Written exams of Robotics 1

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All materials are in English, unless indicated (very old are in Italian)

Year	Date (mm.dd)	Number of exercises	Topics	Notes
2025	11.21 (Midterm Test)	5	Axis/angle representation(s) from a rotation matrix (singular case), with corresponding quaternion; Motor torque for a given load acceleration in a transmission with three gears, with optimization of one reduction ratio; DH frame assignment for a 3R spatial robot, given the DH table (plus last DH to end-effector frame transformation); Definition and kinematic equation of a robotic task using homogeneous matrices; DH frame and table assignment for a RRP planar robot, with special conditions, direct kinematics (including orientation), inverse kinematics in closed form, and set-up of the Gradient method for the numerical inverse kinematics (with analysis of stucking conditions)	solutions
2025	09.19	3	Homogeneous transformations of non-DH frames for a 2R planar robot and their relations with DH frames/table; A PRP planar robot: direct and inverse kinematics in closed form, Jacobian and its singularities; Planning of a PTP joint trajectory with zero initial velocity and acceleration and evaluation of maximum velocity and acceleration	---

2025	07.08	3	DH frames and tables for 7-dof Siemens Artis Zeego robot, with non-negative constant linear DH parameters; Geometric analysis of 2-dim inverse Jacobian solutions with task scaling, including numerical examples; Trajectory planning for a planar RP robot on an arc of a circle, with bang-bang acceleration as time profile (with evaluation of relevant path and trajectory quantities)	solutions
2025	06.11	4	Analysis at phi=0 of the 12 sequences of Euler angles, plus an inverse mapping between omega and Euler XYZ phidot; DH assignment and table for the ABB CBR15000 (GoFa) robot, with special requirements on twist angles and positive sense of joint rotation; Partial inverse kinematics of the first three joints of the ABB CBR 15000 robot for the positioning of the origin O4, starting from a desired end-effector pose with fixed last three joints; Minimum time rest-to-rest motion of a 2R robot under joint velocity and acceleration bounds, interrupted by the need of a sudded stop in minimum time	solutions
2025	02.07	4	DH frame assignment and table of parameters for the 6R YASKAWA Motoman UP500 robot; Inverse kinematics in closed form for a planar RRP robot (with last link twisted by $\pi/2$); Cooperative motion task of two planar robots (RP and PP) along an arc of a circle, with coordinated rest-to-rest motion in minimum time (under joint velocity/acceleration bounds); Bang-coast-bang minimum time motion of a joint under velocity/acceleration bounds, with non-zero initial and final speed	---
2025	01.13	5	1. Transformation from RPY-type YXY angles to axis-angle representation (singular case); 2. Choice of an Harmonic Drive for a desired link resolution with a given incremental encoder on motor side;	---

			3. DH assignment for a 2P3R spatial manipulator and geometric Jacobian derivation; 4. Iterations of Newton method for the numerical Inverse kinematics of a PRP cylindical robot; 5. Trajectory planning for a 2R planar robot in a rendex-vous task with a moving target (and kinematic control)	
	01.13 (with midterm)	3	1-mid: Differential analysis of a PRR planar robot on a 3-dimensional and 2-dimensional task: singularities, null spaces, range spaces; 2-mid: Same as 4 above, with two starting points; 3-mid: Same as 5 above	---
2024	11.22 (Midterm Test)	5	Use of different representations for orientation/rotation: ZXY Euler, axis-angle, RPY-type YXY angles*; Homogeneous transformation matrix for the final pose of a cylindric body after a rolling motion, followed by two rotations about a fixed and a current axis*; DH frames and table for a PPR planar robot under special requirement, with world and end-effector transformations and direct kinematics*, plus primary workspace; Analysis of a transmission with 3 toothed gears and using an incremental encoder; Assignment of DH frames from a DH table for a RPPR spatial robot plus inverse task kinematics in analytical and numeric form*	solutions; MATLAB codes
2024	11.07	4	DH frames and table for a PPR planar robot; Inverse kinematics of the PPR planar robot for end-effector position and orientation; Geometric Jacobian of the PPR planar robot with three inverse velocity problems; Planning of a circular trajectory for the end-effector of the PPR robot	---

2024	09.19	4	DH frame assignment and Table of parameters of Kawasaki RS010N 6R robot with sperical wrist; Velocity of the center of wrist and angular velocity of last DH frame for different combinations of (max) joint velocities, using the data of the Kawasaki RS010N robot; Rest-to-rest minimum time transfer for joint 2 and 3, using the data of the Kawasaki RS010N robot; End-effector resolution of Kawasaki RS010N robot, based on resolution of incremental encoders at joints 2 and 3	solutions
2024	07.08	4	DH frames and table assignment for 7R Franka Research 3 robot with all constant parameters being non-negative; RPY-type YZX angles and their time derivatives from an orientation matrix and an angular velocity; Collaborative task with two 2R planar robots, one with a sliding base on a tilded line; Rest-to-rest trajectory planning on a linear PTP Cartesian path, using a trapezoidal speed with bounds in mixed space	solutions
2024	06.12	5	PTP orientation trajectory using ZXZ angles from rotation matrices with boundary conditions on angular velocity; Sketch of a robot with frames from a given DH table, computing the end-effector position with Jacobian and its singularity analysis; Spatial resolution and field of view of a camera-lens system; Singularity-free placement of a circular path for a 3R planar robot; PTP path and trajectory planning in the joint space for a 2R planar robot with data specified in Cartesian space	solutions
2024	02.16	4	Visual localization of an object from 2 (planar) cameras and pickup via inverse kinematics of a PPR planar robot (5 items); D-H frame assignment and table for 3-dof cylindrical robot, with workspace, Jacobian, singularities, and null space directions;	---

