Robotics I

June 10, 2013

Table 1 contains the Denavit-Hartenberg parameters of a robot with four revolute joints.

i	α_i	a_i	d_i	θ_i
1	$\frac{\pi}{2}$	0	0	θ_1
2	$\frac{\pi}{2}$	0	0	θ_2
3	$-\frac{\pi}{2}$	0	d_3	θ_3
4	0	a_4	0	θ_4

Table 1: Denavit-Hartenberg parameters of a 4R robot

1. Draw a kinematic sketch of the robot, including the associated Denavit-Hartenberg frames according to Tab. 1.
2. Draw the two robot configurations corresponding to $\boldsymbol{\theta} = \mathbf{0}$ and $\boldsymbol{\theta} = \begin{pmatrix} 0 & \pi/2 & \pi & 0 \end{pmatrix}^T$ [rad].
3. Find a singular configuration for the 3×4 Jacobian $\boldsymbol{J}(\boldsymbol{\theta})$ relating $\dot{\boldsymbol{\theta}}$ to the linear velocity $\boldsymbol{v}$ of the origin of frame 4.
4. In such a singular configuration $\boldsymbol{\theta}^*$, consider as numerical data $d_3 = a_4 = 0.5$ [m].
   a) Provide the numerical value of a *feasible* $\boldsymbol{v}_f$ and determine a minimum norm joint velocity $\dot{\boldsymbol{\theta}}_f$ such that $\boldsymbol{J}(\boldsymbol{\theta}^*)\dot{\boldsymbol{\theta}}_f = \boldsymbol{v}_f$. Is this minimum norm solution unique?
   b) Provide the numerical value of an *unfeasible* $\boldsymbol{v}_u$ and use the Jacobian pseudoinverse to compute the joint velocity $\dot{\boldsymbol{\theta}}_u = \boldsymbol{J}^{\#}(\boldsymbol{\theta}^*)\boldsymbol{v}_u$. Which are the properties of $\dot{\boldsymbol{\theta}}_u$?

[120 minutes; open books]