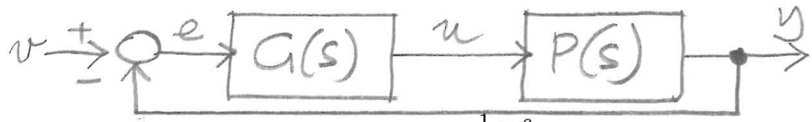
CONTROL SYSTEMS - 4/6/2020

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

1) Consider



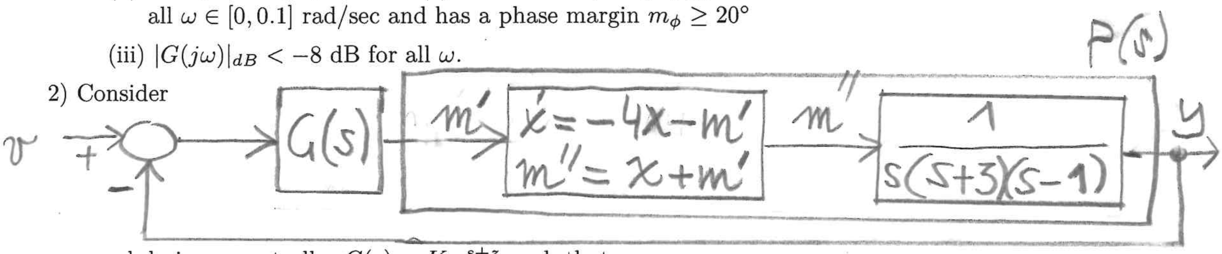
with input v, error e, output y and process $P(s) = \dfrac{1-s}{s^2}$. Design a one dimensional controller $G(s)$ in such a way that

(i) the closed-loop system is asymptotically stable (use Nyquist criterion with approximate Bode plots)

(ii) the open loop system $PG(s)$ is such that $|PG(j\omega)|_{dB} \geq 15.6$ dB for all $\omega \in [0, 0.1]$ rad/sec and has a phase margin $m_\phi \geq 20°$

(iii) $|G(j\omega)|_{dB} < -8$ dB for all ω.

2) Consider



and design a controller $G(s) = K_G \frac{s+z}{s+p}$ such that

(i) the closed-loop system is asymptotically stable

(ii) the steady state error $e_{ss}(t)$ to inputs $v(t) = t$ satisfies $|e_{ss}(t)| \leq 0.1$.

Draw the root locus for $G(s)P(s)$ and determine the values of K in the locus for which the poles of $\frac{P(s)KG(s)}{1+P(s)KG(s)}$ are in $\mathbb{C}^-$.

3) Given the system $\dot{x} = Ax + Bu$, $y = Cx$, where

$$A = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 \end{pmatrix} \tag{1}$$

decompose the system into observable and unobservable subsystems, discuss the stability of these subsystems and compute the state response $x(t)$ with $u(t) \equiv 0$ and $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.