			Rest-to-rest trajectory planning for a 2R planar robot between singular configurations, with specified tangents and uniform scaling; Kinematic control law in the Frenet space for a 3R spatial robot on helical path, traced with constant speed	
2024	01.24	6	Direct and inverse problem with ZXY angles around fixed axes (RPY-type); DH frames and table for the Yaskawa Motoman GP7 robot, with computation of e-e position; Solution of a trigonometric equation in two variables; Geometric Jacobian of a 3R robot known by its DH table, with singularity analysis and computation of the velocity of a point known in e-e frame; Inversion of a desired task trajectory for a 3R planar robot at the position, velocity, and acceleration level; Trajectory planning from rest to nonzero final velocity and maximum velocity computation	solutions
2023	11.15 (Midterm Test)	7	Euler XYZ angles to return a given Euler ZY rotation (symbolic/numeric)*; Axis-angle rotation to return two rotations about fixed axes x and v (symbolic/numeric)*; 2R workspace with limited joint ranges; 2R D-H table from unusual frame assignment*; Steady-state velocity and torque of DC motor with constant voltage, unloaded or with inertial load and reduction; D-H frame assignment and D-H table for a 5R robot, satisfying specifications on q=0 configuration and on positive direction of joint rotations*; RPR planar robot: direct kinematics, inverse kinematics for a 3-dimensional task defined with respect to a 2R robot*	solutions; MATLAB codes
2023	09.11	1 (8 parts)	4P planar robot: -DH frames and table; -homogeneous transformations with world and end-effector frames;	---

			-direct kinematics from world to end-effector; -task Jacobian; -null space of task Jacobian and range space of its transpose; -minimum norm joint velocity for a given task velocity; -rest-to-rest linear trajectory planning between two task points with max joint velocity and acceleration bounds	

2023	07.10	1 (8 parts)	RPR planar robot:
			-workspace with prismatic joint limits;
			-direct kinematics with non-DH (beta) joint variables;
			-inverse kinematics in closed form for the beta joint variables;
			-assignment of DH frames & table of parameters (with q joint variables);
			-direct kinematics with the DH (q) joint variables, with direct and inverse
			  transformation between beta and q;
			-task Jacobian in q and its singularities;
			-analysis of range and null subspaces of J and J transpose in a singularity;
			-rest-to-rest trajectory planning between two task points
			  (without violating limits of joint 2) --is a linear Cartesian path possible?
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2023	06.12	3	DH frame assignment and complete table for the ABB CBR 15000 robot; Minimum time cubic rest-to-rest timing law on a circular path with bound on the Cartesian acceleration norm; Geometric Jacobian, singularity analysis and velocity pseudoinversion for a 4R spatial robot	solutions
2023	03.24	2	DH frame assignment and table for a 6-dof 3P-3R robot, geometric Jacobian with singularity check, computation of the positional direct kinematics and inversion of the velocity mapping at a given configuration; Minimum time rest-to-rest motion of a 2P Cartesian robot under bounded force inputs	---

2023	02.13	4	DH frames and table of parameters for a planar RPPR arm (with outreach computation)*; Compare the difference of ZYZ Euler angles from two rotation matrices and the ZYZ Euler angles extracted from the relative rotation matrix*; Numerical inverse kinematics solution by Newton method (for a 2R planar arm)*; Analysis of the singularities and of the relevant subspaces for the 6x4 geometric Jacobian of a 4-dof robot characterized by its DH table	solutions; MATLAB codes
2023	01.23	5	DH frames and table of parameters for the PAL TIAGo 8-dof arm*; Direct kinematics of the wrist center for the PAL TIAGo 8-dof arm with respect to a world frame*; Algebraic solution of a single kinematic equation in two unknowns; Complete analysis of the Jacobian subspaces for a planar PRPR arm*; Minimum time rest-to-rest motion along a parametrized helix, under velocity/acceleration bounds in the tangent/normal directions of the Frenet frame, with a good placement of a 3R robot to perform the task*	solutions; MATLAB codes
2022	11.18 (Midterm Test)	6	Extraction of an angle and axis from a rotation matrix (singular case)*; Inverse representation of relative rotation matrix with YXY Euler angles*; Analysis of a transmission/reduction system with HD and spur gear*; Definition of a task kinematic equation with homogeneous transformations*; DH frames, table and direct kinematics of a spatial RPR robot*; Inverse kinematics for the position of a spatial RPR robot*	solutions; MATLAB codes
2022	10.21	4	For a spatial RPR robot: - DH frames, table and direct kinematics; - Jacobian and complete singularity analysis (with subspaces); - Kinematic control for regulation without planning; Minimum time rest-to-rest motion for a 2R planar with bang-bang acceleration inputs under maximum joint velocity bounds	solutions

