

Autonomous and Mobile Robotics

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Humanoid Robots 3: Balance

DIPARTIMENTO DI INGEGNERIA INFORMATICA
AUTOMATICA E GESTIONALE ANTONIO RUBERTI



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recap

- the Newton-Euler equations can be used to derive a relation between **CoM** and **ZMP**
- the ZMP represents the point of application of the **resultant ground reaction force**
- sufficient condition for balance: ZMP inside **support polygon**
- approximate model: **Linear Inverted Pendulum (LIP)**

$$\ddot{\mathbf{c}}^{x,y} = \frac{g^z}{c^z} (\mathbf{c}^{x,y} - \mathbf{z}^{x,y})$$

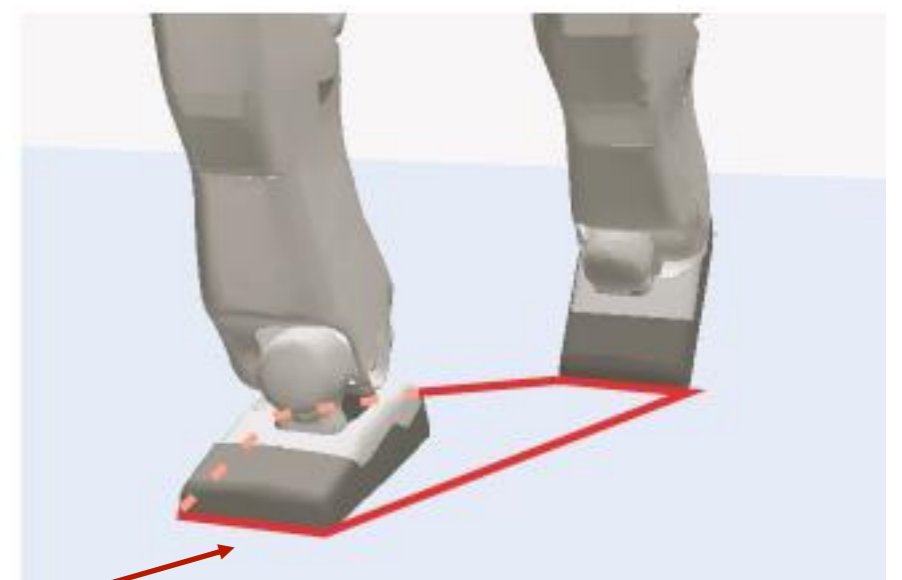
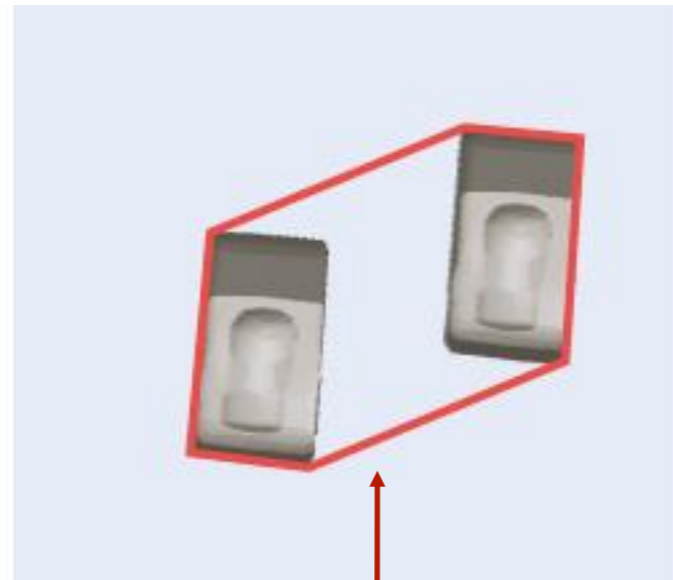
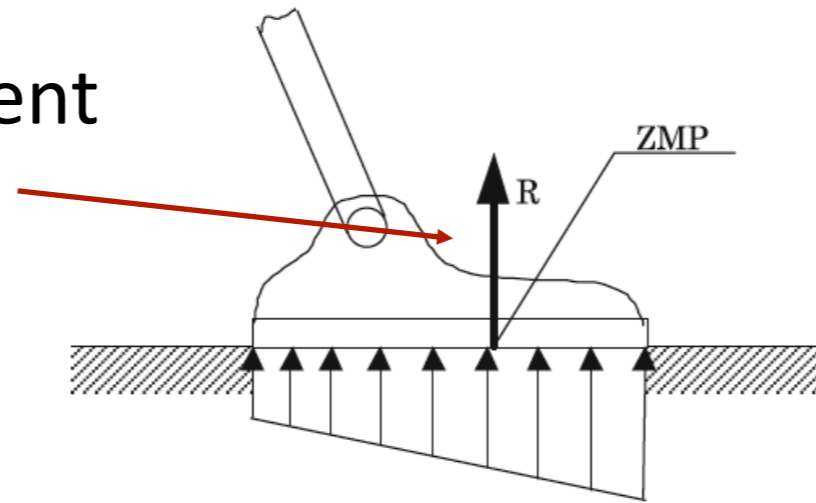
linear inverted pendulum: basic scope

$$\ddot{\mathbf{c}}^{x,y} = \frac{g^z}{c^z} (\mathbf{c}^{x,y} - \mathbf{z}^{x,y})$$

- although extremely simplified, the LIP equation describes in first approximation the time evolution of the CoM trajectory
- it defines a **differential relationship** between the CoM trajectory and the ZMP time evolution
- a suitable ZMP trajectory can be chosen such to satisfy **dynamic balance** by avoiding tilting
- the associated CoM trajectory can then be **tracked** with a complete robot model

