Robotics I

January 11, 2016

Exercise 1

The 5-dof KUKA KR60 L45 robot is shown in Fig. 1. It has all revolute joints and a spherical wrist. The base has no rotation around the vertical axis (and this makes it a robot with 5-dof only). Assign the Denavit-Hartenberg frames and define the associated table of parameters, complying with the positive sense of joint rotation as shown in the left picture. Use the data in the right picture for the constant parameters.

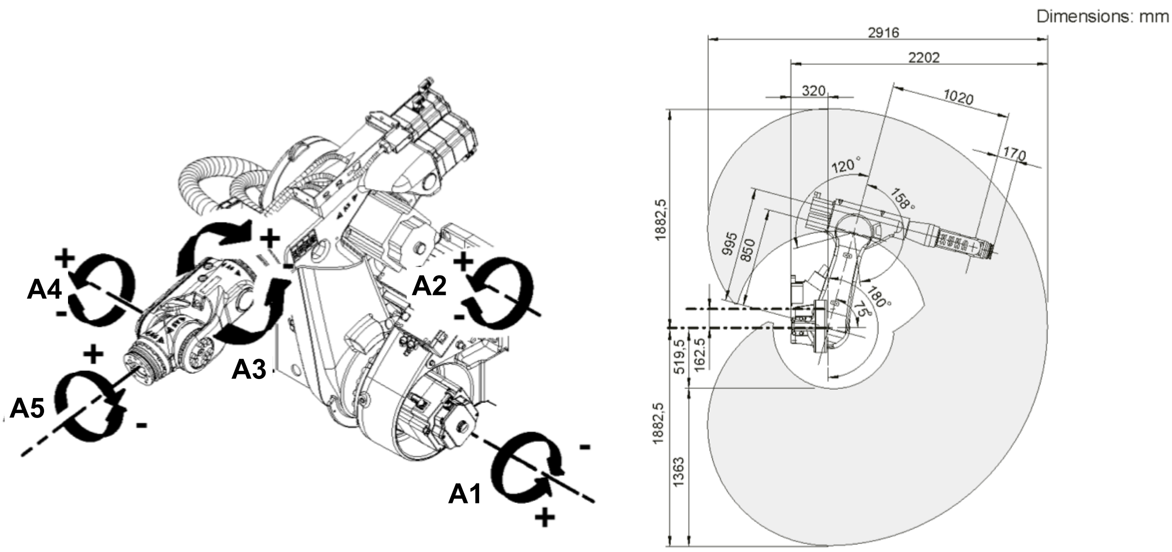

Figure 1: The KUKA KR60 L45 robot and its workspace

Exercise 2

Consider a planar 3R robot with links of equal lengths $\ell_1 = \ell_2 = \ell_3 = 0.5$ [m]. Assuming a Denavit-Hartenberg convention for the definition of the joint angles, consider the robot in the configuration $\boldsymbol{q} = (30°, 30°, 120°)$.

a) Compute a joint velocity vector $\dot{\boldsymbol{q}} = (\dot{q}_1, \dot{q}_2, \dot{q}_3)$ that realizes, if possible, the robot end-effector instantaneous motion specified by the velocity components

$$v_x = 0, \qquad v_y = 1 \text{ [m/s]}, \qquad \omega_z = 0.$$

In case a solution exists, are there multiple possible solutions to this problem?

b) Compute a joint torque $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ that balances a force $\boldsymbol{F}_e = (F_x, F_y) = (-5, 0)$ [N] applied to the robot end-effector, so that the robot remains in static conditions? Is such a $\boldsymbol{\tau}$ unique?

Exercise 3

Consider a planar 2R robot with links of lengths $\ell_1 = 0.1$ and $\ell_2 = 0.2$ [m]. The end-effector should trace the desired Cartesian trajectory

$$\boldsymbol{p}_d(t) = \begin{pmatrix} 0.15 + 0.05 \cos 5\pi t \\ 0.05 \sin 5\pi t \end{pmatrix}, \qquad t \in [0, T],$$

for some arbitrarily large period of time T.

a) At time $t = 0.2$ [s], which robot configuration $\boldsymbol{q}_d$ and joint velocity $\dot{\boldsymbol{q}}_d$ would instantaneously realize the desired trajectory? Do such numerical values $\boldsymbol{q}_d$ and $\dot{\boldsymbol{q}}_d$ exist? If so, are they unique?