2022	09.09	4	DH frame assignment and table of parameter for the 6R Fanuc cr15ia robot with offsets and spherical wrist; Questionnaire (10 true/false, explain) on the inverse kinematics problem; Kinematic analysis in velocity and acceleration of a 3dof robot*; Planning minimum time motions of a single joint with velocity/acceleration bounds in the rest-to-rest and state-to-rest case*	solutions; MATLAB codes
2022	07.08	4	Relationship between derivative of XZY angles w.r.t. fixed axes and angular velocity of an end-effector, with analysis in singularity*; Closed form inverse kinematics for a 3R spatial robot with offset*; Newton iterative method for the same problem of the previous exercise*; Point-to-point path in joint space for same robot of the previous two exercises, continuous up to the acceleration and with initial velocity coming from Cartesian space*	solutions; MATLAB codes
2022	06.10	4	DH frames and table for a 3R spatial robot with offset, its primary workspace, its square Jacobian with analysis of singularities, and admissible end-effector velocity in a double singularity*; Computation of an instantaneous joint acceleration for an RP robot having a non-zero joint velocity, so as to zero the end-effector acceleration; Static equilibrium for the RP robot with a linear force applied at the tip under bounds on the joint generalized forces*; Trajectory tracking in the Cartesian space for a planar 2R robot, with error dynamics that complies with maximum joint velocity limits*	solutions; MATLAB codes
2022	04.05	2	DH frames and table for 4R spatial robot, with end-effector homogenous transformation, direct kinematics for position, angular part of the geometric Jacobian and its associated null-space joint velocity; Analysis of a double bang-bang jerk profile of motion	---
2022	02.03	5	DH frames and table for the Crane-X7 robot (7R); Axis-angle extraction from relative rotation between initial orientation expressed by YXY Euler angles and final orientation expressed by rotation matrix;	solutions

			Task nominal inversion and feedback control at the acceleration level with singularity analysis for planar 3R arm; Trajectory planning for a planar PR robot using a two-cubic spline and a via point so as to avoid an obstacle; Transmission/reduction system with gears and pulleys	
2022	01.11	7	D-H frames and table for a planar RPR robot with L-shaped forearm, draw the robot for q=0 and compute e-e position and orientation, find the rotation matrix ^3R_e and extract the XYX Euler angles; Inverse task kinematics for the above RPR robot; Task Jacobian for the above RPR robot and singularity analysis; Wrench transformation of F/T sensor measures and joint torques for static equilibrium for the above RPR robot; Cartesian trajectory planning on a parametrized elliptic path; Tracing the elliptic trajectory with joint velocity commands in the nominal case and with feedback control; Minimum number of bits for a multi-turn absolute encoder	solutions
2021	11.19 (Midterm Test)	9	Questionnaire with 9 items: sequence of axis-angle and elementary y-rotation around fixes axes; ZYZ Euler inverse formulas to match a desired relative rotation; DH frames, table and gripper frame for a 4R spatial robot with spherical shoulder; primary workspace of a 2R robot with joint limits; task kinematics of a planar 5-dof bi-manual robot; analytical inverse kinematics for the RRP planar robot, with a numerical example; motor torque computation for a desired link acceleration in a geared wheel transmission; minimum resolution of an absolute encoder in a motor-transmission-link system; three sentences to describe SCARA robots	solutions
2021	10.19	2	For a spatial PPR robot: DH frame assignment and table, gripper orientation, direct kinematics for position, Jacobian computation, singularities and Cartesian mobility;	---

			Smooth coordinated rest-to-rest joint trajectory planning for the same PPR robot	
2021	09.10	2	For a planar PRR robot: direct task kinematics, task Jacobian and singularities, null space/range space analysis in a singularity providing numerical examples, inverse task kinematics in closed form, primary and secondary workspaces*; For the same planar PRR robot, a joint trajectory planning problem for a specific position/orientation task*	solutions; MATLAB codes
2021	07.12	3	DH frame assignment and table for a 4-dof spatial RRRP robot, with direct kinematics in position, Jacobian, and singularity analysis*; Analysis of a joint axis having a DC motor with double reduction gear and encoder, with maximum torque for a bang-bang link acceleration motion*; Smooth trajectory planning in orientation, interpolating with splines the Euler ZYX angles of three given rotation matrices*	solutions; MATLAB codes
2021	06.11	3	DH frame assignment and table for a portal robot (3P) with a spherical wrist (3R); A motion task and a static balancing task for a planar RPR robot (with inverse kinematics)*; Smooth coordinated rest-to-rest trajectory planning in position and orientation, with continuity up to the acceleration*	solutions; MATLAB codes
2021	02.04 (Remote)	8	Questionnaire with 8 items: from an axis-angle representation to XYZ RPY angles and their singularity analysis; a two-jaw 5-dof gripper (DH, kinematics, and definition of task variables); small encoder errors and their effects on Cartesian accuracy for a 2R robot; primary and secondary workspace of a planar PPR robot with bounded prismatic joints; steps of Newton and Gradient methods for the inverse kinematics of a RP robot near a singularity; 3R pointing device and singularity of its angular geometric Jacobian; trajectory planning between two given orientations, with final non-zero angular velocity	solutions