ZMP

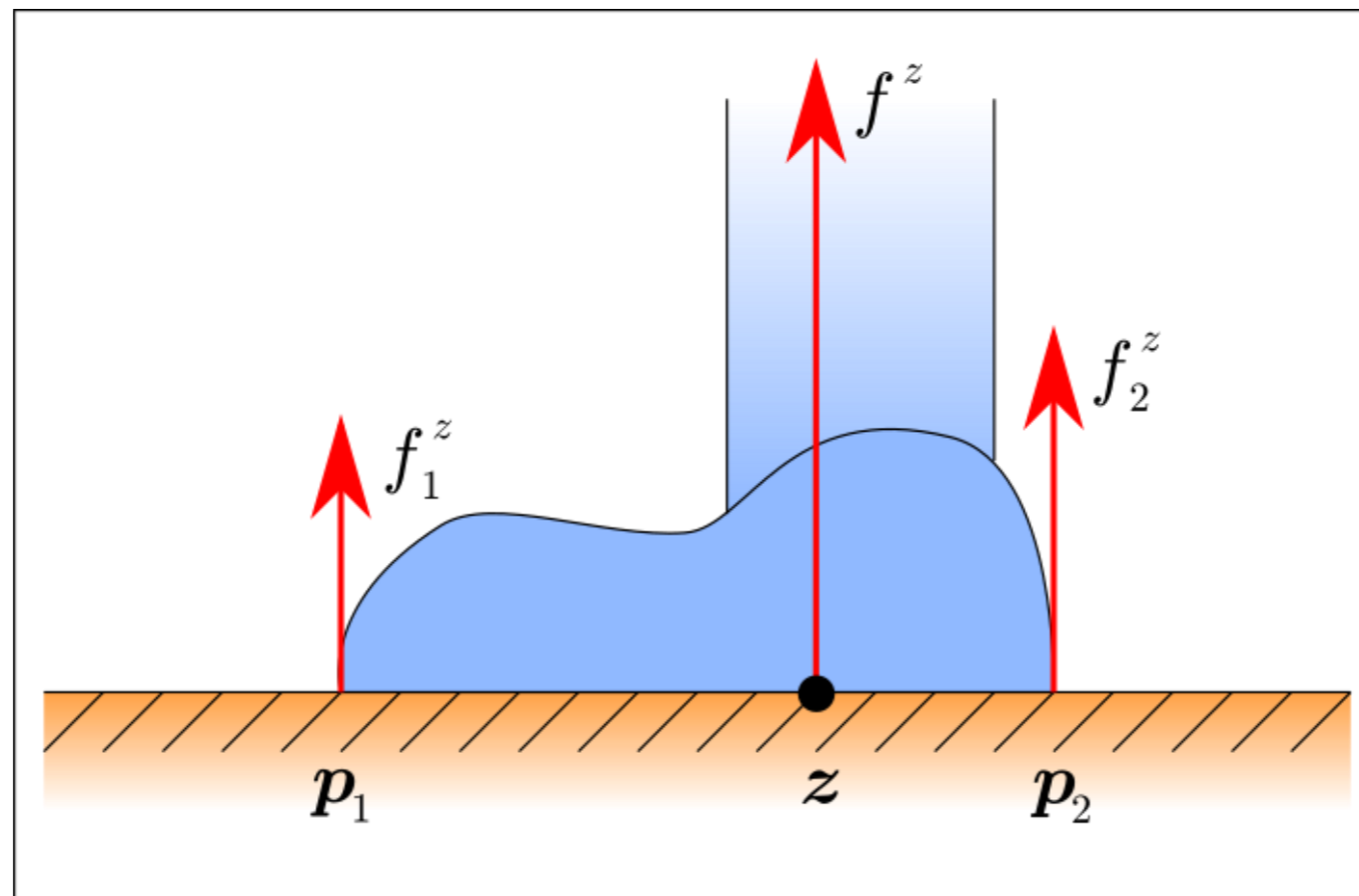
vertical component
of the GRF



Support Polygons during
Double Support

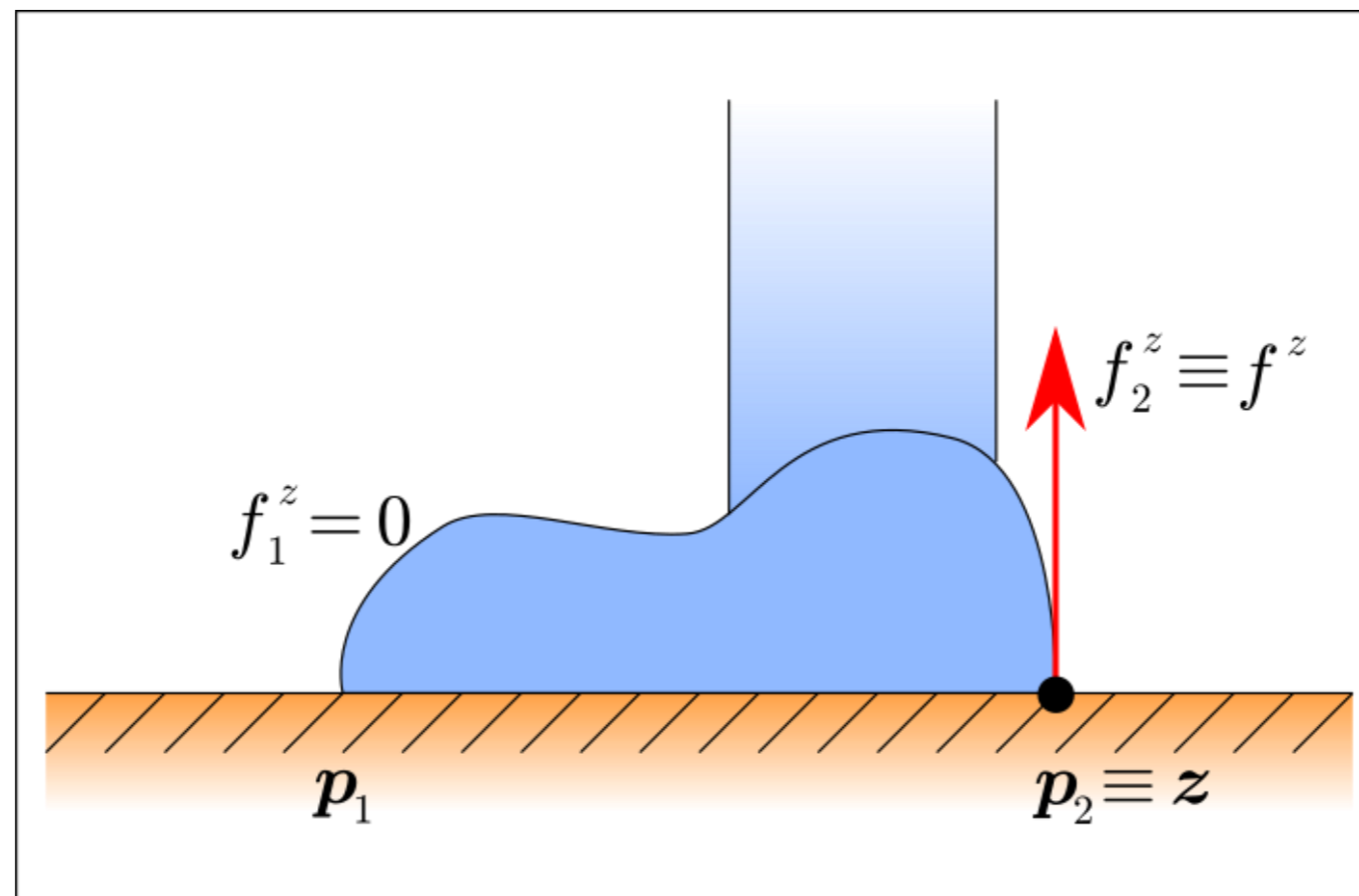
ZMP – simplified case

- let us consider a simplified example: two vertical ground reaction forces in the x - z plane, applied at p_1 and p_2
- the ZMP is somewhere in between p_1 and p_2 , shifted towards the larger of the two forces



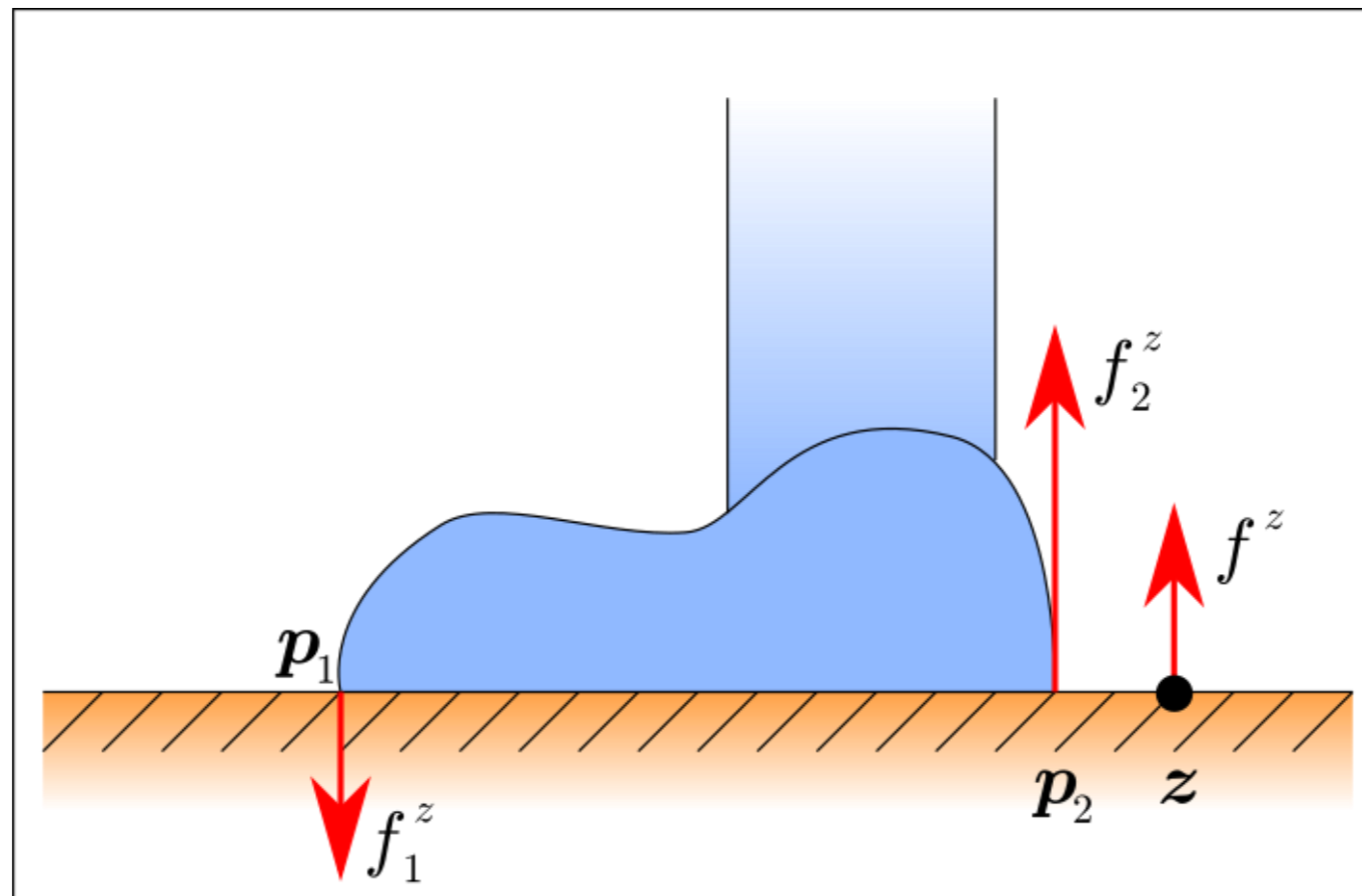
ZMP – simplified case

- if we reduce the magnitude of f_1^z , the resultant force moves towards p_2 , up until the point when f_1^z equals zero
- at this point the ZMP exactly coincides with p_2

