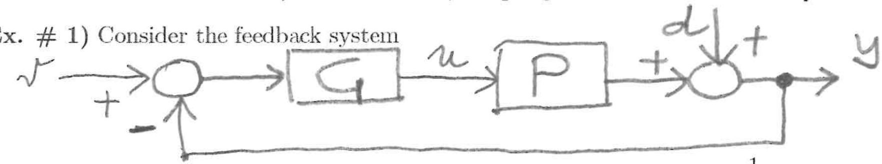
CONTROL SYSTEMS - 1/9/2020

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

Ex. # 1) Consider the feedback system



with input v, output y, disturbance d and controlled process $P(s) = \frac{1}{s^3}$. Design a 2-dimensional controller $G(s)$ such that

(i) the closed-loop system is asymptotically stable (use Nyquist criterion with approximate Bode plots)

(ii) the open loop system $PG(s)$ has largest as possible phase margin and $|PG(j\omega)|_{dB} \geqslant 20$ dB for all $\omega \in [0, 0.1]$ rad/sec,

(iii) $|G(j\omega)|_{dB} \leqslant 0$ dB for all ω.

Ex. # 2) Consider the feedback system of **Ex.#1** with controlled process

$$P : \dot{x} = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u, \ y = \begin{pmatrix} -1 & -2 \end{pmatrix} x \tag{1}$$

Design a controller $G(s)$ such that

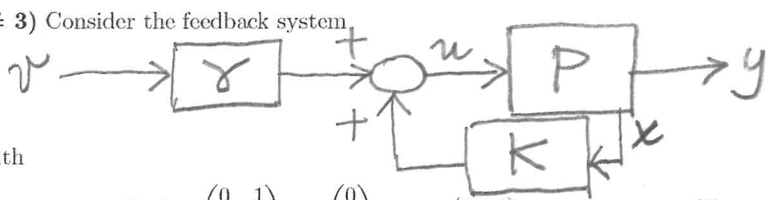
(i) the closed-loop system is asymptotically stable with steady state output response $y_{ss}(t) \equiv 0$ to constant disturbances $d(t)$ and cosinusoidal disturbances $d(t) = \cos(t)$,

(ii) the closed-loop eigenvalues have real part $\leqslant -0.3$,

(iii) $G(s)$ has minimal dimension.

Draw as precisely as possible the root locus of $PG(s)$.

Ex. # 3) Consider the feedback system



with

$$P : \dot{x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \ y = \begin{pmatrix} 1 & 0 \end{pmatrix} x \tag{2}$$

Find $\gamma > 0$ and $K \in \mathbb{R}^{1\times 2}$ such that

(i) the closed-loop system is asymptotically stable with steady state output response $y_{ss}(t) = \delta_{-1}(t)$ to inputs $v(t) = \delta_{-1}(t)$,

(ii) the closed-loop eigenvalues are all equal to -2.