			assigned; kinematic control for the self-motion of a planar 3R (using null-space projection or joint space decomposition)	
2021	01.12 (Remote)	10	Questionnaire with 10 items: inverse problem and singularity example with Euler YXZ sequence; time derivative of an orientation matrix from the angular velocity in body frame; recognize the inverse of the DH homogeneous matrix; DH frame assignment and table, with direct position Kinematics of a spatial 2R robot; analysis of a multi-turn absolute encoder; singularity analysis of a planar RRPR robot in a 3-dimensional task; joint torques for two 2R robots in equilibrium when exchanging a Cartesian force; coordinated joint trajectory planning for a 2R robot in a rendez-vous task with a moving target; minimum time for given trajectory profiles of a 2R robot under joint velocity and acceleration bounds; Cartesian kinematic control in rotated frame for the same previous rendez-vous task	solutions
2020	11.20 (Remote Midterm Test)	10	Questionnaire with 10 items: ZYX sequence around fixed axes*; axis/angle from a relative rotation*; why 4 parameters only in DH frame transformations; computational issues in products; why kinematic vs. torque commands; DH frame assignment and parameter table for a spatial PRPR robot*; inverse kinematics in analytic form for this PRPR robot*; numerical derivative of position measures by 1-step BDF (Euler) formula; link displacement and resolution in a 2R arm with a transmission belt and an incremental encoder; kinematic definition of a task*	solutions; MATLAB codes
2020	10.27 (Remote)	2	Analysis of the kinematics (direct, inverse, and differential with singularities) of a spatial RRPR robot without using DH variables; Motion computation with a jerk profile of the bang-coast-bang type	---
2020	09.11 (Remote)	5	Angular acceleration from second time derivative of a rotation matrix; DH table associated to an assigned set of frames for the UR5 manipulator;	solutions

			Cooperative task (handing over of an object) of two planar 2R and 3R robots; Singularity analysis of a 3x3 robot Jacobian matrix, with computation of linear subspaces in the rank 1 case; Minimum time in a state-to-rest task for a single mass under bounded force input	
2020	07.15 (Remote)	5	DH frames assignment and related table for a 4-dof (PRRR) planar robot; Trajectory planning on a eight-shaped path for a Cartesian planar robot under velocity and acceleration bounds; Euler YXY rotation matrix and relation between the time derivative of these angles and the angular velocity; Inverse kinematics and inverse differential kinematics for a planar 3R robot on a linear path with specified end-effector orientation; Questionnaire with 2 questions	solutions
2020	06.05 (Remote)	5	Complete a DH frame assignment and related table for a 4R spatial robot; Direct and differential kinematics for a planar RRP robot and joint torques balancing a Cartesian force in regular or singular configurations; Analysis of range and null spaces of the 3x3 Jacobian of a spatial 3R robot, with inverse differential solution in a singularity; Minimum-time smooth rest-to-rest trajectory planning for a 2R robot with joint velocity and acceleration bounds; Questionnaire with 3 questions	solutions
2020	02.12	4	DH frame assignment and table of parameters for a spatial 4-dof PRRR robot; For the same robot above: direct positional kinematics, balancing joint torque of a Cartesian force, angular Jacobian with its singularities and null space; Trajectory planning in the Cartesian space for the end-effector of a PPR planar robot moving in contact with a circle with bounds on velocity and acceleration in the joint space;	solutions

				Questionnaire with 8 questions (very mixed in nature)	
2020	01.07	4		DH frame assignment and table of parameters for the 7R Cesar arm*; Linear part of the geometric Jacobian of the 7R Cesar arm and its use for the numerical solution of an inverse kinematics problem with the Gradient method; Trajectory planning in the Cartesian space for the end-effector position and orientation of an RPR planar robot moving in contact with a linear surface*; Questionnaire with 8 questions (very mixed in nature)	solutions; MATLAB codes
2019	11.29 (Midterm Test in classroom)	5		Computation of orientation using ZYX angles w.r.t. fixed axes (RPY) and axis-angle methods, in a rotated and or in the base frame; DH table from a given assignment of frames for the 6R UR10 manipulator; Workspace analysis, DH frame assignment, and three inverse kinematics problems for a planar 2R robot with a L-shaped second link; Iterative numerical step/solution with Newton method for the inverse solution of a 3-dimensional kinematic task; Questionnaire with 7 questions (mostly on sensing and actuation)	solutions
2019	09.11	3		Joint acceleration command for zeroing the end-effector acceleration in a state (position/velocity) of a 3R planar robot (with analytic Jacobian computation); Assigned bang-(coast-)bang profiles for a RP planar robot: sketch time evolution in joint space and compute end-effector velocity and acceleration (with norms); Analysis of the resolution of a laser sensor mounted on the tip of a rotating link, driven by a DC motor with reduction and incremental encoder	solutions
2019	07.11	3		Static balancing torque for an off-centered payload in a 3R planar robot; Base placement for a RP planar robot with limited joint range in order to execute a linear path within its workspace*;	solutions; MATLAB codes

			Time-optimal motion of a joint with a prescribed structure of bang-coast-bang acceleration profile, under velocity and acceleration bounds*	
2019	06.17	2	DH assignment, joint range matching, direct kinematics, inverse kinematics of the wrist center, and linear Jacobian of the wrist center for the 6R Kawasaki S030 robot*; Specifying the end-effector velocity in different ways/representations and their relationships, with a simple numerical example*	solutions; MATLAB codes
2019	02.05	3	DH assignment and direct kinematics for a 4R spatial robot*; Geometric Jacobian and singularity analysis*; Kinematic control of a 2R planar robot along a circular trajectory in the Cartesian space, with inverse kinematics initialization*;	solutions; MATLAB codes
2019	01.11	3	DH assignment for a RRPR spatial robot*; Its geometric Jacobian and force/velocity analysis in Cartesian/joint space*; Rest-to-rest minimum time trajectory planning for a PR planar robot under joint acceleration and Cartesian acceleration norm bounds*	solutions; MATLAB codes
2018	11.16 (Midterm Test in classroom)	5	Computation on orientations using various representations*; Generating a DH homogeneous matrix; DH table from frames for a 7R anthropomorphic manipulator*; Analysis of a DC motor servo drive; Iterative step of a numerical solution of the inverse kinematics of a RP planar robot*	solutions; MATLAB codes
2018	07.11	2	Geometric analysis and direct and inverse differential mappings for a minimal representation of orientation by rotations around the sequence of fixed axes YXZ; Trajectory planning/control at the acceleration level for a RP robot executing a circular motion in the Cartesian plane	solutions
2018	06.11	2	Planar 2R robot with L-shaped second link: DH frames and table, direct kinematics; special configurations and primary workspace, inverse kinematics, analytic Jacobian, singularities and range/null spaces, inverse	solutions