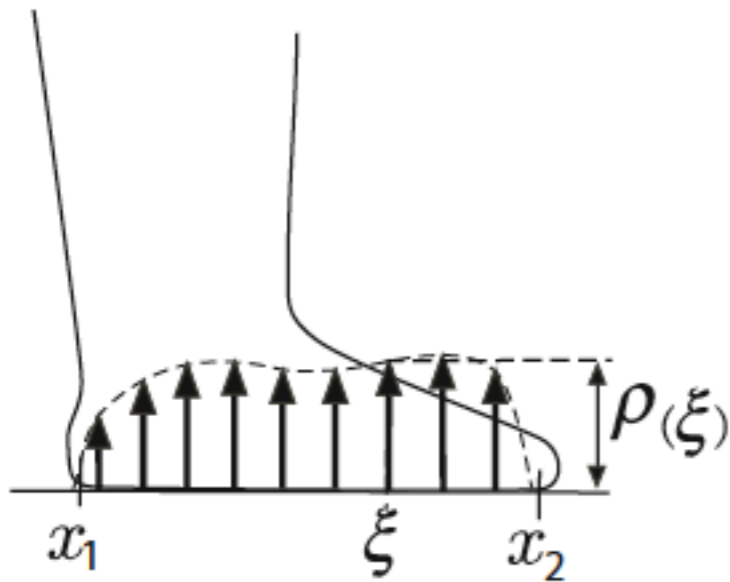


ZMP – simplified case

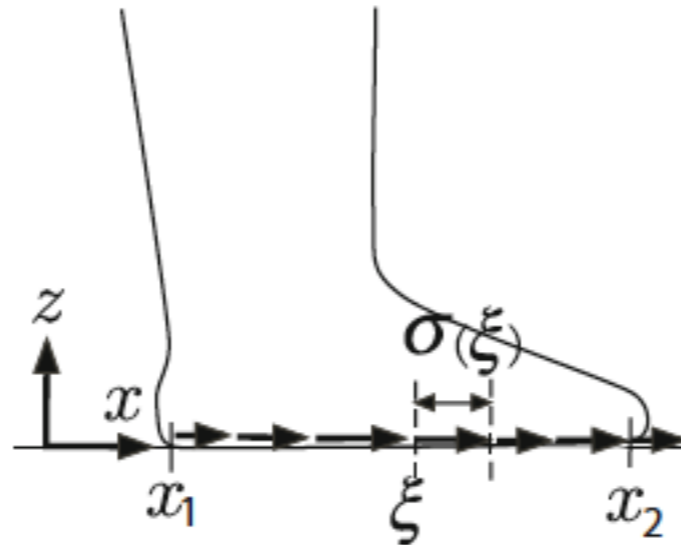
- imagine we could keep moving the ZMP to the right: it would end up outside the support polygon, requiring a negative f_1^z
- this is impossible because the vertical ground reaction force is **unilateral**: the ground can only push on the robot and not pull



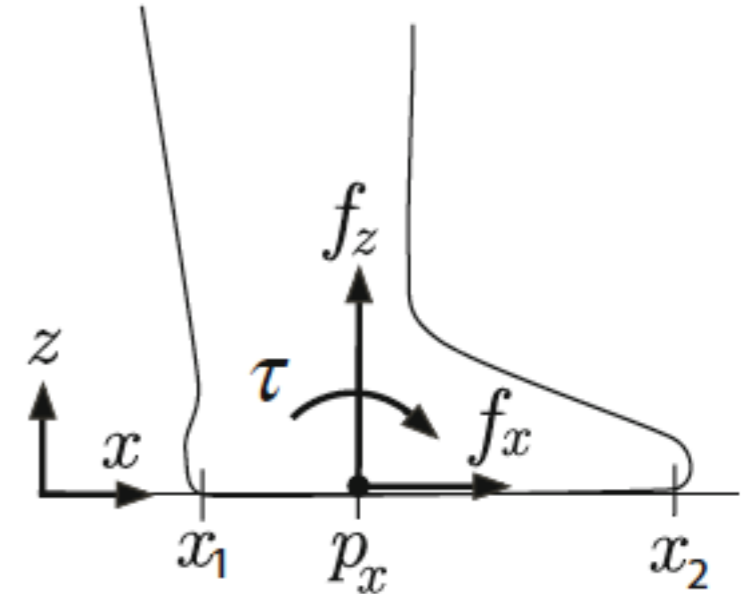
ZMP – 2D case



(a) Vertical force



(b) Horizontal force



horizontal force $f_x = \int_{x_1}^{x_2} \sigma(\xi) d\xi$

vertical force $f_z = \int_{x_1}^{x_2} \rho(\xi) d\xi$

torque $\tau(p_x) = - \int_{x_1}^{x_2} (\xi - p_x) \rho(\xi) d\xi$

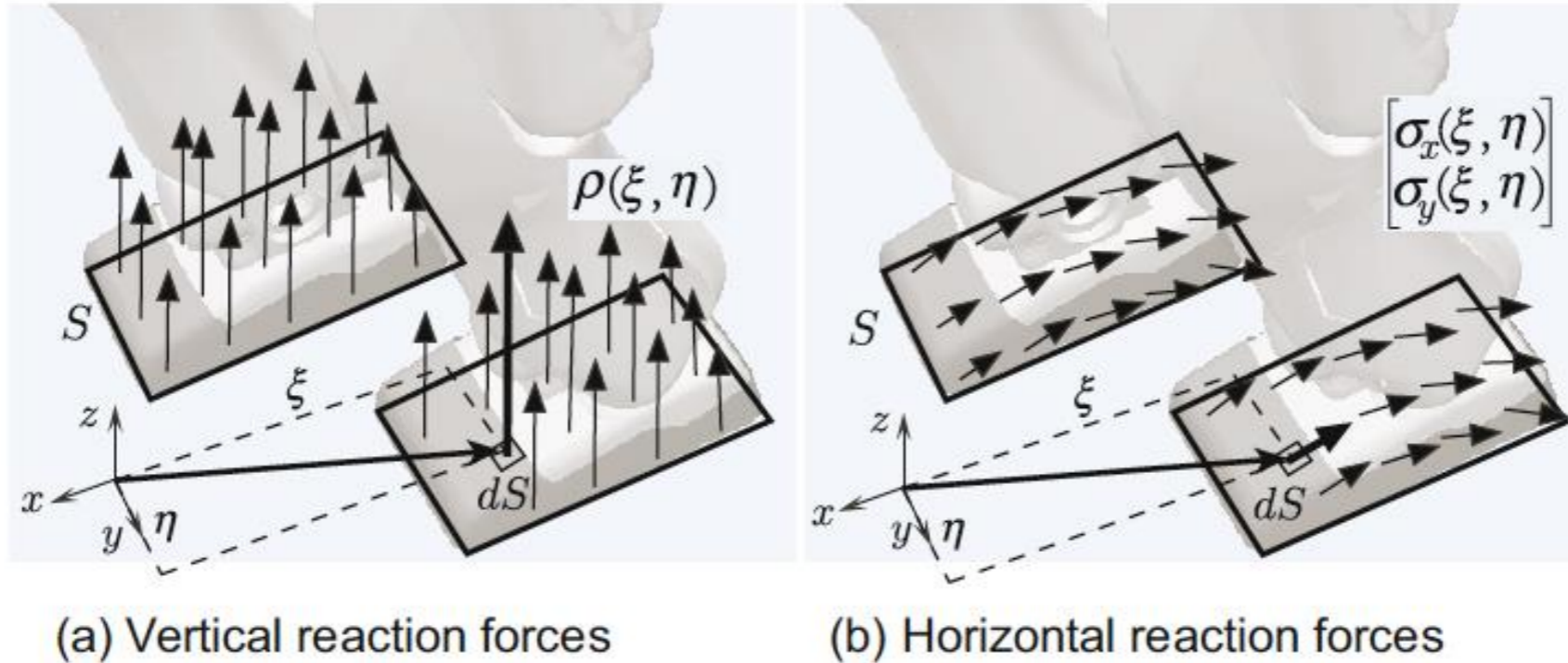
(computed wrt to a generic point p_x)

