Exercise 1

The kinematics of a 3R robot is defined by the following Denavit-Hartenberg table (units in [m] or [rad]):

<table>
<thead>
<tr>
<th>i</th>
<th>$\alpha_i$</th>
<th>$a_i$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$d_1 = 5$</td>
<td>$q_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$a_2 = 4$</td>
<td>0</td>
<td>$q_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$a_3 = 3$</td>
<td>0</td>
<td>$q_3$</td>
</tr>
</tbody>
</table>

Determine the $3 \times 3$ linear part of the geometric Jacobian $J(q)$ of this robot. When the robot is in the configuration $q_0 = (\pi/2, \pi/4, \pi/2)$ [rad] and has a joint velocity $\dot{q}_0 = (1, 2, -2)$ [rad/s], determine, if possible, a joint acceleration $\ddot{q}$ that realizes a zero end-effector acceleration, i.e., $\ddot{p} = 0$. [Bonus: What if the second link parameter is changed to $a_2 = 3$?]

Exercise 2

The RP robot shown in Fig. 1 starts from rest at time $t = 0$ in the configuration $q(0) = (0, 1)$ [rad, m] and moves under the action of the following discontinuous joint acceleration commands for a time $T = 2$ [s]:

$$\ddot{q}_1(t) = \begin{cases} A_1 = 2 \text{ [rad/s}^2], & t \in [0, T/4], \\ 0, & t \in [T/4, 3T/4], \\ -A_1 = -2 \text{ [rad/s}^2], & t \in [3T/4, T]; \end{cases}$$

$$\ddot{q}_2(t) = \begin{cases} -A_2 = -0.5 \text{ [m/s}^2], & t \in [0, T/2], \\ A_2 = 0.5 \text{ [m/s}^2], & t \in [T/2, T]. \end{cases}$$

a. Plot the time profiles of $q_i(t)$, $\dot{q}_i(t)$ and $\ddot{q}_i(t)$, for $i = 1, 2$.
b. Does the robot cross a singularity during this motion?
c. Compute the mid time configuration $q(T/2)$ and the final configuration $q(T)$ reached in this motion. Sketch the robot in these two configurations, as well as in the initial one.
d. Provide the analytic expressions of the end-effector velocity and acceleration norms, i.e., $\|\dot{p}\|$ and $\|\ddot{p}\|$.
e. Draw the end-effector velocity and acceleration vectors $\dot{p}(T/2)$, $\dot{p}(T/2)^-$ and $\ddot{p}(T/2)^+$ on the mid time configuration of the robot sketched at item c. Compute the numerical values of $\|\dot{p}(T/2)\|$, $\|\ddot{p}(T/2)^-\|$ and $\|\ddot{p}(T/2)^+\|$.

Exercise 3

A link of length $L = 1.5$ m is rotated by a DC motor mounted at the base through a gear with reduction ratio $N_r = 4$. The motor has a quadrature incremental encoder with $N_p = 250$ pulses/turn and a digital counter of $n = 10$ bits. The link carries a laser scanner at its other end. The laser measures distances from the link tip to obstacles (in the same plane of link motion) up to a maximum distance of $d = 5$ m. The laser has a depth resolution $\Delta \rho = 12$ mm and an angular resolution $\delta_\alpha = 0.2^\circ$ in the range $\alpha = \pm 90^\circ$ (the zero is when the scanning ray is aligned with the link). Sketch the setup and analyze the resolution of this system for measuring the position of objects in the environment. In the worst-case condition, which is the largest possible Cartesian displacement $\Delta$ of an object that provides no change in the output readings?