			kinematics and inverse differential kinematics solutions on numerical data; Minimum time rest-to-rest trajectory planning, with joint velocity and acceleration bounds and joint coordination	
2018	03.27	2	DH frames/table for a 5-dof spatial RRPRP robot, with all non-negative constant parameters, sketch of two configurations, its geometric Jacobian, and a basis for null-space wrenches (forces/moments) in a given configuration; Questionnaire on singularity issues in 6-dof manipulators	solutions
2018	02.05	4	DH frames/table for 4R Comau e.Do robot, with all non-negative constant parameters; Questionnaire on sensors for manipulators and related measurements issues; Geometric Jacobian derivation for the 4R Comau e.Do. robot, with analysis of the singularities and computation of a null-space joint velocity; Cubic spline interpolation of four knots in time, check of velocity/acceleration limits and with uniform time scaling*	solutions, MATLAB code
2018	01.11	4	DH frames/table for 7R Franka Emika (Panda) robot, with evaluation of elementary operations in direct kinematics; Questionnaire on numerical methods for inverse kinematics; Definition of a coordinated task (position, orientation, and linear velocity of the end-effectors) for two planar 3R manipulators; Smooth rest-to-rest trajectory planning for a RP robot, with uniform time scaling to satisfy at best joint velocity and acceleration limits	solutions
2017	11.24 (Midterm Test in classroom)	4	DH frames/table for a planar RPR robot, direct kinematics with two different sets of coordinates and their relation; Analysis of a transmission/reduction assembly (incremental encoder choice to guarantee a Cartesian resolution); DH frame assignment associated to a given DH table for Stäubli robot RX 160;	solutions

			Inverse problem for an axis-angle representation of a relative rotation matrix	
2017	10.27	1	DH frame assignment and table of parameters for Stäubli robot RX 160, with comparison to joint angles and limits of the manufacturer, and computation of the position of the wrist center and of the angular part of the geometric Jacobian for the first three joints	---
2017	09.21	3	Rotations, final orientation, angular velocity, and linear velocity of the tip for a thin rod*; Rest-to-rest cubic trajectories in minimum time for a planar 2R robot under maximum joint velocity bounds; Analysis of a transmission/reduction assembly	solutions; MATLAB code
2017	07.11	3	DH assignment for a 5-dof cylindrical robot and geometric Jacobian; Cubic, quintic, and seventh-degree polynomial trajectories for a rest-to-rest motion, and their minimum time under maximum velocity or acceleration bounds; Discuss incremental vs absolute encoders, their mounting, and ways to measure the robot end-effector position	solutions
2017	06.06	3	Direct kinematics of a planar PRPR robot in different coordinates (manufacturer and DH) and their mapping; Definition of kinematic control laws for a 3R elbow-type robot in reaction to human presence in the Cartesian space sensed by laser scanning; Geometric cubic spline through four knots (for a single joint) and time properties of the associated trajectory executed with constant speed*	solutions; MATLAB code
2017	04.11	2	Inverse kinematics of a spatial 3R (elbow-type) robot in analytic form (with numerical example); Minimum-time motion of a joint with generic non-zero boundary velocities under velocity and acceleration bounds (with numerical example)	solutions
2017	02.03	3	DH frames assignment and table of parameters for the left arm of the NAO humanoid robot;	solutions

			Minimum-time rest-to-rest motion between two Cartesian points for a RP planar robot under joint velocity/acceleration bounds, followed by manipulability/singularity analysis; Kinematic control of a 3R planar robot on a linear Cartesian trajectory with continuous acceleration, imposing specified transients along the tangent and normal to the trajectory	
2017	01.11	3	DH frames assignment, table of parameters, and task Jacobian for a planar RPRP robot, with singularity analysis; Rest-to-move Cartesian planning of path and timing law for a planar 2R robot, with continuity of velocity; Completing the geometric Jacobian of a 3R spatial robot, with rank analysis, and its use for static balance of forces/torques applied to end-effector	solutions
2016	11.18 (Midterm Test in classroom)	4	Specific DH frame assignment for Universal Robot UR5; Use of homogeneous transformation matrix, with ZYX Euler angles representation; DH frames/table for a 2R robot moving in 3D, with direct kinematics computation; Inverse problem for an axis-angle representation of a (rotation?) matrix	solutions
2016	10.28	2	Interpreting a given rotation matrix parametrized by two angles in fixed and moving axes; Second-order inverse differential kinematics and control for a planar 2R robot*	solutions; MATLAB code
2016	09.12	2	Inverse kinematics for the wrist of the UR10 robot; Jacobian, singularities and inverse differential solutions for a planar RPR robot with skewed prismatic joint	solutions
2016	07.11	3	Analysis of a single cubic joint trajectory with non-zero final velocity*; Angular Jacobian from a DH table of a 3R arm and its singularities*;	solutions; MATLAB codes