ZMP definition: $\tau(z_x) = 0$

ZMP

$$z_x = \frac{\int_{x_1}^{x_2} \xi \rho(\xi) d\xi}{f_z}$$

ZMP – 3D case



vertical component of the GRF

$$f_z = \int_S \rho(\xi, \eta) dS$$

$$\boldsymbol{\tau}_n(\mathbf{p}) \equiv [\tau_{nx} \ \tau_{ny} \ \tau_{nz}]^T$$

$$\tau_{nx} = \int_S (\eta - p_y) \rho(\xi, \eta) dS$$

$$\tau_{ny} = - \int_S (\xi - p_x) \rho(\xi, \eta) dS$$

$$\tau_{nz} = 0.$$

$$\tau_{nx} = 0$$

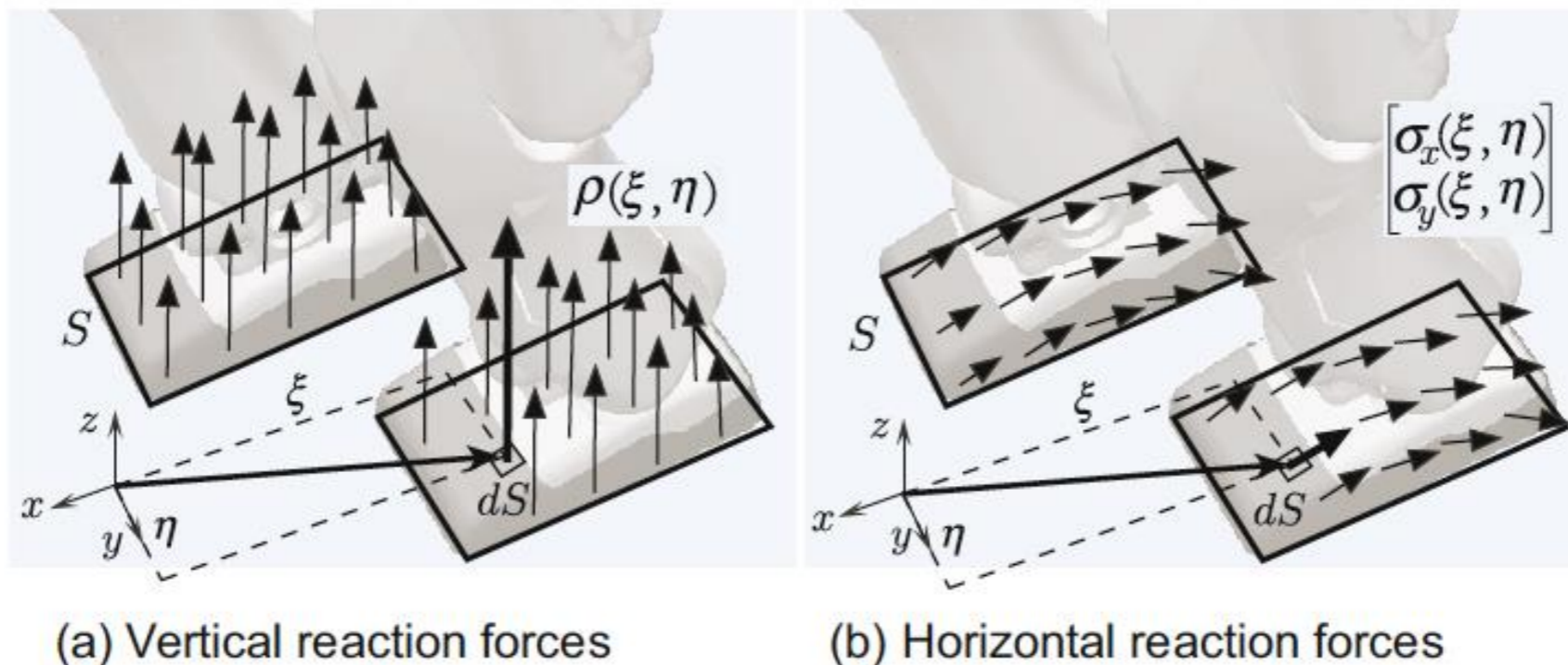
$$\tau_{ny} = 0$$

ZMP

$$p_x = \frac{\int_S \xi \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}$$

$$p_y = \frac{\int_S \eta \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}.$$

ZMP – 3D case



horizontal component of the GRF

$$f_x = \int_S \sigma_x(\xi, \eta) dS$$

$$f_y = \int_S \sigma_y(\xi, \eta) dS.$$

$$\boldsymbol{\tau}_t(\mathbf{p}) \equiv [\tau_{tx} \ \tau_{ty} \ \tau_{tz}]^T$$

$$\tau_{tx} = 0$$

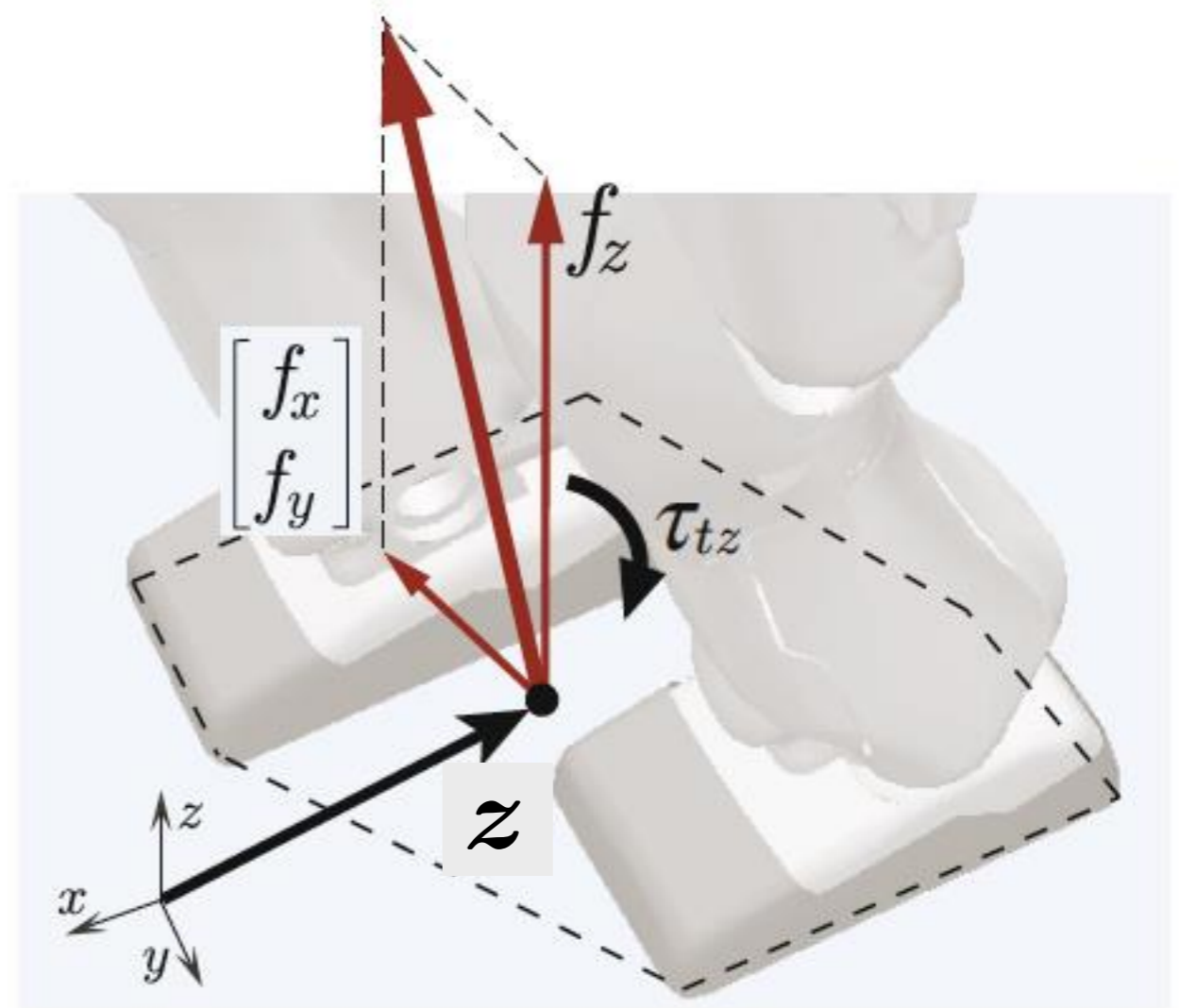
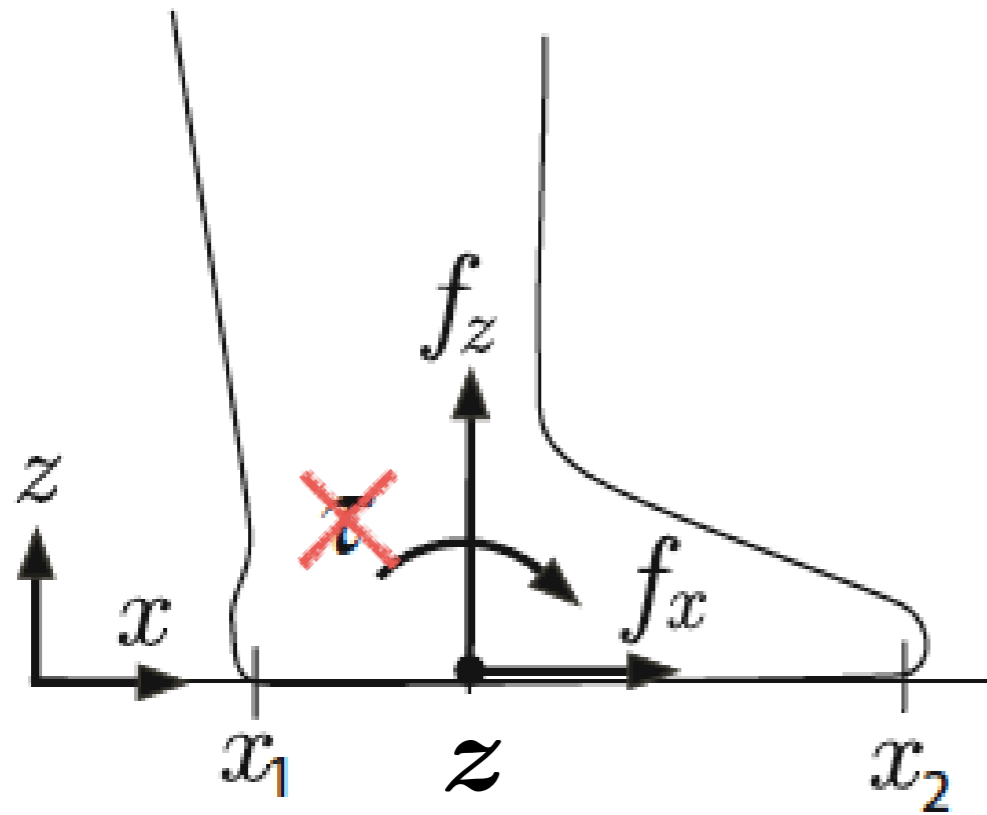
$$\tau_{ty} = 0$$

$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi, \eta) - (\eta - p_y)\sigma_x(\xi, \eta)\} dS$$

$$\begin{aligned} \boldsymbol{\tau}_p &= \boldsymbol{\tau}_n(\mathbf{p}) + \boldsymbol{\tau}_t(\mathbf{p}) \\ &= [0 \ 0 \ \tau_{tz}]^T, \end{aligned}$$