b) Suppose that the robot motion is controlled by the kinematic control law

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{-1}(\boldsymbol{q})\big[\dot{\boldsymbol{p}}_d + \boldsymbol{K}_p\,(\boldsymbol{p}_d - \boldsymbol{f}(\boldsymbol{q}))\big], \qquad \boldsymbol{K}_p = 10 \cdot \boldsymbol{I}_{2\times 2}, \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{q})$ is the direct kinematics for this task, and that at time $t = 1.8$ [s] the robot is in the configuration $\boldsymbol{q} = (-\pi/2, \pi/2)$. Provide the value of the command $\dot{\boldsymbol{q}}$ given by (1) in such a condition. Compute also the associated end-effector velocity and sketch it on the robot. Where is this end-effector velocity vector pointing?

c) When $\boldsymbol{f}(\boldsymbol{q}) = \boldsymbol{p}_d(0.2)$, how can the control law (1) be modified so as to generate $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_d$ as in item a), if this is at all possible for some configuration $\boldsymbol{q}$?

[240 minutes; open books]

Solution

January 11, 2016

Exercise 1

Figures 2–3 show two views of a possible DH frame assignment consistent with the requested sense of joint rotation (counterclockwise = positive). The associated parameters are given in Table 1.

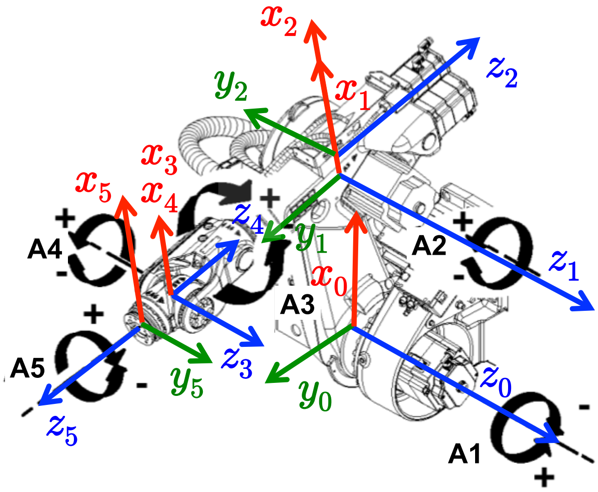

Figure 2: Denavit-Hartenberg frame assignment (perspective view from the right side)

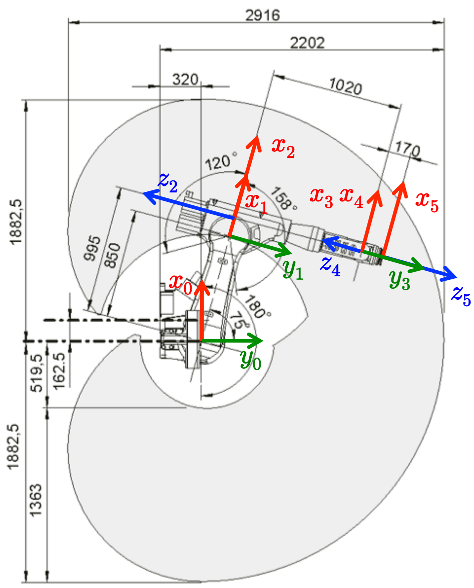

Figure 3: Denavit-Hartenberg frame assignment (lateral view from the left side)

i	α_i	a_i	d_i	θ_i
1	0	a_1	0	q_1
2	$\pi/2$	a_2	0	q_2
3	$-\pi/2$	0	d_3	q_3
4	$\pi/2$	0	0	q_4
5	π	0	d_5	q_5

$$a_1 = 850 \text{ mm}$$
$$a_2 = 145 \text{ mm}$$
$$d_3 = -1020 \text{ mm}$$
$$d_5 = -170 \text{ mm}.$$

Table 1: Denavit-Hartenberg table of parameters associated with the frame assignment in Figs. 2–3 for the KUKA KR60 L45 robot.

Exercise 2

For a), we consider the direct kinematics for the three-dimensional task vector $\boldsymbol{r}$ associated to the end-effector position (p_x, p_y) in the plane and the angle α_z of the DH end-effector frame with respect to the $\boldsymbol{x}_0$ axis, i.e.,

$$\boldsymbol{r} = \begin{pmatrix} p_x \\ p_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ q_1 + q_2 + q_3 \end{pmatrix} = \boldsymbol{f}(\boldsymbol{q}), \tag{2}$$

where we used the usual shorthand notation $c_{123} = \cos(q_1+q_2+q_3)$ and similar. Differentiating (2), we get

$$\dot{\boldsymbol{r}} = \begin{pmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\alpha}_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ \omega_z \end{pmatrix} = \frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}},$$

with the (3×3) Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ expressed by

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{pmatrix} -(l_1 s_1 + l_2 s_{12} + l_3 s_{123}) & -(l_2 s_{12} l_3 s_{123}) & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{pmatrix}. \tag{3}$$