			Inverse (differential) kinematics of a planar 2R arm to match a desired Cartesian velocity and design of a kinematic control law to recover initial errors*	
2016	06.06	2	DH frame assignment and table of parameters for the 6R Universal Robot UR5; (Pseudo-)code for the iterative numerical solution to the inverse kinematics of a planar 3R robot in positioning tasks	solution of Ex #1 only
2016	04.01	1	Rest-to-rest smooth and coordinated trajectory planning in minimum time for a 2R robot moving between two Cartesian positions under joint velocity, acceleration, and jerk limits	solution
2016	02.04	4	Completing the definition of a rotation matrix; Inverse kinematics of a 2-dof robot, specified by its DH table, with joint range; 4R planar robot performing a simultaneous double velocity task; Minimum time trajectory planning on a rectangular path, with bounds on the norms of the Cartesian velocity and acceleration and continuity of the velocity solution.	solutions
2016	01.11	3	Denavit-Hartenberg frame assignment for 5-dof KUKA KR60 L45; Inverse differential kinematics and static solution for a planar 3R robot*; Planning through a singularity and Cartesian kinematic control of a planar 2R robot*.	solutions; MATLAB codes
2015	10.27	2	Joint acceleration command for obtaining a desired Cartesian acceleration, at its numerical evaluation for a planar 2R robot; Analysis of a multiple-gear transmission.	solutions
2015	09.11	3	Angular velocity of a spherical wrist; Inverse kinematics in closed form for a spatial RPR robot; Singularities and null/range space analysis of the task Jacobian for a planar 3R robot.	solutions
2015	07.10	2	Analysis and displacement computation for an assigned bang-bang type profile of the snap (4th time derivative);	---

			Placing of the base of a planar 2R robot for executing a straight line in its workspace and joint velocity computation at a singular configuration.	
2015	06.05	2	Path planning with an helix in 3D and minimum time rest-to-rest motion with cubic timing profile and bounded norm of Cartesian velocity; Placing of the base of an elbow-type 3R robot for executing a straight line in its workspace and joint velocity computation at a specific configuration.	---
2015	04.01	1	Minimum-time trajectory planning between two Cartesian points for a planar 2R robot under joint velocity and joint acceleration constraints.	same as 2006.07.13 (in Italian), with modified data
2015	02.06	2	Complete inverse kinematics analysis in orientation for a 3-dof robot, including singular or regular numerical cases and an inverse differential problem; Planning a Cartesian trajectory on a circular path of given radius between two points, with trapezoidal speed and bounds on the norms of the velocity, of the acceleration, and of the normal acceleration.	solutions (also longer version available)
2015	01.09	3	Effect of incremental encoder resolution on the accuracy of end-effector position measure for a planar 2R robot*; Planning of a Cartesian straight-line trajectory for a RP planar robot, to be executed in minimum time under joint range and joint velocity limits*; Kinematic control with prescribed Cartesian transient error for a 3R anthropomorphic robot*.	solutions; MATLAB codes
2014	11.21 (Test in classroom)	4	Reduction ratio and optimal inertia/acceleration of joint 2 of PUMA 560 robot; DH table of parameters from assigned frames of a PUMA 560 robot; Primary workspace of a generic planar 3R manipulator; Inverse kinematics in closed form of a 3P-3R spatial robot with spherical wrist	solutions

2014	10.27	2	Inverse representation problem and analysis of relation between angular velocity and derivative of Euler angles XYZ; Geometric Jacobian of SCARA-type robot and solution of a problem of inverse differential kinematics in a singularity	solutions
2014	09.22	1	DH frames and table for the Siemens Artis Zeego medical robot, having 7 DOFs (one prismatic and six revolute joints)	---
2014	07.15	1	7R KUKA LWR robot, with frozen last three joints: direct kinematics of the tool center point and related Jacobian, solution to the inverse kinematics when one joint angle is assigned, singularity analysis	---
2014	06.10	1	DH frames assignment and table for the COMAU RACER 7-1.4 robot, and mapping by comparison with the one used by the robot manufacturer	solution
2014	04.02	3	Draw the DH frames of a 4R robot and the direct kinematics (position only), given the DH table; For the same robot, static torques balancing a desired force; Smooth minimum time rest-to-rest motion of a single joint under velocity and acceleration bounds	---
2014	02.06	3	Definition and use of the Jacobian transpose for force transformations; A 4-3-4 trajectory planning problem: formulation and solution*; Proof of Cartesian trajectory tracking using both the Jacobian transpose (feedback) and the Jacobian inverse (feedforward)	solutions; MATLAB code
2014	01.09	3	PPR planar robot: DH frame assignment and table, primary and secondary workspace for bounded range of prismatic joints; Planning of rest-to-rest orientation trajectory using YZY Euler angles, with motion time satisfying a bound on the norm of angular velocity*; Joint velocity commands in a 6R robot with spherical wrist for planning or tracking (kinematic control) end-effector trajectories with zero desired angular velocity	solutions; MATLAB code
2013	11.29 (Test in classroom)	3	Optimal reduction ratio of a cascaded spur gear and harmonic drive transmission; K-1207 7-dof robot: DH frames and table of parameters;	solutions