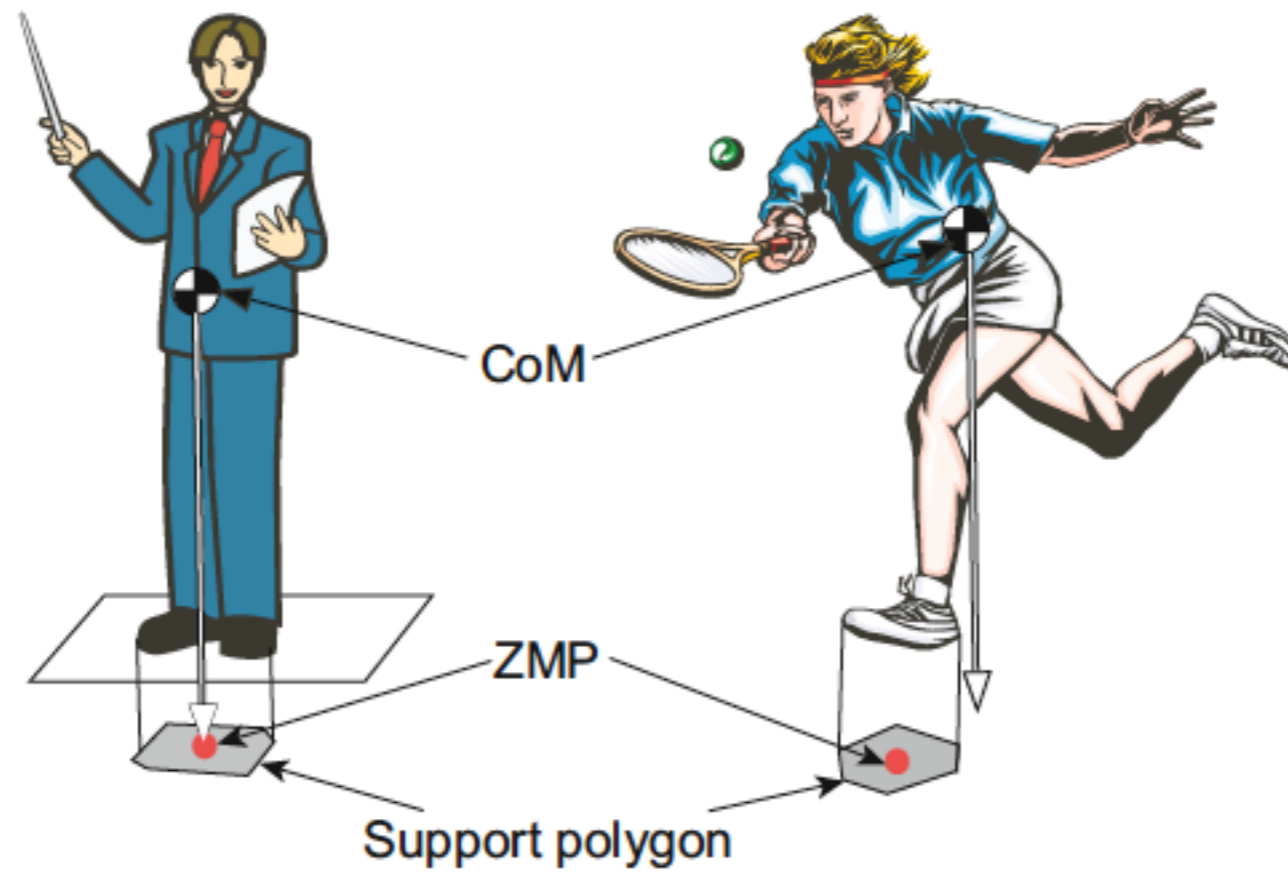
if robot moves, the z component will be different from 0

ZMP



as long as the ZMP is **in** the Support Polygon,
the support foot will **not** rotate

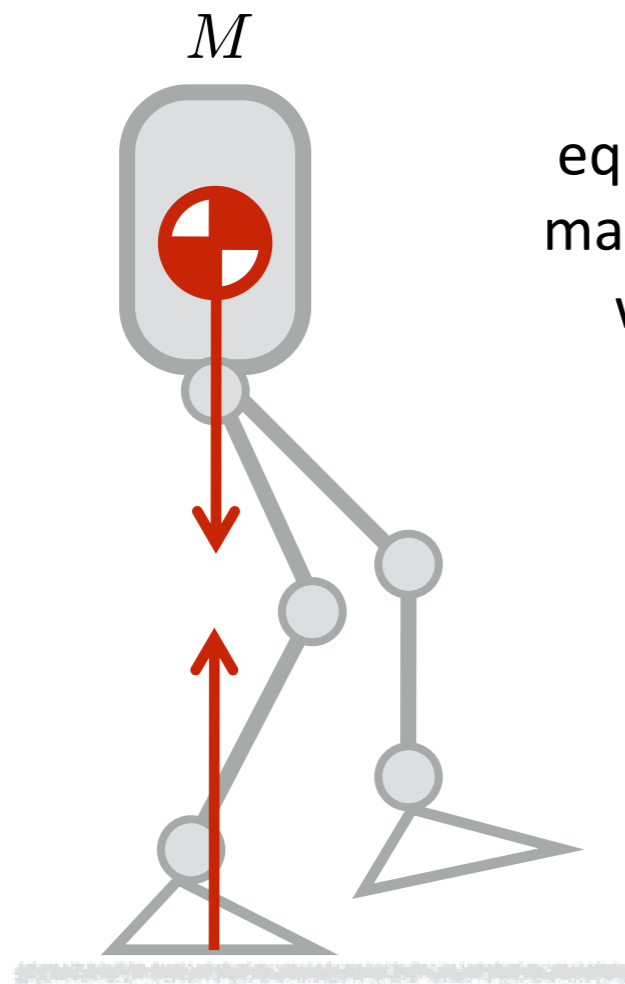
ZMP



static balance

humanoid motionless:

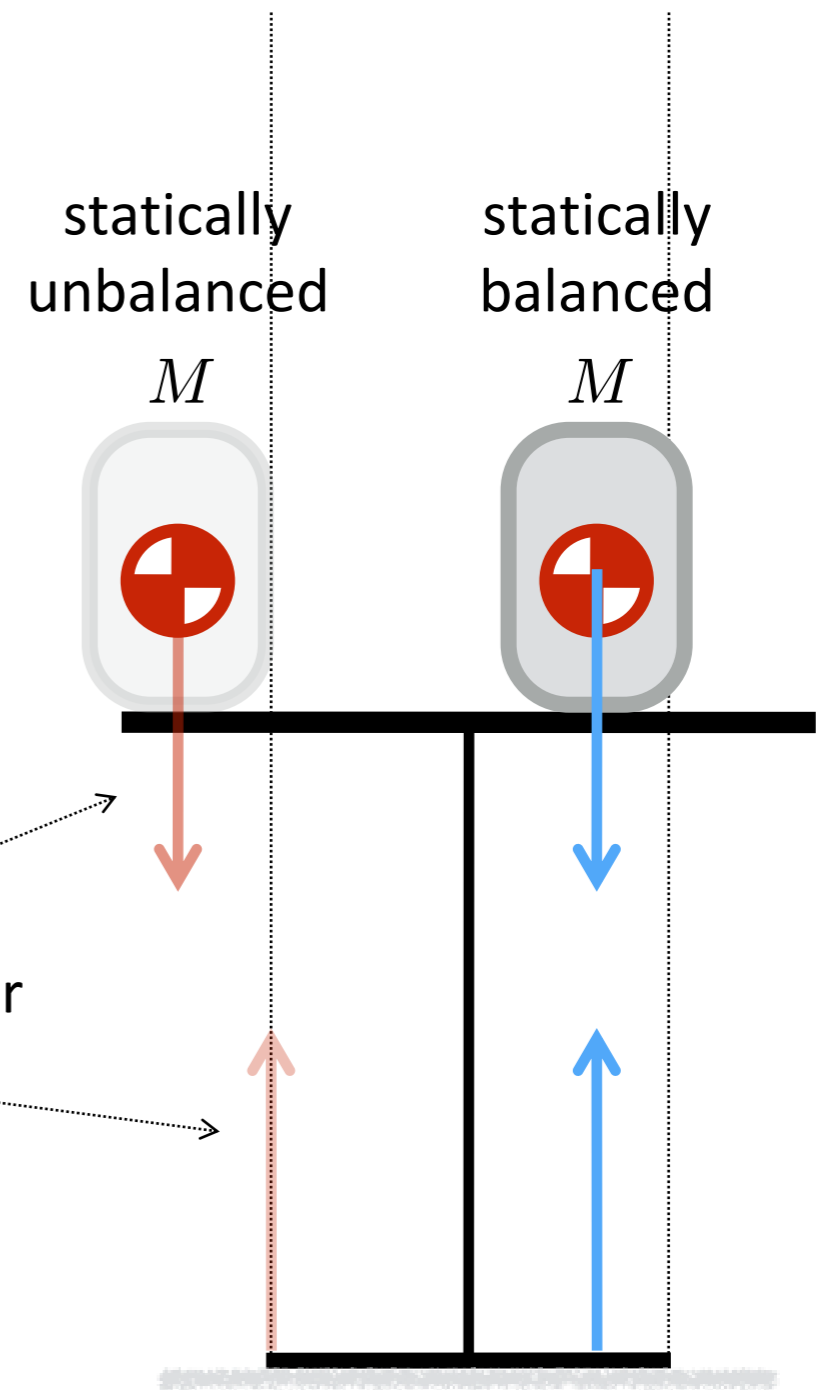
statically balanced robots keep the center of mass within the polygon of support in order to maintain postural stability (sufficient when the robot moves slow enough so all the inertial forces are negligible)