It is easy to see that the Jacobian in (3) is nonsingular at the given configuration ($\det \boldsymbol{J} = 0.125$). Therefore, the numerical solution (computed in Matlab) is

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{-1}\dot{\boldsymbol{r}} = \begin{pmatrix} -0.6830 & -0.4330 & 0.0000 \\ 0.1830 & -0.2500 & -0.5000 \\ 1.0000 & 1.0000 & 1.0000 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.4641 \\ -5.4641 \\ 2.0000 \end{pmatrix} = \begin{pmatrix} 2\sqrt{3} \\ -2(1+\sqrt{3}) \\ 2 \end{pmatrix} \text{ rad/s}.$$

Similarly for b), considering that the external is no external moment M_z applied to the end-effector, the joint torque *balancing* (thus, having the opposite sign due to the principle of action and reaction) the external pure force $\boldsymbol{F}_e$ is

$$\boldsymbol{\tau} = -\boldsymbol{J}^T \begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} = -\boldsymbol{J}^T \begin{pmatrix} -5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3.4151 \\ -2.1651 \\ 0.0000 \end{pmatrix} = \begin{pmatrix} -5(1+\sqrt{3})/4 \\ -5\sqrt{3}/4 \\ 0 \end{pmatrix} \text{ Nm}.$$

This torque is unique. Note that the torque component at the third joint is zero, since the line of action of the force $\boldsymbol{F}_e$ in the given robot configuration passes through the axis of this joint.

Exercise 3

The desired end-effector position and velocity are

$$\boldsymbol{p}_d(t) = \begin{pmatrix} 0.15 + 0.05\cos 5\pi t \\ 0.05 \sin 5\pi t \end{pmatrix}, \qquad \boldsymbol{v}_d(t) = \dot{\boldsymbol{p}}_d(t) = \begin{pmatrix} -0.25\pi \sin 5\pi t \\ 0.25\pi \cos 5\pi t \end{pmatrix}.$$

With reference to Fig. 4, we see that the desired path is tangent to the inner boundary of the robot workspace.

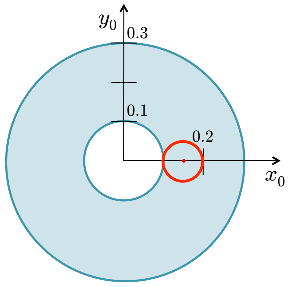

Figure 4: The desired Cartesian path and the robot workspace

In case a), when $t = 0.2$ it is

$$\boldsymbol{p}_d(0.2) = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix} \text{[m]}, \qquad \boldsymbol{v}_d(0.2) = \begin{pmatrix} 0 \\ -\pi/4 \end{pmatrix} \text{[m/s]} \tag{4}$$

and the end-effector touches the boundary so that the robot can be there in the *unique* configuration $\boldsymbol{q}_d = (\pi, \pi)$. Moreover, the desired velocity at this point is feasible since it belongs to the range space of the (singular) robot Jacobian at that configuration,

$$\boldsymbol{J}(\boldsymbol{q}_d) = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.2 \end{pmatrix}, \qquad \boldsymbol{v}_d(0.2) = \begin{pmatrix} 0 \\ -\pi/4 \end{pmatrix} \in \mathcal{R}\left\{\boldsymbol{J}(\boldsymbol{q}_d)\right\}.$$

In fact, there is an *infinite* number of combinations for the velocities of the two joints that realize the desired Cartesian velocity. The joint velocity solution with minimum norm is obtained as

$$\dot{\boldsymbol{q}}_{d,min} = \boldsymbol{J}^{\#}(\boldsymbol{q}_d)\boldsymbol{v}_d = \begin{pmatrix} 0 & 2 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ -\pi/4 \end{pmatrix} = -\begin{pmatrix} \pi/2 \\ \pi \end{pmatrix}.$$

All other solutions can be written as

$$0.1\,\dot{q}_{d1} + 0.2\,\dot{q}_{d2} = v_{d2} = -\pi/4 \qquad \Rightarrow \qquad \dot{\boldsymbol{q}}_d = \dot{\boldsymbol{q}}_{d,min} + \begin{pmatrix} -2\alpha \\ \alpha \end{pmatrix}, \text{ for any } \alpha \in \mathbb{R}.$$

