Exercise 1  Given the following \( \text{ALC} \) TBox:

\[
\begin{align*}
F & \sqsubseteq D \sqcup E \\
E & \sqsubseteq \exists r . C \\
D & \sqsubseteq \forall r . B \\
D \cap A & \sqsubseteq \neg E \\
C & \sqsubseteq A \\
B & \sqsubseteq \neg C
\end{align*}
\]

(a) tell whether the TBox \( T \) is satisfiable, and if so, show a model for \( T \);
(b) tell whether the concept \( D \) is satisfiable with respect to \( T \), and if so, show a model for \( T \) where \( D \) is satisfiable;
(c) tell whether the concept \( D \cap E \) is satisfiable with respect to \( T \), and if so, show a model for \( T \) where \( A \cap D \) is satisfiable;
(d) given the ABox \( A = \{ F(a), r(a, b) \} \), use the tableau method to establish whether the knowledge base \( \langle T, A \rangle \) entails the assertion \( B(b) \).

Exercise 2  Given the following ASP program \( P \):

\[
\begin{align*}
\text{r}(X,Y) :& \leftarrow \text{q}(X,Y). \\
\text{r}(X,Y) :& \leftarrow \text{q}(X,Z), \text{r}(Z,Y). \\
\text{s}(X,Y) :& \leftarrow \text{p}(X), \text{p}(Y), \text{r}(X,Y). \\
\text{t}(X,Y) :& \leftarrow \text{r}(X,Y), \neg \text{r}(X,X). \\
\text{v}(X,Y) :& \leftarrow \text{p}(X), \text{p}(Y), \text{t}(X,Y). \\
\text{v}(X,Y) :& \leftarrow \text{p}(X), \text{p}(Y), \neg \text{s}(X,Y). \\
\text{v}(X,Y) :& \leftarrow \text{q}(X,Y), \neg \text{t}(X,Y). \\
\text{p}(a). \text{p}(c). \text{p}(d). \\
\text{q}(a,b). \text{q}(b,c). \text{q}(c,a). \text{q}(c,d).
\end{align*}
\]

(a) tell whether \( P \) is stratified;
(b) compute the answer sets of \( P \).

Exercise 3

We want to formalize knowledge about persons and kinship relationships. In particular, we want to formalize the following statements:

1. every woman is a person;
2. every man is a person;
3. for every \( x, y, z \), if \( x \) has child \( y \) and \( x \) is a woman, then \( y \) has mother \( x \);
4. for every \( x, y, z \), if \( x \) has child \( y \) and \( x \) is a man, then \( y \) has father \( x \);
5. for every \( x, y \), if \( x \) has mother \( y \), then \( x \) does not have father \( y \);
6. for every \( x, y \), if \( x \) has father \( y \), then \( x \) does not have mother \( y \);
7. for every \( x, y \), if \( x \) has mother \( y \), then \( y \) does not have mother \( x \);
8. for every \( x, y \), if \( x \) has father \( y \), then \( y \) does not have father \( x \);
9. for every \( x, y, z \), if \( x \) has mother \( y \) and \( y \) has mother \( z \), then \( x \) has grandmother \( z \).

(a) Choose the most appropriate knowledge representation language for expressing the above knowledge among the following ones: \( \text{ALC} \), Datalog, Datalog with constraints, ASP, OWL, DL-Lite, \( \mathcal{EL} \), RL, RDFS, motivating your choice;
(b) express the above knowledge in the formalism chosen at the previous point.

Exercise 4

(a) Write an RDF/RDFS model representing the following statements about URIs Person, HasParent, HasMother, HasFather, Man, Woman, City, livesIn, Ann, Bob, Jane, Mary, Paul, Sandy, Rome, Milan,

1. Person, Man, Woman, and City are classes;
2. Man and Woman are subclasses of Person;
3. HasParent, HasMother, HasFather, livesIn, are properties;
4. IsMother and HasFather are subproperties of HasParent;
5. HasParent has domain Person and range Person; 
6. HasMother has domain Person and range Woman; 
7. HasFather has domain Person and range Man; 
8. livesIn has domain Person and range City; 
9. Jane is a woman; 
10. Jane has father Bob; 
11. Paul is the son of Ann; 
12. Mary and Bob are the children of Paul and Sandy; 

(b) Write SPARQL queries corresponding to the following requests: (b1) return all the pairs of siblings (i.e., the pairs of persons who have the same parents); (b2) return all the grandparents of Jane and, optionally, the city where they live; (b3) return the men who live in the cities where at least a grandchild of Paul lives.

Exercise 5
Given the RL knowledge base \(\langle T, A \rangle\), where \(T\) is the following TBox:
\[
\begin{align*}
C \cap D & \sqsubseteq G \\
E \cap D & \sqsubseteq H \\
s & \sqsubseteq r \\
t & \sqsubseteq r \\
r & \sqsubseteq u \\
\exists u. T & \sqsubseteq F \\
\exists u. F & \sqsubseteq D \\
\exists u. T & \sqsubseteq C \\
\exists s. T & \sqsubseteq E
\end{align*}
\]
and \(A\) is the following ABox:
\[
t(a_5, a_4), \quad s(a_5, a_3), \quad t(a_4, a_2), \quad s(a_4, a_1), \quad t(a_3, a_7), \quad s(a_2, a_6)
\]
1. compute the materialization of the ABox \(A\) with respect to the TBox \(T\);
2. tell whether the concept assertion \(G(a_7)\) is entailed by \(\langle T, A \rangle\);
3. write a Datalog program corresponding to the above TBox.