[180 minutes; open books]
Indeed, this solution is valid as long as the robot is out of singularities. To evaluate (5), we need first to end-effector acceleration:

\[
J = \begin{bmatrix}
J_{11} = 1 \\
J_{21} = 0 \\
J_{31} = 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos q_1 (a_2 \cos q_2 + a_3 \cos (q_2 + q_3)) \\
\sin q_1 (a_2 \cos q_2 + a_3 \cos (q_2 + q_3)) \\
d_1 + a_2 \sin q_2 + a_3 \sin (q_2 + q_3)
\end{bmatrix}.
\]

Therefore, using the usual compact notation for trigonometric functions, we have

\[
J(q) = \frac{\partial f(q)}{\partial q} = \begin{bmatrix}
-s_1 (a_2 s_2 + a_3 s_{23}) \\
c_1 (a_2 c_2 + a_3 c_{23}) - s_1 (a_2 s_2 + a_3 s_{23}) \\
0
\end{bmatrix}.
\]

The Jacobian is singular when

\[
\det J(q) = -a_2 a_3 s_3 (a_2 c_2 + a_3 c_{23}) = 0.
\]

The end-effector acceleration \(\vec{p}\) is computed as

\[
\vec{p} = J(q) \vec{q} + J(q) \dot{\vec{q}}.
\]

Thus, in order to realize a zero end-effector acceleration, we need to set \(\vec{p} = 0\) in (4) and solve for \(\vec{q}\) or

\[
\dot{\vec{q}} = -J^{-1}(q) J(q) \dot{\vec{q}}.
\]

Indeed, this solution is valid as long as the robot is out of singularities. To evaluate (5), we need first to derive the time derivative of the robot Jacobian. Let \(J_i(q)\) be the \(i\)th column of the Jacobian \(J(q)\), for \(i = 1, 2, 3\). We compute

\[
J_i(q) = \frac{dJ_i(q)}{dt} = \sum_{j=1}^{3} \frac{\partial J_i(q)}{\partial q_j} \dot{q}_j
\]

\[
= \begin{bmatrix}
-c_1 q_1 (a_2 c_2 + a_3 c_{23}) + s_1 (a_2 s_2 q_2 + a_3 s_{23} (q_2 + q_3)) \\
s_1 q_1 (a_2 c_2 + a_3 c_{23}) - c_1 (a_2 s_2 q_2 + a_3 s_{23} (q_2 + q_3)) \\
0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-s_1 (a_2 s_2 + a_3 s_{23}) - c_1 (a_2 c_2 q_2 + a_3 c_{23} (q_2 + q_3)) \\
0
\end{bmatrix}.
\]

When the robot is in the configuration \(q_0 = (\pi/2, \pi/4, \pi/2)\) [rad] and has a joint velocity \(\dot{q}_0 = (1, 2, -2)\) [rad/s], evaluation of (2) and (5) gives

\[
J_0 = J(q_0) = \begin{bmatrix}
-0.7071 & 0 & 0 \\
0 & -4.9497 & -2.1213 \\
0 & 0.7071 & -2.1213
\end{bmatrix}.
\]

\[
J_0 = J(q_0) = \begin{bmatrix}
5.6569 & 4.9497 & 2.1213 \\
-0.7071 & -5.6569 & 0 \\
0 & -5.6569 & 0
\end{bmatrix}.
\]

Since \(\det J_0 = -8.4853 \neq 0\), we can use eq. (5) for computing the joint acceleration \(\ddot{q}\) that realizes a zero end-effector acceleration:

\[
\ddot{q} = -J_0^{-1} (J_0 \dot{q}_0) = \begin{bmatrix}
-1.4142 & 0 & 0 \\
0 & -0.1768 & 0.1768 \\
0 & -0.0589 & -0.4125
\end{bmatrix} \begin{bmatrix}
11.3137 \\
-12.0208 \\
-11.3137
\end{bmatrix} = \begin{bmatrix}
16 \\
-0.1250 \\
-3.5750
\end{bmatrix} \text{[rad/s$^2$].}
\]

\[\text{(7)}\]

\[\text{This computation is made easier when expressing the Jacobian in frame 1, i.e., using } J(q) = R^T (q_1) J(q).\]
**Bonus part.** If we change the second link parameter from \( a_2 = 4 \) to \( a_2 = 3 \), a singular configuration will be encountered. This can be recognized already from the direct kinematics \( \mathbf{J} \); in fact, we have in this case

\[
\dot{p}' = p'_{a_2=3} = \begin{pmatrix} 0 & 0 & 9.2426 \end{pmatrix}^T,
\]

namely the end-effector is placed on the axis of joint 1: any rotation \( \dot{q}_1 \) will not move the end-effector — a situation of singularity. Re-evaluating then the Jacobian \( \mathbf{J} \) yields

\[
\mathbf{J}'_0 = \mathbf{J}(q_0)_{a_2=3} = \begin{pmatrix} 0 & 0 & 0 \\ -4.2426 & -2.1213 & 0 \\ 0 & 0 & -2.1213 \end{pmatrix} \Rightarrow \det \mathbf{J}'_0 = 0, \quad R \{ \mathbf{J}'_0 \} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.
\]

