Exercise 1

Consider a helix path whose parametrization is given by

\[ \mathbf{p}(s) = \begin{pmatrix} x(s) \\ y(s) \\ z(s) \end{pmatrix} = \begin{pmatrix} r (\cos s - 1) + x_0 \\ r \sin s + y_0 \\ ks + z_0 \end{pmatrix}, \quad s \in \mathbb{R}, \quad (1) \]

and let two Cartesian points \( \mathbf{P}_A = (p_{Ax}, p_{Ay}, p_{Az})^T \) and \( \mathbf{P}_B = (p_{Bx}, p_{By}, p_{Bz})^T \) be assigned. Define an interval \( s \in [0, s_{\text{max}}] \) and scalar values \( r, k, x_0, y_0, \) and \( z_0 \) in (1) such that \( \mathbf{p}(0) = \mathbf{P}_A \) and \( \mathbf{p}(s_{\text{max}}) = \mathbf{P}_B. \) Moreover, associate to this path a rest-to-rest timing law given by a cubic polynomial \( s = s(t), \) \( t \in [0, T] \), where \( T \) is the total motion time.

- Does the trajectory interpolation problem always have a solution? Is the solution unique?

- Determine a path (1) that solves the above problem for the numerical data \( \mathbf{P}_A = (0, 2, -10)^T \) and \( \mathbf{P}_B = (-2, 0, 10)^T. \) Compute the expression of the curvature \( \kappa(s) \) of this path.

- For the chosen timing law, provide the expressions of \( \dot{\mathbf{p}}(t) \) and \( \ddot{\mathbf{p}}(t) \), and determine the minimum time \( T \) that realizes the interpolation under the constraint \( \| \dot{\mathbf{p}}(t) \| \leq V_{\text{max}}. \)

Exercise 2

Consider a 3R elbow-type robot having its base mounted on the plane \( z = 0. \) The shoulder joint is at a height \( \ell_1 = 5. \) The links 2 and 3 have equal lengths \( \ell_2 = \ell_3 = 10. \)

- Place the robot base at a point \( (x_b, y_b) \) on the plane \( z = 0 \) so that the end-effector is capable of executing the solution path of Exercise 1.

- Find a robot configuration \( \mathbf{q} = \mathbf{q}^* \) at which the end-effector is placed in the (single) point of path (1) where the norm of the Cartesian velocity \( \dot{\mathbf{p}} \) in the minimum time trajectory of Exercise 1 has its maximum value.

- Compute at \( \mathbf{q}^* \) the joint velocity \( \dot{\mathbf{q}} \in \mathbb{R}^3 \) of the robot that realizes the desired velocity \( \dot{\mathbf{p}} \) of the above minimum time trajectory.

[150 minutes; open books]