			Planar RPR manipulator: inverse kinematics for planar pose, primary workspace for limited range of prismatic joint	
2013	09.19	1	Planar RPPR manipulator: DH frames and table of parameters, analysis of maximum reach with limits on the prismatic joints	---
2013	07.15	1	Analysis of a joint velocity motion of trapezoidal type for a planar 2R arm, with evaluation of selected Cartesian quantities (displacement, velocity, acceleration)	---
2013	06.10	1	4R spatial manipulator: assignment of DH frames, Jacobian for the linear velocity, and analysis of feasible motion at a singularity	---
2013	04.10	1	Minimum time trajectory planning for planar 3R manipulator on a three-dimensional rest-to-rest task, with joint velocity and acceleration bounds*	solution; MATLAB code
2013	02.06	2	DH assignment and geometric Jacobian of a 4-dof robotic finger; Trajectory interpolation with a class of trigonometric functions, with analysis of wandering*	solutions; MATLAB code
2013	01.09	3	Definition of a minimal representation of orientation, and singularities of the associated differential relation; Singularities and minimum norm joint velocity solution for a planar 4R arm; Effect of encoder errors on the end-effector position estimate of a 3R anthropomorphic robot	solutions
2012	09.10	1	DH frame assignment to elbow-type 3R robot, with analysis of linear and angular velocities of the end-effector in a given configuration	solution
2012	07.05	1 (4 parts)	6-dof portal robot for aeronautical industry: pointing task; inverse kinematics; positioning task and its inverse kinematics; solution for numerical data*	solutions; MATLAB code
2012	06.11	3	Derivative of a rotation matrix in fixed or rotated frame; Jacobian, singularities, and null/range spaces analysis of planar RPR arm; Resolution of incremental encoders for a Cartesian task of a 2R robot	solutions
2012	04.26	2	DH assignment and Jacobian expressed in camera frame of 3R articulated arm (symbolic MATLAB code included);	solutions; MATLAB code

			Rest-to-rest orientation planning with axis-angle method and cubic timing law*	
2012	02.09	3	Angular velocity of the COMAU NJ4 170 robot with non-spherical wrist; DH assignment, Jacobian, and singularities of RRP (polar) arm; Planning of straight Cartesian paths, singularity handling, and joint vs. Cartesian kinematic control for the RRP arm	solutions
2012	01.11	2 + bonus	Primary and secondary workspace of a planar 3R arm, singularities, and manipulability index H (bonus: write a MATLAB* program plotting H); Rest-to-rest minimum time motion between two Cartesian poses, with bounds on joint velocity and acceleration	solutions; MATLAB code
2011	09.12	1	Inverse differential kinematics for a SCARA-type robot for two 6-dimensional desired task velocities	solution
2011	07.04	1	Barrett 4-dof WAM: D-H frame check, direct kinematics, actuator transformation, linear velocity Jacobian*, singularity and joint limit check	solution; MATLAB code
2011	06.17	1	Polytopes of feasible Cartesian velocity for a 2R planar robot with joint velocity bounds in different configurations*	solution; MATLAB code
2011	02.25	1	Cyclic joint trajectory design, singularity crossing and time scaling for a 3R anthropomorphic robot*	solution; MATLAB code
2011	02.03	1	Various Jacobians with their analysis and a joint acceleration synthesis for a 3R anthropomorphic robot	solution
2010	09.15	1	Trajectory definition with double symmetric bang-coast-bang jerk profile*	solution; MATLAB code
2010	07.07	1	DH assignment for the 6R KUKA KR-30-3 robot and direct kinematics of the center of its spherical wrist	solution
2010	06.15	2	Singularities for a RP planar robot in a one-dimensional task and kinematic control at the joint acceleration level; Relation between angular velocity and derivative of Euler angles YXZ	solutions
2010	02.11	1	Path planning for a 2R planar robot among obstacles with singularity crossing*	solution; MATLAB code

2010	01.12	2 (one in common, option A or B for the other)	Cartesian trajectory planning on spiral path for position and orientation with velocity/acceleration constraints and trapezoidal speed profile*; (A) Input-output linearization control for front-wheel drive car-like; or (B) Geometric Jacobian for a cylindrical robot, singularities, and kinematic Cartesian control in acceleration	solutions (with options A and B); MATLAB code
2009	12.17 (Test in classroom)	1	Geometric Jacobian for a 4R spatial robot, feasibility of a Cartesian linear/angular velocity, minimum norm joint velocity solution, and joint torque balancing a Cartesian force/torque*	solution; MATLAB code
2009	11.10 (Test in classroom)	2	Minimal representation of orientation around fixed YXZ axes; DH assignment for a spatial 3R arm pointing a head camera, direct kinematics for the orientation, and condition for an infinite number of inverse solutions	solutions
2009	09.10	1	Jacobian of mobile manipulator, with Nomad base (unicycle) and 3R anthropomorphic manipulator (Puma, with frozen wrist)	solution
2009	07.10	2	Inverse kinematics of a RP robot, workspace with limited joint range, and number and type of inverse solutions in the workspace; Planning of a coordinated roto-translation in the Cartesian space	solutions
2009	06.10	1 (3 parts)	DH assignment for a planar PRP robot; Singularities and linear subspaces associated to the Jacobian for a planar positioning task; Kinematic control in the task space (planar position and orientation) with two case studies of feasibility with respect to joint velocity bounds	solution
2009	02.09	2	Kinematic control in the Cartesian space in acceleration; Placing the base of a planar 2R robot so as to maximize manipulability and task velocity in a given direction **(in Italian)**	solutions **(in Italian)**
2009	01.08	2	Angular velocity for an axis/angle rotation: general proof and computation of a trajectory for end-effector orientation; Direct kinematics, Jacobian and singularity analysis for a 3R supporting leg of the SmartEE parallel platform **(in Italian)**	solutions **(in Italian)**