In these cases, compensating with a suitable joint acceleration a drift of the end-effector acceleration due to the current joint velocity is still possible, provided that the Cartesian drift is in the span of the Jacobian, Moreover, the inversion in \( \mathbf{J} \) should be replaced by a pseudoinversion of the Jacobian matrix, namely

\[
\dot{q} = -\mathbf{J}^+(q)\dot{\mathbf{J}}(q)\dot{q}.
\]

To check if this is the case, we re-evaluate with \( \dot{\mathbf{J}} \) the time derivative of the Jacobian, and then the drift term:

\[
\dot{\mathbf{J}}'_0 = \dot{\mathbf{J}}_0(q_0)_{a_2=3} = \begin{pmatrix} 4.2426 & 4.2426 & 2.1213 \\ -4.2426 & 0 & 0 \\ -4.2426 & 0 & 0 \end{pmatrix} \Rightarrow \dot{\mathbf{J}}'_0 q_0 = \dot{\mathbf{J}}'_0 \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 8.4853 \\ -8.4853 \\ -8.4853 \end{pmatrix} \notin R \{ \mathbf{J}'_0 \}.
\]

Therefore, even with the use of a pseudoinverse, we will not be able to impose the desired (zero) end-effector acceleration: an error (of minimum possible norm) will result. Computing the pseudoinverse and evaluating \( \mathbf{J} \) gives

\[
\mathbf{J}''_0 = \mathbf{J}^+(q_0)_{a_2=3} = \begin{pmatrix} 0 & 0 & 0 \\ -0.2357 & 0.2357 & 0 \\ 0 & 0 & -0.4714 \end{pmatrix} \Rightarrow \dot{q} = \mathbf{J}''_0 \dot{\mathbf{J}}_0 q_0 = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}.
\]

Checking the end-effector acceleration obtained with this joint solution,

\[
\ddot{p} = \mathbf{J}'_0 \dot{\mathbf{J}}'_0 q_0 = \begin{pmatrix} 8.4853 \\ 0 \\ 0 \end{pmatrix}^T \neq \mathbf{0}^T,
\]

confirms that the component that is outside the range of the Jacobian (i.e., in the \( z \) direction) is not canceled, while the task is achieved for the remaining part.

**Exercise 2**

We proceed by integrating twice the joint acceleration commands, taking into account the initial state of the robot at time \( t = 0 \) (\( \mathbf{q}(0) = (0, 1) \) [rad; m], \( \dot{\mathbf{q}}(0) = \mathbf{0} \)) and the total motion time \( T = 2 \) [s]. For the joint velocities, we have obtain

\[
\dot{q}_1(t) = \begin{cases} A_1 t = 2 t \text{ [rad/s]}, & t \in [0, 0.5], \\
V_1 = 1 \text{ [rad/s]}, & t \in [0.5, 1.5], \\
V_1 - A_1 (t - 1.5) = 1 - 2(t - 1.5) = 4 - 2 t \text{ [rad/s]}, & t \in [1.5, 2] \end{cases}
\]

and

\[
\dot{q}_2(t) = \begin{cases} -A_2 t = -0.5 t \text{ [m/s]}, & t \in [0, 1], \\
-V_2 + A_2 (t - 1) = -0.5 + 0.5(t - 1) = 0.5 t - 1 \text{ [m/s]}, & t \in [1, 2]. \end{cases}
\]