equivalent representation:
mass M on a massless table
with finite length base

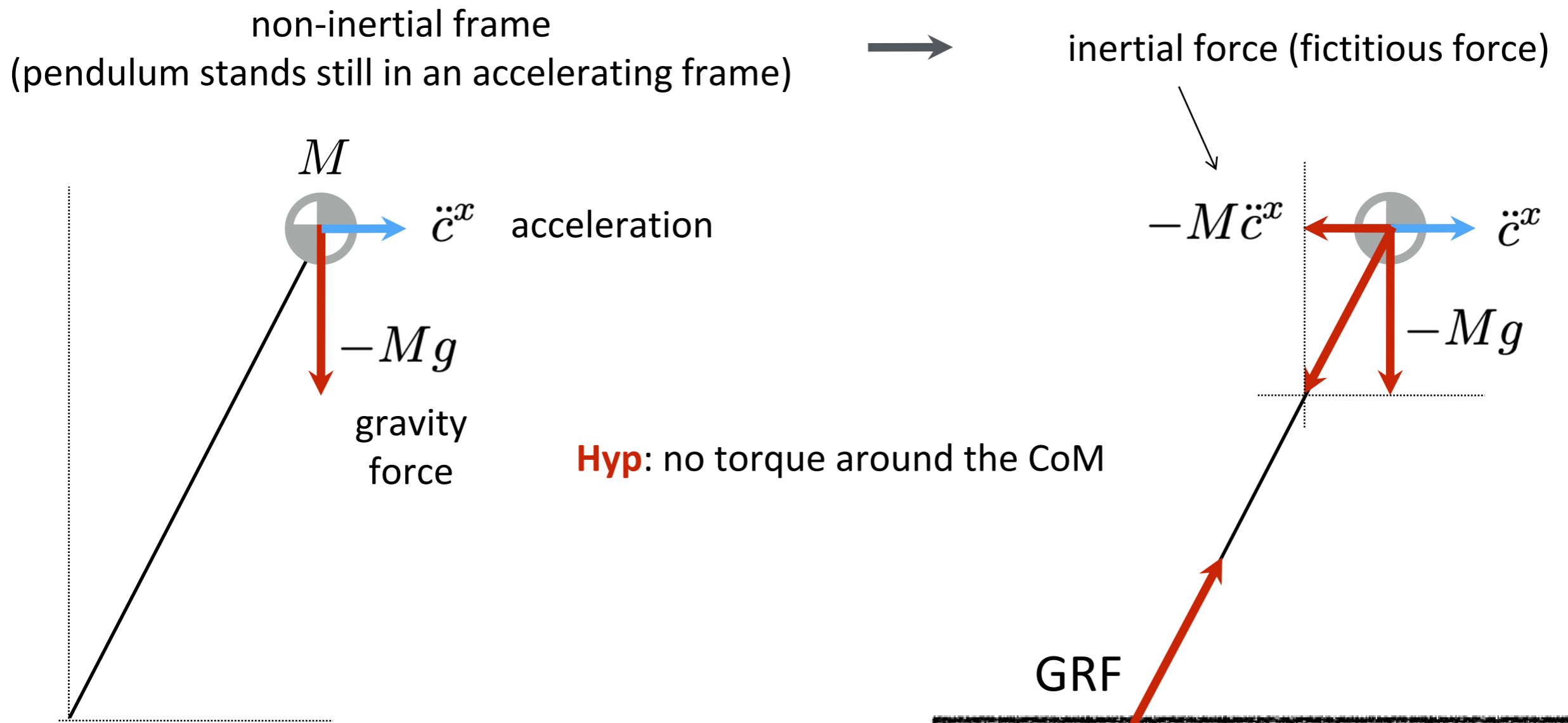
the table starts tipping over

if the CoM stays within these
boundaries no tipping over occurs



dynamic balance

- we can analyze dynamic balance in the same way by adding an **inertial force** (fictitious force in an accelerating frame)



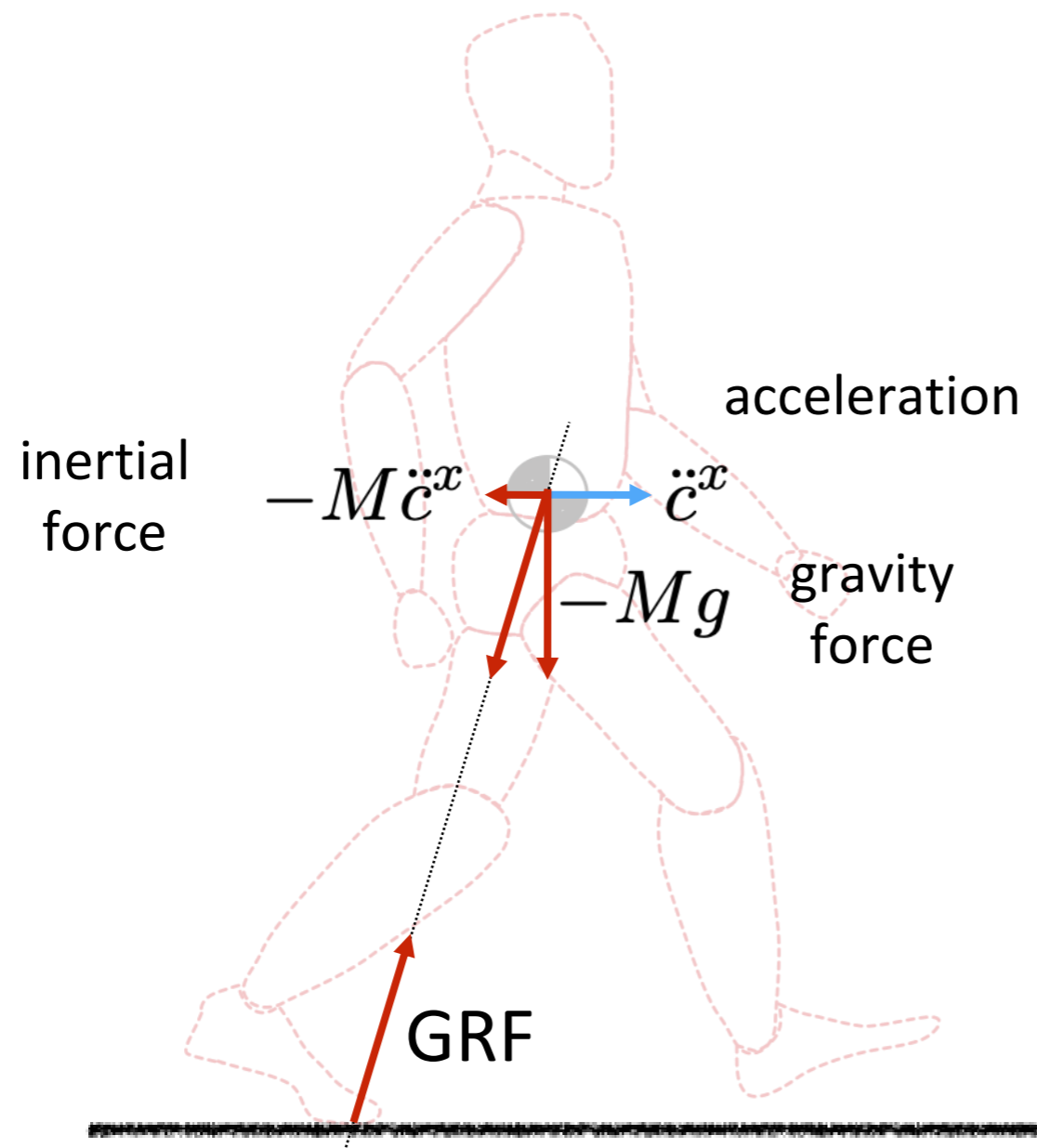
dynamic balance

humanoid walking:

the GRF will also have a component parallel to the ground;