For case b), when $t = 1.8$ the position and velocity of the desired trajectory are again as in (4). With the $2R$ robot in the configuration $\boldsymbol{q} = (-\pi/2, \pi/2)$, the Jacobian is out of singularities and can be safely inverted. From the direct kinematics, the robot end-effector position is

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{pmatrix}\Bigg|_{\boldsymbol{q}=(-\pi/2,\pi/2)} = \begin{pmatrix} 0.2 \\ -0.1 \end{pmatrix}$$

with a Cartesian position error

$$\boldsymbol{e}_p = \boldsymbol{p}_d - \boldsymbol{p} = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0.2 \\ -0.1 \end{pmatrix} = \begin{pmatrix} -0.1 \\ 0.1 \end{pmatrix}.$$

The Cartesian kinematic controller provides then

$$\begin{aligned} \dot{\boldsymbol{q}} &= \boldsymbol{J}^{-1}(\boldsymbol{q})\left[\dot{\boldsymbol{p}}_d + \boldsymbol{K}_p\left(\boldsymbol{p}_d - \boldsymbol{f}(\boldsymbol{q})\right)\right] = \boldsymbol{J}^{-1}(\boldsymbol{q})\left[\boldsymbol{v}_d + \boldsymbol{K}_p \boldsymbol{e}_p\right] \\ &= \begin{pmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{pmatrix}^{-1} \left[\begin{pmatrix} 0 \\ -\pi/4 \end{pmatrix} + 10 \cdot \boldsymbol{I}_{2\times 2} \begin{pmatrix} -0.1 \\ 0.1 \end{pmatrix}\right] \\ &= \begin{pmatrix} -10.0000 \\ 11.0730 \end{pmatrix} = \begin{pmatrix} -10 \\ 15 - 5\pi/4 \end{pmatrix} \text{ [rad/s]}. \end{aligned} \tag{5}$$

The instantaneous Cartesian velocity associated to (5) is

$$\boldsymbol{v} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} = \begin{pmatrix} -1 \\ 0.2146 \end{pmatrix} \text{ [m/s]}.$$

The two situations at $t = 0.2$ s (motion in nominal conditions) and at $t = 1.8$ s (motion with tracking error) are illustrated in Fig. 5.

Finally, the answer to c) is that it is certainly possible to obtain the solution in the conditions of case a) using the control law (1), by replacing the inverse of the Jacobian with its pseudoinverse, possibly specifying also a generic term in the null space of the Jacobian

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\#}(\boldsymbol{q})\left[\boldsymbol{v}_d + \boldsymbol{K}_p \boldsymbol{e}_p\right] + \left(\boldsymbol{I} - \boldsymbol{J}^{\#}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\right)\dot{\boldsymbol{q}}_0. \tag{6}$$

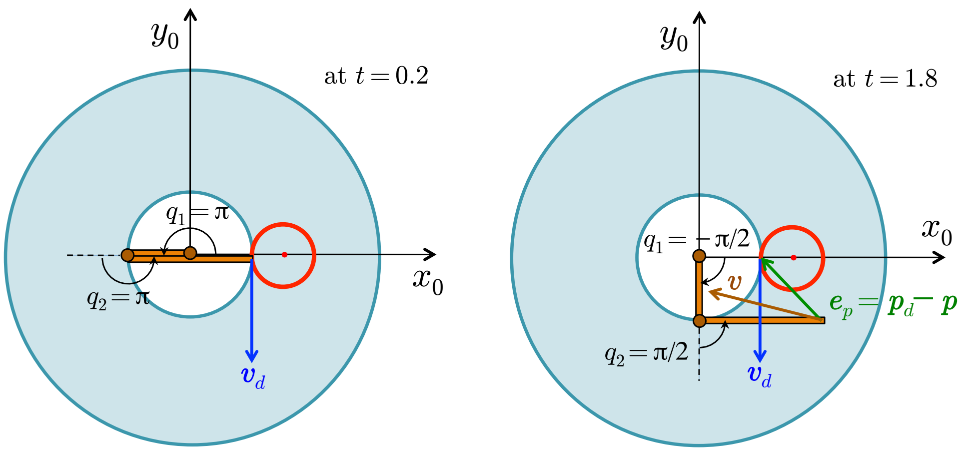

Figure 5: The solution at $t = 0.2$ s [left] and at $t = 1.8$ s [left] (for the sake of illustration, vectors are not represented in scale: only their correct direction is preserved)

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