2008	09.11	1	Second-order kinematic model of a nonholonomic mobile manipulator, a car-like mobile base with a planar 2R arm (optional: singularity analysis) **(in Italian)**	solution **(in Italian)**
2008	07.02	2	Statics of a planar 2R robot with two force applied along the links; Nonholonomic constraints of a fire-truck mobile robot (optional: kinematic model) **(in Italian)**	solutions **(in Italian)**
2008	03.20	2	Optimal planning of a trajectory composed by three velocity pieces, with initial/final sinusoidal profiles and acceleration constraint; Differential kinematics of a spatial 3R robot with eye-in-hand camera **(in Italian)**	---
2008	01.07	2	Linear Cartesian motion of a planar 3R robot and singularities; Kinematic model of a WMR with two steering wheels **(in Italian)**	solutions **(in Italian)**
2007	12.03	3	Inverse kinematics of a planar 2R robot with test on the joint range feasibility*; Angular resolution of a servo-drive with incremental encoder and sizing of the motion reduction element; Optimal trajectory planning with velocity/acceleration constraints and continuity up to acceleration* **(in Italian)**	solutions **(in Italian)**; MATLAB code
2007	09.13	1 (2 parts)	DH assignment for a spatial 3R robot and computation of the end-effector linear and angular velocity; Pseudo-code of an algorithm for numerical inverse kinematics **(in Italian)**	---
2007	06.28	2	Geometric Jacobian for the wrist frame of a KUKA KR6 Sixx robot with last three joints frozen; Planning of a piecewise polynomial trajectory through four point with boundary conditions up to the jerk and continuity in acceleration **(in Italian)**	---
2007	03.23	1	DH assignment for the KUKA KR150K robot and relationship with the "zero" configuration from the industrial robot data sheet **(in Italian)**	---
2007	01.08	3	Singularity analysis and analytical inverse kinematics for a planar RRP robot;	solutions **(in Italian)**

			Use of kinematic redundancy for handling joint range limits; Pros and cons of the use of vision in robot motion control **(in Italian)**	
2006	12.04	2	DH assignment, direct kinematics, and workspace of a spatial RRPR robot; Optimal planning of Cartesian straight-line trajectory for a planar RP robot with velocity/acceleration constraints and use of uniform time scaling to satisfy maximum joint velocity bounds **(in Italian)**	solutions **(in Italian)**
2006	09.11	1 (3 parts)	DH assignment for a planar 3R robot; Jacobian and its singularities; Planning of a trajectory between two Cartesian points where the robot is in a singularity, with acceleration continuity **(in Italian)**	---
2006	07.13	1	Minimum-time trajectory planning between two Cartesian points for a planar 2R robot under joint velocity and joint acceleration constraints **(in Italian)**	---
2006	06.30	1	DH assignment for the DLR LWR-III 7R robot **(in Italian)**	---
2006	04.03	1 (2 parts)	Robot-excavator: Direct kinematics; Inverse kinematics, statics, placement of robot base in the workspace (choose one) **(in Italian)**	solution **(in Italian)**
2006	01.09	1 (3 parts)	Mobile base moving in circle with a planar 2R manipulator on board: Inverse kinematics; Differential kinematics; Singularity analysis **(in Italian)**	solution **(in Italian)**
2005	12.16	1 (3 parts)	"Painting" RPPR robot: DH assignment; Direct kinematics; Minimum-time cyclic Cartesian trajectory under joint velocity constraints **(in Italian)**	solution **(in Italian)**
2005	09.22	1	DH assignment for the Comau Smart Six robot **(in Italian)**	---
2005	04.05	2	Statics and computation of joint accelerations for a constrained planar 3R robot; Computing wheel velocities so as to assign a given linear velocity to a point on the chassis of the SuperMario mobile robot **(in Italian)**	solutions **(in Italian)**
2005	01.12	2	Direct and differential kinematics, singularity analysis and control of a mobile manipulator --unicycle base with spatial 3R robot; Minimum-time trajectory planning between two Cartesian points under acceleration and, possibly, velocity constraints for a planar 2P robot (multiple solution paths) **(in Italian)**	solutions **(in Italian)**

2004	12.16	2	DH assignment, direct kinematics, singularity analysis, trajectory planning without singularities, and workspace for a spatial RPR robot; Path planning in the joint space, with given initial and final Cartesian tangents and an obstacle to be avoided* **(in Italian)**	solutions **(in Italian)**; MATLAB code
2004	04.06	1	Planning of a cyclic joint trajectory passing through three Cartesian points for a planar 2R robot **(in Italian)**	solution **(in Italian)**
2004	03.25	2	Odometry computation and minimum-time motion for the SuperMario wheeled mobile robot; Singularities, workspace, and manipulability for a planar 4R robot **(in Italian)**	solutions **(in Italian)**
2004	01.08	2	DH assignment, direct kinematics, statics, and minimum norm joint velocity computation for a (redundant) planar RRP robot; Planning of an orientation trajectory using the axis/angle method or with the YZY Euler angles* **(in Italian)**	solutions **(in Italian)**; MATLAB code
2003	12.11	2	DH assignment for a 3R pointing structure and its direct kinematics; Trajectory planning for a planar RP robot under bounds on the Cartesian acceleration norm and on the joint velocities* **(in Italian)**	solutions **(in Italian)**; MATLAB code

Note: For these* problems, MATLAB codes for computing solutions and/or for graphics are either embedded in the solution text or available to the students of the course upon request (contact deluca@diag.uniroma1.it).