Control Systems
23/3/2019

Exercise 1 Bode plots of $P(s)$ are drawn in Fig. 1. For (i) we must have $G(s) = G_1(s)G_2(s)$ with $G_1(s) = K_{G,1}$ and $G_2(0) = 1$ such that the steady state error e_0 to inputs $v(t) = \delta_1(t)$ satisfies

$$|e_0| = |W_e(0)| = \frac{1}{1 + P(0)K_{G,1}}| \leq 1 \Rightarrow |K_{G,1}| \geq 2$$

Set $K_{G,1} = 2$. For maximizing the phase margin since $G(s)$ must be one-dimensional and since the phase of $G_1(s)P(s)$ is $-180°$ for all ω (see Bode plot of $G_1(s)P(s)$ in Fig. 2), we use an anticipative action $R_a(s)$ with $m_a = 16$ at $\omega_t^* = 4$ rad/sec and $\omega_N = 4$ rad/sec (maximum phase increase $=\approx 62°$ with magnitude increase $\approx 12dB$. Notice that in this way we maximize the phase margin, having at hand only a one-dimensional $G(s)$, with a certain crossover frequency ω_t^*. Notice that $\omega_t^* = 4$ rad/sec has been chosen on account of the fact that the magnitude of $|G_1P(j\omega_t^*)|_{dB} \leq -12dB$ which is the magnitude increase given by the anticipative action at ω_N but $|G_1P(j\omega_t^*)|_{dB} \geq -24dB$ where 24 dB is the maximum magnitude increase which can be given by a single anticipative action ($m_a = 16$). We get $\tau_a = 1$ and

$$G_2(s) = K_{G,2}R_a(s) = K_{G,2}\frac{1 + s}{1 + \frac{s}{16}}$$

where $K_{G,2}$ is to be selected. In order to place the crossover frequency exactly at $\omega_t^* = 5$ rad/sec, notice that $|G_1P(j\omega_t^*)|_{dB} \approx -18dB$ and $|G_1R_aP(j\omega_t^*)|_{dB} \approx -5dB$ ($R - a$ provides a magnitude increase $\approx 12dB$). Therefore, we choose $K_{G,2} = 6dB \approx 2$ and finally the controller $G(s)$ is

$$G(s) = 4\frac{1 + s}{1 + \frac{s}{16}}$$

The Bode plots of $G(s)P(s)$ and its Nyquist plot are drawn in Fig. 3 and 4. The Nyquist plot shows that the closed-loop system is asymptotically stable (we have 1 counterclockwise tour around the point $-1 + 0j$ and the number of poles with positive real part of $G(s)P(s)$ is 1).

Exercise 2 We have for the output w with inputs d and m in Laplace domain

$$w(s) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 \\ 0 & s+2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} m(s) + \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 \\ 0 & s+2 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} d(s) = \frac{1}{s}m(s)$$

For the output $y(s)$ we have with inputs v and d

$$y(s) = \frac{G(s)P_2(s)\frac{w(s)}{m(s)}}{1 + G(s)P_2(s)\frac{w(s)}{m(s)}}v(s) = \frac{G(s)F(s)}{1 + G(s)F(s)}v(s)$$

where $F(s) = \frac{s-1}{s(s+1)}$. Therefore, y is not influenced by d and (ii) is trivially satisfied. Next, design $G(s)$ so that to satisfy (i). It is possible to satisfy (i) and (iii) at the same time.