For the joint positions, we obtain

\[
q_1(t) = \begin{cases} q_1(0) + \frac{1}{2} A_1 t^2 = t^2 \text{ [rad]}, & t \in [0, 0.5], \\
q_1(0.5) + V_1 (t - 0.5) = 0.25 + (t - 0.5) = t - 0.25 \text{ [rad]}, & t \in [0.5, 1.5], \\
q_1(1.5) + V_1 (t - 1.5) - \frac{1}{2} A_1 (t - 1.5)^2 = 1.25 + (t - 1.5) - (t - 1.5)^2 \text{ [rad]}, & t \in [1.5, 2] \end{cases}
\]

and

\[
q_2(t) = \begin{cases} q_2(0) - \frac{1}{2} A_2 t^2 = 1 - 0.25 t^2[m], & t \in [0, 1], \\
q_2(1) - V_2 (t - 1) + \frac{1}{2} A_2 (t - 1)^2 = 0.75 - 0.5(t - 1) + 0.25(t - 1)^2 [m], & t \in [1, 2]. \end{cases}
\]

The qualitative time profiles of \( q_i(t) \), \( \dot{q}_i(t) \) and \( \ddot{q}_i(t) \), for \( i = 1, 2 \), are shown in Fig. 2 with joint variations \( \Delta q_1 = 1.5 \) [rad] and \( \Delta q_2 = -0.5 \) [m]. The robot never crosses its kinematic singularities, i.e., any
configuration \( q_s = (\ast, 0) \) with the second link fully retracted. The initial, mid time and final configurations are
\[
q(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad q(1) = \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix}, \quad q(2) = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} \quad \text{[rad; m]}. \]

Fig. 3 sketches the RP robot in these configurations.

The end-effector velocity and acceleration of the RP robot are computed by differentiation of its direct kinematics
\[
p = \begin{pmatrix} q_2 \cos q_1 \\ q_2 \sin q_1 \end{pmatrix}. \]

We have
\[
\dot{p} = \begin{pmatrix} q_2 \cos q_1 - q_1 q_2 \sin q_1 \\ q_2 \sin q_1 + q_1 q_2 \cos q_1 \end{pmatrix} = \begin{pmatrix} \cos q_1 & -\sin q_1 \\ \sin q_1 & \cos q_1 \end{pmatrix} \begin{pmatrix} \dot{q}_2 \\ \dot{q}_2 \dot{q}_1 \end{pmatrix} = R(q_1) \begin{pmatrix} \dot{q}_2 \\ \dot{q}_2 \dot{q}_1 \end{pmatrix}, \tag{9}
\]
where a planar \( 2 \times 2 \) rotation matrix \( R \) by an angle \( q_1 \) has been put in evidence. The last vector in (9) is the end-effector velocity expressed in the frame rotated by the angle \( q_1 \), i.e., \( \dot{p} \). The norm of vector \( p \) is computed as follows:
\[
||\dot{p}||^2 = \dot{p}^T \dot{p} = \left( \begin{array}{c} \dot{q}_2 \\ q_2 \dot{q}_1 \end{array} \right) R^T(q_1) R(q_1) \left( \begin{array}{c} \dot{q}_2 \\ q_2 \dot{q}_1 \end{array} \right) = \left( \begin{array}{c} \dot{q}_2 \\ q_2 \dot{q}_1 \end{array} \right) ^2 = \dot{q}_2^2 + q_2^2 \Rightarrow ||\dot{p}|| = \sqrt{\dot{q}_2^2 + q_2^2}. \tag{10}
\]
Finally, Fig. 4 shows the vectors $\dot{\mathbf{p}}\parallel$ and $\ddot{\mathbf{p}}\parallel$ for computing the acceleration $\ddot{\mathbf{p}}$, note first that

$$R(q_1) = \begin{pmatrix} -\sin q_1 & -\cos q_1 \\ \cos q_1 & -\sin q_1 \end{pmatrix}, \quad \dot{q}_1 = \begin{pmatrix} \cos q_1 & -\sin q_1 \\ \sin q_1 & \cos q_1 \end{pmatrix}, \quad \dot{q}_1 = R(q_1) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$  