the motion requires the exchange of horizontal frictional force with the ground

hyp: no torque around the CoM

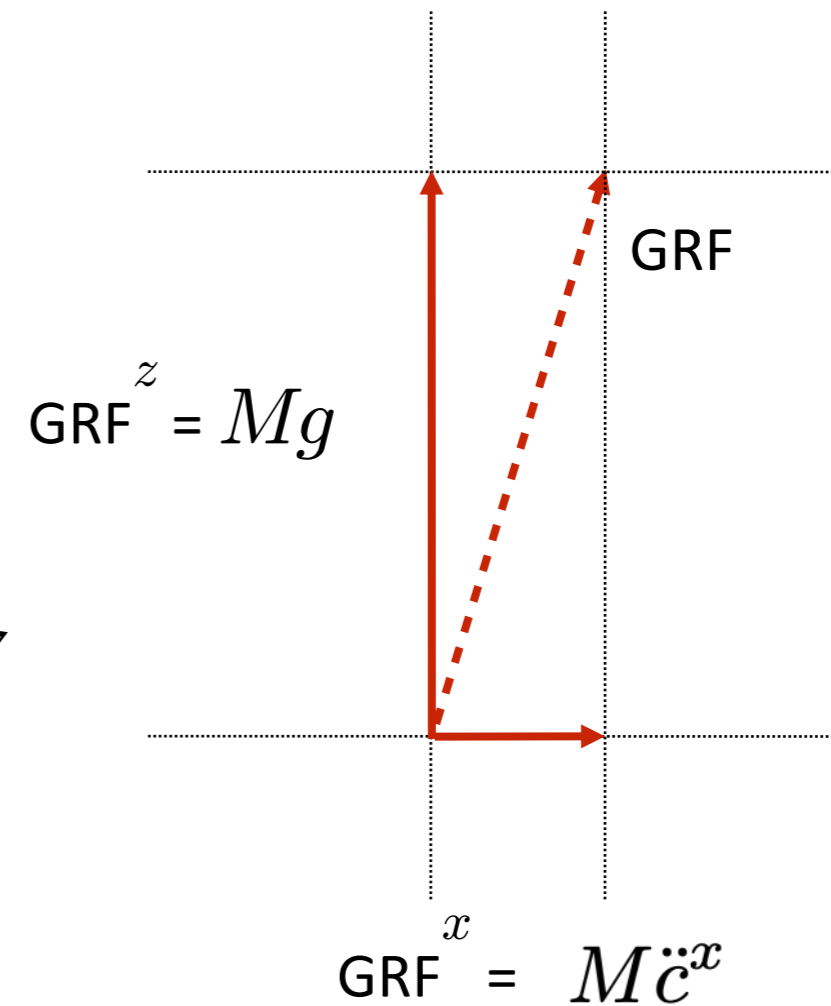
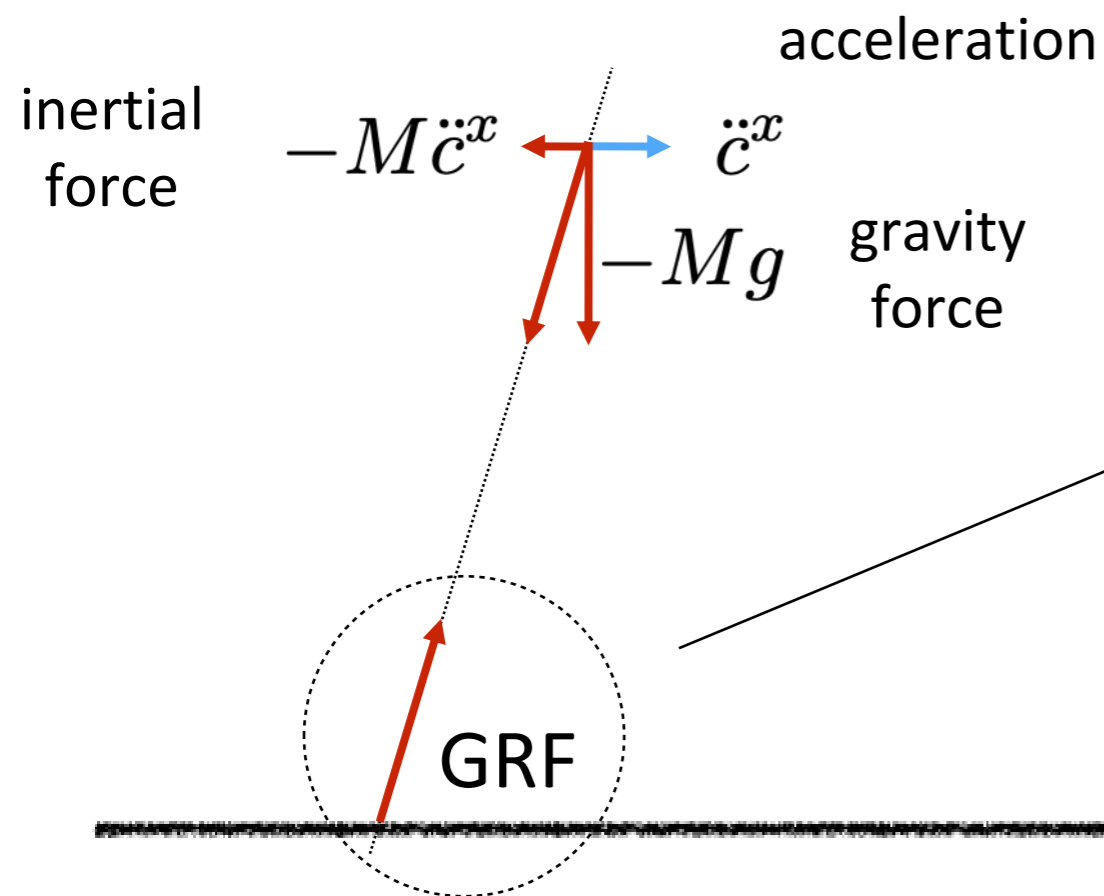


dynamic balance

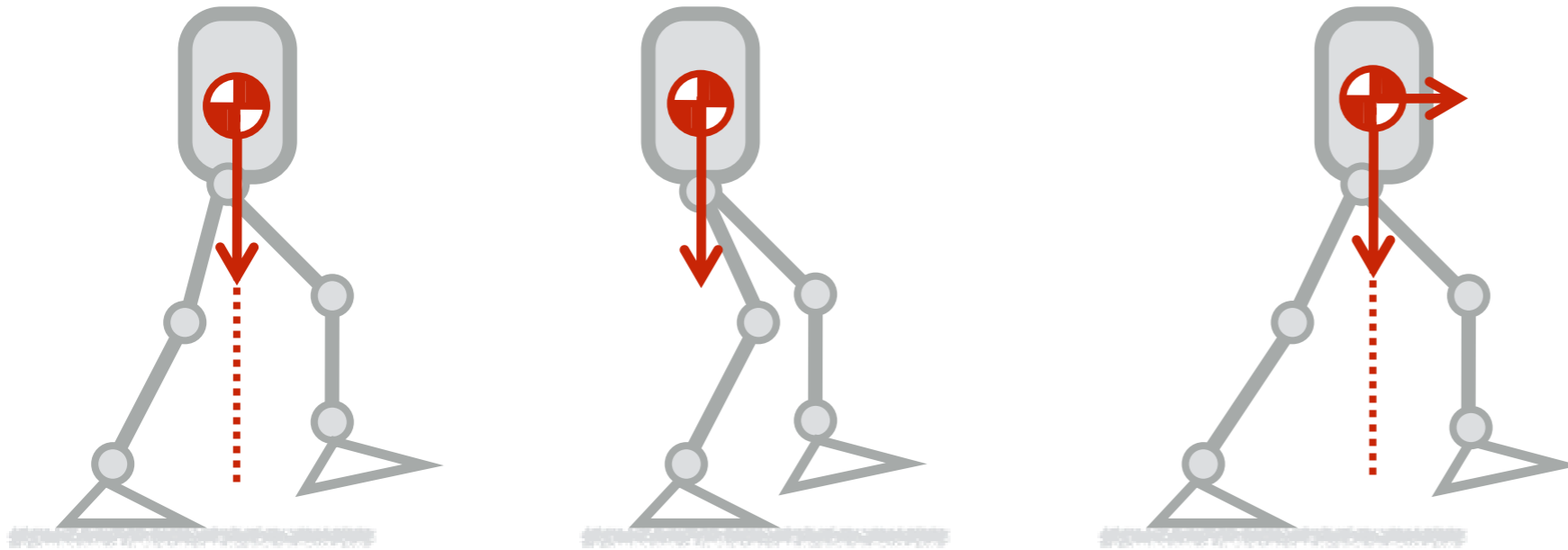
Ground Reaction Force (GRF): 2D components (x, z)

