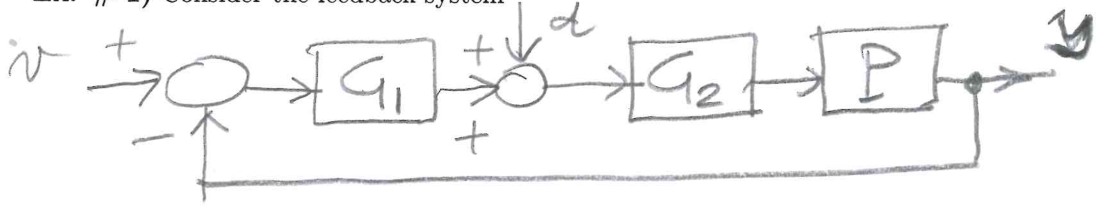
CONTROL SYSTEMS - 22/7/2022

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

Ex. # 1) Consider the feedback system



with $\mathbf{P}(s) = \frac{s+1}{s^2}$. Design controllers $\mathbf{G}_1(s)$ and $\mathbf{G}_2(s)$ such that

(i) $\mathbf{G}_1(s)$ has minimal dimension and $|\mathbf{G}_2(j\omega)|_{dB} \leqslant 36dB$ for all $\omega \geqslant 0$,

(ii) the closed-loop system is asymptotically stable (check with Nyquist criterion) with steady-state error response $\mathbf{e}_{ss}(t) = 0$ to ramp inputs $\mathbf{v}(t) = t$ and steady-state output response $\mathbf{y}_{ss}(t) = 0$ to constant disturbances $\mathbf{d}(t)$,

(ii) the open loop system $\mathbf{PG}_1\mathbf{G}_2(s)$ has crossover frequency $\omega_t^* \geqslant 5$ rad/sec and phase margin $m_\phi^* \geqslant 30°$

Ex. # 2) Given the plant $\mathbf{P}(s) = \frac{s+5}{(s^2+1)(s-1)}$:

(i) draw the root locus using the Routh criterion to determine the exact picture on the imaginary axis

(ii) determine a controller $G(s)$ with dimension 2 such that the feedback system $W(s) = \frac{PG(s)}{1+PG(s)}$ has zero steady state error to constant inputs and it is asymptotically stable with poles having negative real part $\leqslant -3$.

Ex. # 3) Given the system $\dot{x} = Ax + Bu$, $y = Cx$, where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \end{pmatrix}, \qquad (1)$$

and $x(t, x_0, u)$ the state solution with input u and initial condition x_0 and $y(t, x_0, u)$ the corresponding output, determine

(i) if there exists $t_f > 0$ and $x_0 \in \mathbb{R}^2$ such that $x(t_f, x_0) = 2x_0$

(ii) the set $\mathcal{X}$ of initial states $x_0 \in \mathbb{R}^2$ such that there exists a control input u which steers x_0 into the final state $x_f = (2\ 2)^\top$, i.e. $x(t_f, x_0, u) = x_f$ for some $t_f > 0$. If $\mathcal{X}$ is non empty, pick any such $x_0 \in \mathcal{X}$ and determine a control u and $t_f > 0$ for which $x(t_f, x_0, u) = x_f$.

(iii) the set of initial states $x_0 \in \mathbb{R}^2$ such that $y(t, x_0, u) = 0$ for all $t \geqslant 0$ and any control u.