Differentiation of eq. (9) provides

$$\ddot{\mathbf{p}} = R(q_1) \begin{pmatrix} \dddot{q}_2 \\ \dddot{q}_1 \dddot{q}_2 + q_2 \dddot{q}_1 \end{pmatrix} + R(q_1) \begin{pmatrix} \dddot{q}_2 \\ q_2 \dddot{q}_1 \end{pmatrix} = R(q_1) \begin{pmatrix} \dddot{q}_2 - q_2 \dddot{q}_1^2 \\ q_2 \dddot{q}_1 + 2 \dddot{q}_2 q_1 \end{pmatrix} = R(q_1) \dddot{\mathbf{p}}\parallel.$$  

Moreover, the norm of this vector is

$$\|\ddot{\mathbf{p}}\parallel\| = \|\dddot{\mathbf{p}}\parallel\| = \sqrt{(\dddot{q}_2 - q_2 \dddot{q}_1^2)^2 + (q_2 \dddot{q}_1 + 2 \dddot{q}_2 q_1)^2}.$$  

The evaluation of (9) and (10) at $t = T/2 = 1$ s yields

$$\dot{\mathbf{p}}(1) = R(q_1(1)) \begin{pmatrix} \dddot{q}_2(1) \\ q_2(1) \dddot{q}_1(1) \end{pmatrix} = R(q_1(1)) \begin{pmatrix} -0.5 \\ 0.75 \end{pmatrix} = \begin{pmatrix} -0.8771 \\ 0.2079 \end{pmatrix}$$

and

$$\|\dot{\mathbf{p}}(1)\| = \sqrt{\dddot{q}_2(1)^2 + q_2(1) \dddot{q}_1(1)^2} = 0.9014.$$  

The evaluation of (11) at $t = T/2 = 1$ s should take into account the discontinuity of the acceleration of the second joint at the mid time instant:

$$\ddot{\mathbf{p}}(1^-) = R(q_1(1)) \begin{pmatrix} \dddot{q}_2(1^-) - q_2(1) \dddot{q}_1^2(1) \\ q_2(1) \dddot{q}_1(1) + 2 \dddot{q}_1(1) \dddot{q}_2(1) \end{pmatrix} = R(q_1(1)) \begin{pmatrix} -1.25 \\ -1 \end{pmatrix} = \begin{pmatrix} -0.2330 \\ -1.5837 \end{pmatrix}$$

and

$$\ddot{\mathbf{p}}(1^+) = R(q_1(1)) \begin{pmatrix} \dddot{q}_2(1^+) - q_2(1) \dddot{q}_1^2(1) \\ q_2(1) \dddot{q}_1(1) + 2 \dddot{q}_1(1) \dddot{q}_2(1) \end{pmatrix} = R(q_1(1)) \begin{pmatrix} -0.25 \\ -1 \end{pmatrix} = \begin{pmatrix} -0.4987 \\ -0.9021 \end{pmatrix}.$$  

Similarly, for the norm (12) we have

$$\|\ddot{\mathbf{p}}(1^-)\| = \sqrt{(\dddot{q}_2(1^-) - q_2(1) \dddot{q}_1^2(1))^2 + (q_2(1) \dddot{q}_1(1) + 2 \dddot{q}_1(1) \dddot{q}_2(1))^2} = 1.6008$$

and

$$\|\ddot{\mathbf{p}}(1^+)\| = \sqrt{(\dddot{q}_2(1^+) - q_2(1) \dddot{q}_1^2(1))^2 + (q_2(1) \dddot{q}_1(1) + 2 \dddot{q}_1(1) \dddot{q}_2(1))^2} = 1.0308.$$  

Finally, Fig. 4 shows the vectors $\ddot{\mathbf{p}}(1)$, $\ddot{\mathbf{p}}(1^-)$ and $\ddot{\mathbf{p}}(1^+)$ on the R-P robot in the mid time configuration. For this picture, it is more convenient to use the vectors expressed in the rotated frame, i.e., to draw for instance $\ddot{\mathbf{p}}(1)$ on the rotated second link (rather than attempting directly to draw $\ddot{\mathbf{p}}(1)$). Note that the relative scales of these vectors are somewhat arbitrary.