hyp: no torque around the CoM

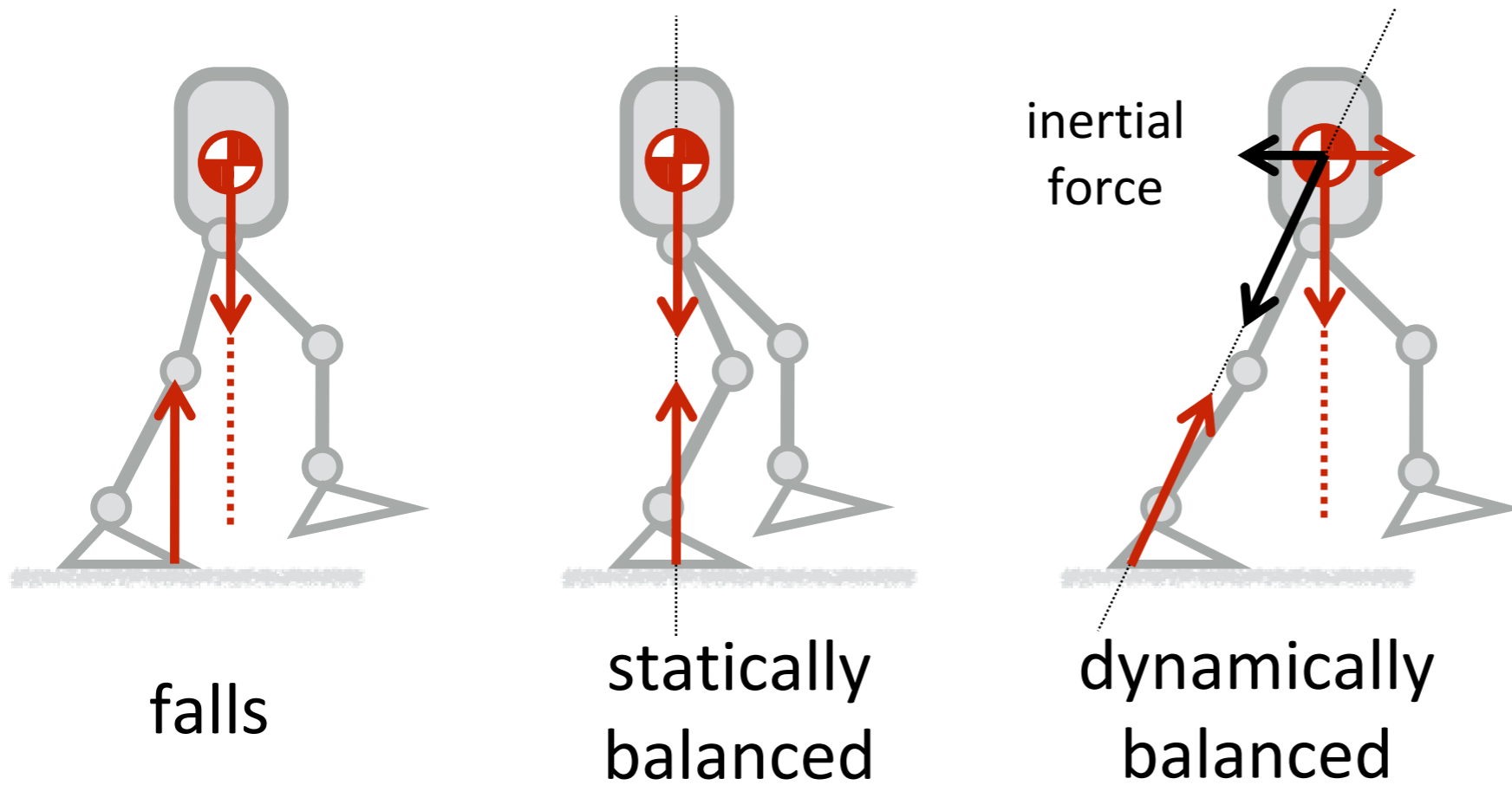
from the previous derivation:



which robot falls down?



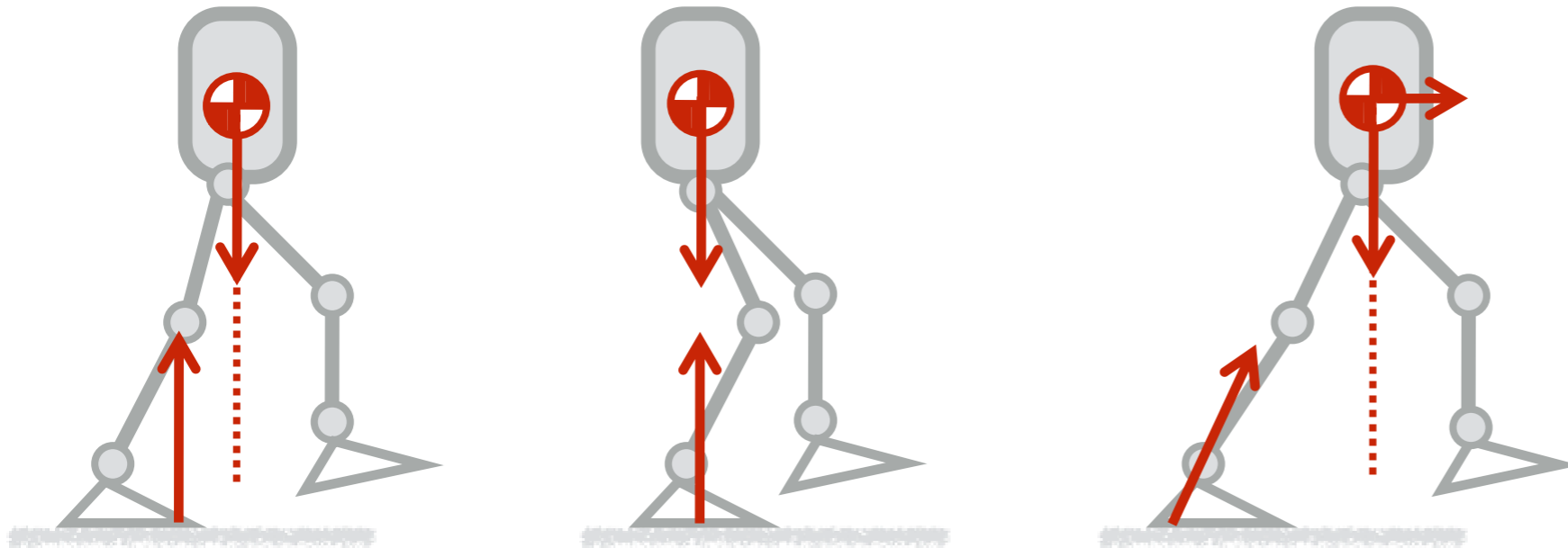
which robot falls down?



where is the ZMP?

z^x (ZMP): point on the ground where the GRF is applied

use the dynamics equation on **horizontal** flat ground and neglect $\dot{\mathbf{L}}^{x,y}$



$$\frac{c^z}{\ddot{c}^z + g^z} (\ddot{\mathbf{c}}^{x,y} + \cancel{\mathbf{g}}^{x,y}) = (\mathbf{c}^{x,y} - \mathbf{z}^{x,y}) + \frac{\cancel{S} \dot{\mathbf{L}}^{x,y}}{M(\ddot{c}^z + g^z)}$$

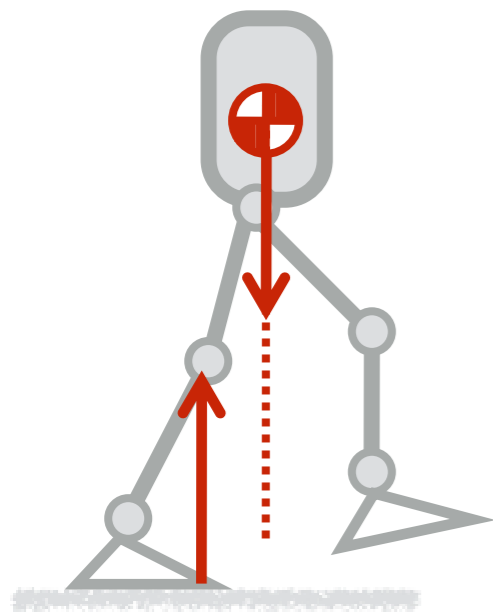
where is the ZMP?

hyp CoM at **constant height**

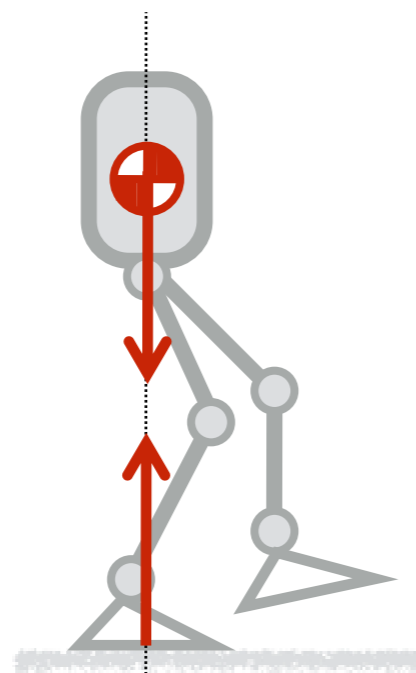
$$c^z = \text{constant}$$



LIP equation
in the
sagittal plane

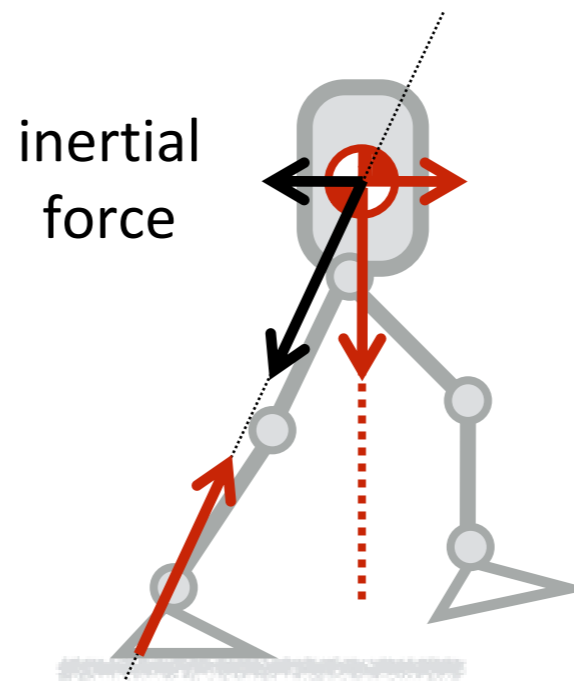


falls



statically
balanced

$$z^x = c^x$$



inertial
force

dynamically
balanced

$$z^x = c^x - \frac{c^z}{g^z} \ddot{c}^x$$

dynamically balanced locomotion

generate a gait for walking while maintaining balance

