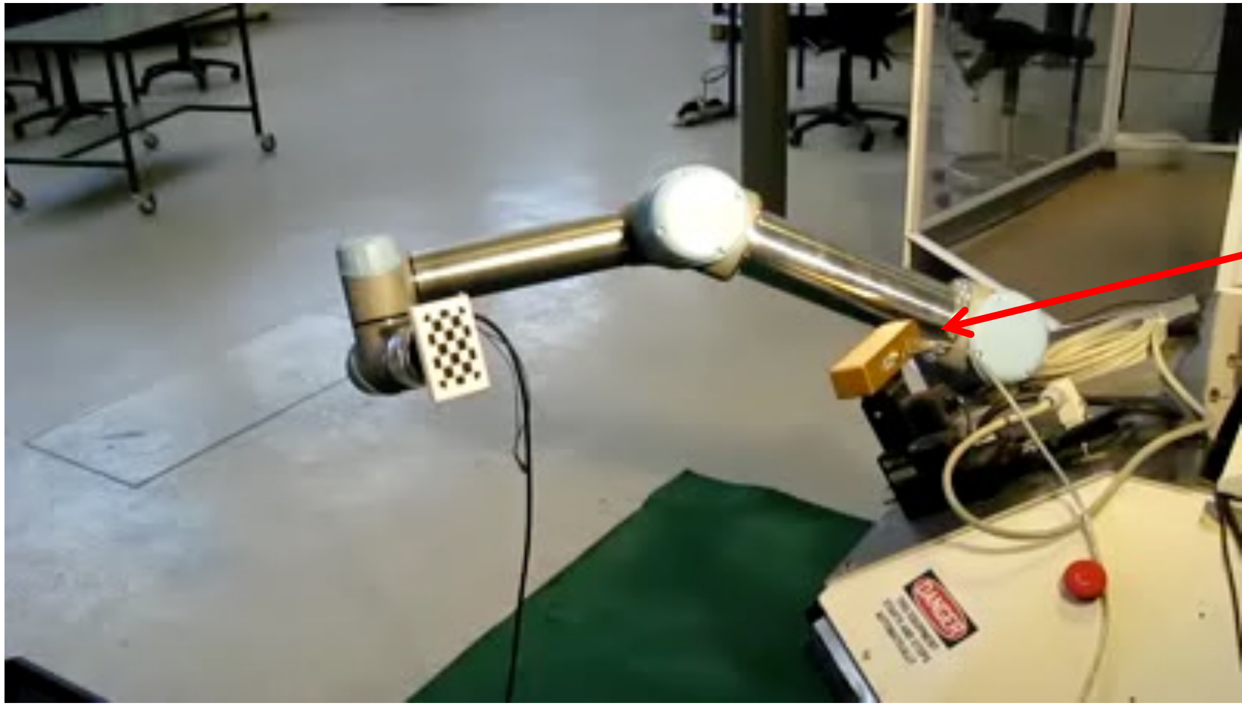
- Improvement by **a factor 8÷10**

Final comments

- an iterative least squares method
  - original problem is nonlinear in the unknowns, then linearized using first-order Taylor expansion
- it is useful to calibrate first and separately those quantities that are less accurate (typically, the encoder bias)
  - keeping the remaining ones at their nominal values
- alternative kinematic descriptions can be used
  - more complex than D-H parameters, but leading to a better numerical conditioning of the regressor matrix in calibration algorithm
  - one such description uses the POE (Product Of Exponential) formula
- more in general, 6 base parameters should also be included
  - to locate 0-th robot frame w.r.t. world coordinate frame (of external sensor)
- accurate calibration/estimation of real parameters is a general problem in robotics (and beyond...)
  - for sensors (e.g., camera calibration)
  - for models (identification of dynamic parameters of a manipulator)

Calibration experiment
in a research environment

video



Videre Design
stereovision
camera

- automatic data acquisition for simultaneous calibration of
  - robot-camera transformation
  - DH parameters of the manipulator

Calibration experiment

in an industrial setting

video

FANUC
3D Laser
calibration
(with iR Vision)