![Figure 4: The mid time robot configuration with: $\ddot{\mathbf{p}}(1)$ [left]; $\ddot{\mathbf{p}}(1^-)$ and $\ddot{\mathbf{p}}(1^+)$ [right].](image-url)
Exercise 3

The measurement system is composed by two parts, the laser scanner and the rotating link carrying it, see Fig. 5. The rotation added by the actuated link is useful because it enlarges the angular range of the sensor. On the other hand, uncertainty is added to the laser measurement, due to the angular resolution of the encoder which translates into an uncertain localization of the base of the sensor. To analyze the overall behavior, we consider first the two systems separately.

For the motor-link assembly, the angular resolution $\delta_m$ of the encoder mounted on the motor shaft (after electronic multiplication by 4) and the lateral uncertainty $\Delta_L$ at the link end are computed as

$$\delta_m = \frac{360^\circ}{4 \times N_p} \left( \frac{\pi}{180^\circ} \right) = \frac{2\pi}{4 \times 250} = 0.00628 \text{ [rad]}, \quad \Delta_L = \frac{\delta_m L}{N_r} = \frac{0.00628}{4} \times 1.5 = 0.0024 \text{ [m] = 2.4 [mm]}.$$

Note that the $n = 10$ bits of the digital counter in the encoder are sufficient to represent the full rotation, since $2^n = 2^{10} = 1024 > 1000$ (the number of electrical pulses per turn).

For the laser scanner, the angular resolution $\delta_s$ corresponds to an uncertainty in the lateral positioning (w.r.t. the pointing ray) of a sensed object. The worst-case situation is when the object is placed at the maximum sensing distance $d$ from the laser source. The (Cartesian) width resolution $\Delta_s$ in this case is

$$\Delta_s = \delta_s d = 0.2^\circ \left( \frac{\pi}{180^\circ} \right) 5 = 0.00349 \times 5 = 0.0175 \text{ [m] = 17.5 [mm]}.$$

Instead, the depth resolution $\Delta_\rho$ is rather independent from the distance. Thus, the region of uncertainty in the scanning process, when the base of the laser sensor is in a fixed, known position, can be approximated by a rectangle of size $\Delta_\rho \times \Delta_s = 12 \times 17.5$ [mm×mm]. A displacement of an object within this small area will not generate any change in the sensor reading. In particular, if it crosses this area in diagonal, we get

$$\Delta \simeq \sqrt{\Delta_\rho^2 + \Delta_s^2} = \sqrt{12^2 + 17.5^2} = 21.2 \text{ [mm]}. \quad (13)$$

When combining the scanning process with the variable orientation of the link, the measurement uncertainty on the object position will increase, due to the uncertainty $\Delta_L$ on the lateral positioning at the link end where the sensor is placed. However, the outcome of this combination will depend on the relative angle
between the direction of the laser ray and the direction of the link that carries the sensor. With reference to Fig. 6, we consider two limit cases: when the ray is aligned with the link, at $\alpha = 0$ [case (a)], and when the ray is at the boundary of its angular sensing range, e.g., at $\alpha = \pi/2$ [case (b)]. In the first case, $\Delta_L$ adds to $\Delta_\phi$ while the depth resolution $\Delta_\rho$ remains unaffected. In the second case, $\Delta_L$ adds to $\Delta_\rho$ while the width resolution $\Delta_\phi$ is unaffected. All other feasible values of $\alpha$ lead to intermediate situations. The largest Cartesian displacement of an object that would provide no change in the output reading is again on the diagonal of the rectangle of uncertainty. We have:

$$\Delta_a \simeq \sqrt{\Delta_\rho^2 + (\Delta_\phi + \Delta_L)^2}, \quad \Delta_b \simeq \sqrt{(\Delta_\rho + \Delta_L)^2 + \Delta_\phi^2}.$$ 

Since for the given data $\Delta_\phi > \Delta_\rho$, it follows that $\Delta_a > \Delta_b$. Thus, the worst increase in uncertainty will happen in case (a):

$$\Delta = \Delta_a \simeq \sqrt{12^2 + (17.5 + 2.4)^2} = 23.2 \text{[mm]}.$$ 

The resolution of the measurement system (or, equivalently, the largest positional uncertainty of the sensed object) in the mobile case has worsened by about 2 mm with respect to the fixed case.

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