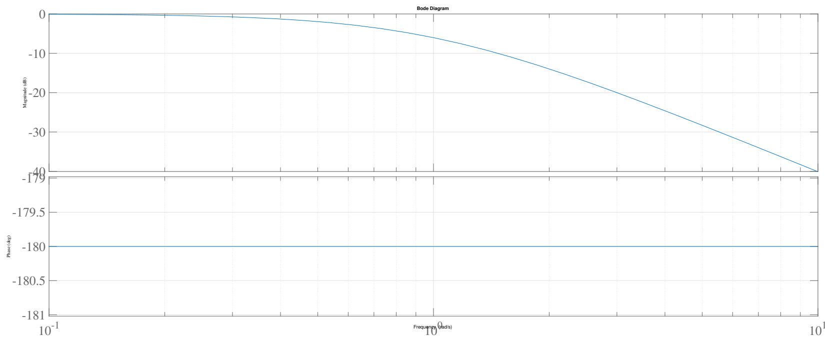

Figure 1: Bode plots of $P(s)$

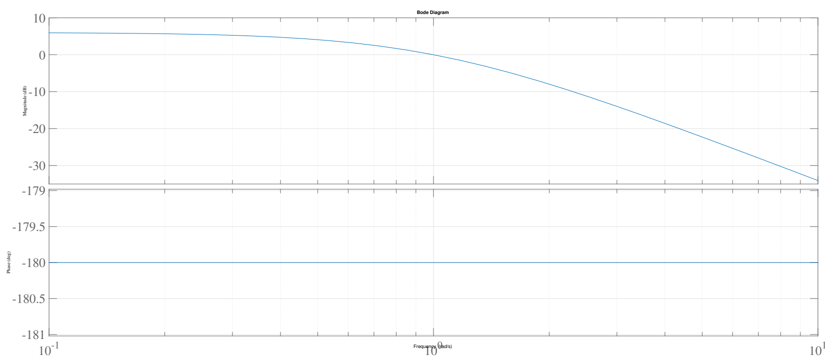

Figure 2: Bode plots of $G_1(s)P(s)$

Consider $G(s)$ of the form

$$G(s) = K\frac{s+z}{s+p}$$

By comparison, requiring that the closed-loop poles are all in -2,

$$\begin{aligned}(s+2)^3 = s^3+6s^2+12s+8 = NUM(1+G(s)F(s)) &= (s+p)(s^2+s)+K[s^2+(z-1)s-z] \\ &= s^3+(p+1+K)s^2+(K(z-1)+p)s-zK\end{aligned}$$

we get $zK=-8$, $p+1+K=6$, $z-1+p=12 \Rightarrow z=4+\sqrt{8}$, $p=5+8/(4+\sqrt{8})$ and $K=-8/(4+\sqrt{8})$.

The root locus of $G(s)F(s)$ is drawn is Figs. 5 and 6.

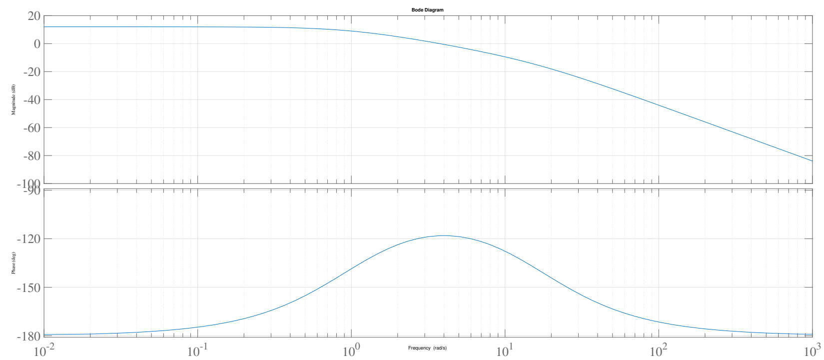

Figure 3: Bode plots of $G(s)P(s)$

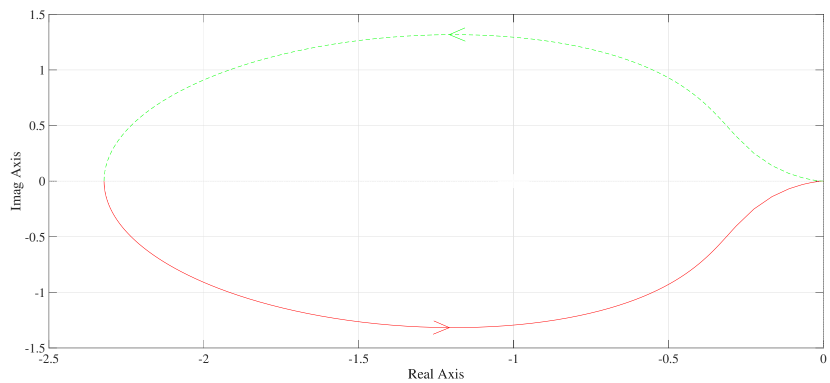

Figure 4: Nyquist plot of $G(s)P(s)$

Exercise 3. We have for the transfer functions P_1 and P_2:

$$P_1(s) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & 10 \\ -1 & s+11 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{s+2}{s^2+11s+10}$$

$$P_2(s) = \frac{1}{s+2}$$

Moreover, the output $Y(s)$ in Laplace domain is

$$Y(s) = \frac{(1+K(s)G(s)P_1(s))P_2(s)}{1+L(s)} d(s) + \frac{L(s)}{1+L(s)} v(s) \tag{1}$$

where $L(s) = G(s)P_1(s)P_2(s)$. The point is to design $G(s)$ and $K(s)$ as required with the constraint that they must be realizable (number of poles greater or equal to the number of zeroes).

(i) Design $G(s)$ first. The closed-loop input-output transfer function is $W(s) = \frac{L)(s)}{1+L(s)}$ (see

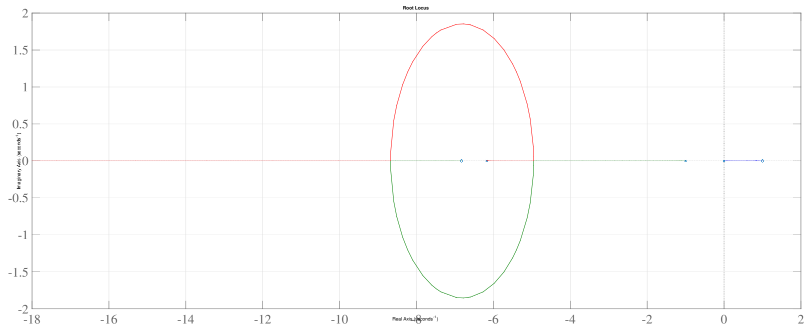

Figure 5: Positive root locus of $G(s)F(s)$

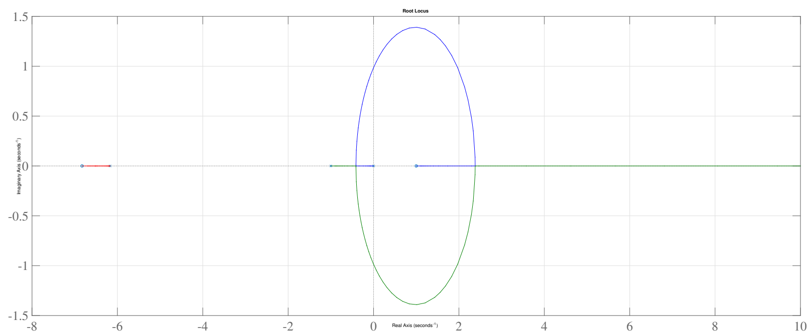

Figure 6: Negative root locus of $G(s)F(s)$

(1)). We choose for $W(s)$ a form

$$W(s) = \frac{2}{(s+1-j)(s+1+j)} = \frac{2}{s^2+2s+2}$$

(two closed-loop poles at $-1 \pm j$). Since

$$G(s)P_1(s)P_2(s) = L(s) = \frac{W(s)}{1-W(s)} = \frac{2}{s^2+2s} := F(s)$$

then

$$G(s) = \frac{F(s)}{P_1(s)P_2(s)} = 2\frac{s^2+11s+10}{s^2+2s}$$

(ii) The closed-loop disturbance-output transfer function is $W_d(s) = \frac{(1+K(s)G(s)P_1(s))P_2(s)}{1+L(s)}$ (see (1)). We design $K(s)$ in such a way that

$$|W_d(j\omega)| \leq 0.1, \forall \omega \in [0, 10] rad/sec. \tag{2}$$

Since

$$K(s) = \frac{(1 + L(s))W_d(s) - P_2(s)}{L(s)} = \frac{1}{2}\Big[(s^2 + 2s + 2)W_d(s) - s\Big]$$

choose

$$W_d(s) = \frac{1}{s + 10}$$

Clearly, $W_d(s)$ satisfies (2) since

$$|W_d(j\omega)| = \frac{1}{\sqrt{100 + \omega^2}} \leq 0.1$$

for all $\omega \in [0, 10]$ rad/sec and, moreover,

$$K(s) = \frac{1}{2}\left[\frac{2 - 8s}{s + 10}\right]$$