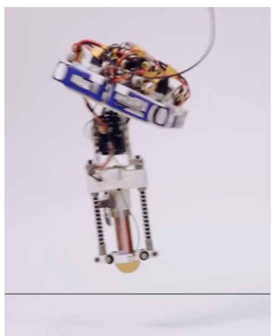
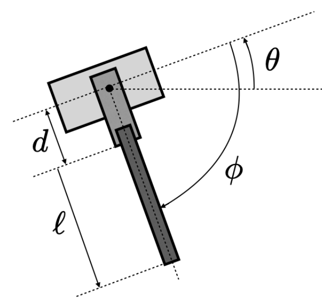
Autonomous and Mobile Robotics
Midterm Class Test, 2022/2023

Problem 1

A simplified model of a single-legged hopping robot consists of a body with inertia moment J and a leg of mass m. The leg is connected to the body through a revolute joint and a prismatic joint, which are separated by a distance d. Both joints are actuated; in particular, the extension of the length can be changed via the prismatic joint actuator.





When the robot is in flight phase, angular momentum is conserved. This leads to the following constraint

$$J\dot{\theta} + m(d+\ell)^2\dot{\phi} = 0$$

where we have assumed that the leg mass is concentrated at its extremity (foot) and the initial momentum is zero.

Let the configuration of the robot be defined as $\boldsymbol{q} = (\phi, \ell, \theta)$ (the translational motion is not of interest).

(a) Discuss the geometry of the robot configuration space.

(b) Derive a kinematic model of the robot for which there is a clear physical interpretation of the associated inputs.

(c) Discuss local and global mobility of the robot in its configuration space.

(d) Using the kinematic model, find a feedback control law for the flight phase that will drive the body orientation θ to zero and the leg length to a constant value h. Provide a block scheme.

(e) Consider for simplicity the case $d = 0$. Show that the system is differentially flat and derive the associated reconstruction formulas. Based on this, devise an algorithm for planning a feasible path between two arbitrary configurations $\boldsymbol{q}_s$ and $\boldsymbol{q}_g$.

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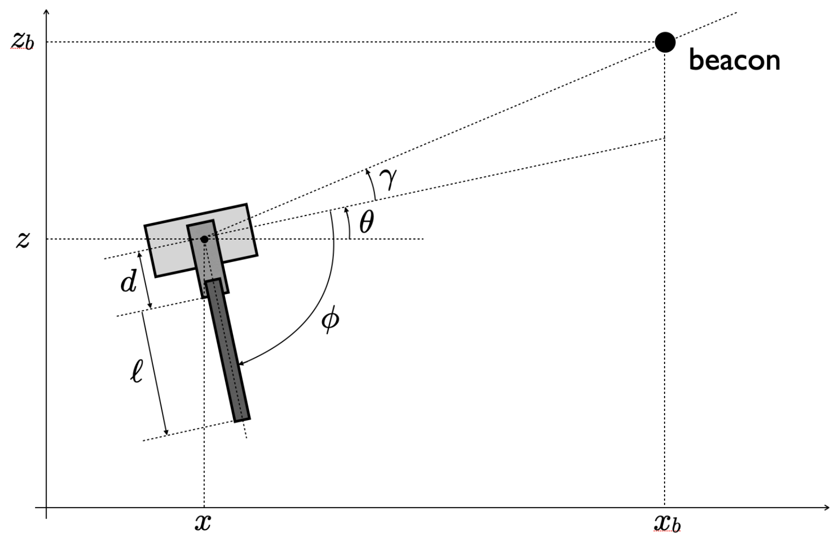
Problem 2

Consider again the hopping robot in Problem 1. Assume it is equipped with the following sensors:

- an incremental rotary encoder at the revolute joint;
- an incremental linear encoder at the prismatic joint;
- a sensor that measures the bearing angle γ of a beacon whose location (x_b, z_b) is known.

Moreover, an external vision system provides at each time a precise measure of the robot Cartesian position (x, z), which can then be considered as an exogenous known variable.



Build a localization system for estimating the robot configuration in real time. Provide equations (be sure to define all symbols) and a block scheme, clearly indicating how each sensor is used.

Problem 3

Are the following claims *true* or *false*? Answer and provide a short explanation.

(a) At the end of a Lie bracket maneuver of finite duration 4ϵ, the displacement of a unicycle will be aligned with the zero motion line at the starting instant.

(b) A car-like vehicle with a trailer has two instantaneous centers of rotation, one for the car and one for the trailer.

(c) In a bicycle, the speed of (the contact point of) the front wheel can be higher than the speed of (the contact point of) the rear wheel.

(d) In a unicycle, trajectory tracking controllers based on output error require only the measurements of the robot Cartesian coordinates for implementation.

(e) When proving convergence of the controller for Cartesian regulation of the unicycle, Barbalat's lemma is used in place of LaSalle's theorem because the time derivative $\dot{V}$ of the Lyapunov-like function $V = (x^2 + y^2)/2$ is negative *semidefinite* rather than *definite*.

[3 h]