Autonomous and Mobile Robotics
Solution of Final Class Test, 2014/2015

Solution of Problem 1

Letting $\boldsymbol{q} = (x, y, \theta)$, the kinematic model is readily expressed as

$$\dot{\boldsymbol{q}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta \\ \frac{r}{d} \end{pmatrix} \omega_R + \begin{pmatrix} \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta \\ -\frac{r}{d} \end{pmatrix} \omega_L = \boldsymbol{g}_1(\boldsymbol{q})\omega_R + \boldsymbol{g}_2(\boldsymbol{q})\omega_L,$$

where r is the radius of the wheels and d is the distance between their centers.

As for system controllability, the Lie Bracket of the two input vector fields is easily obtained as

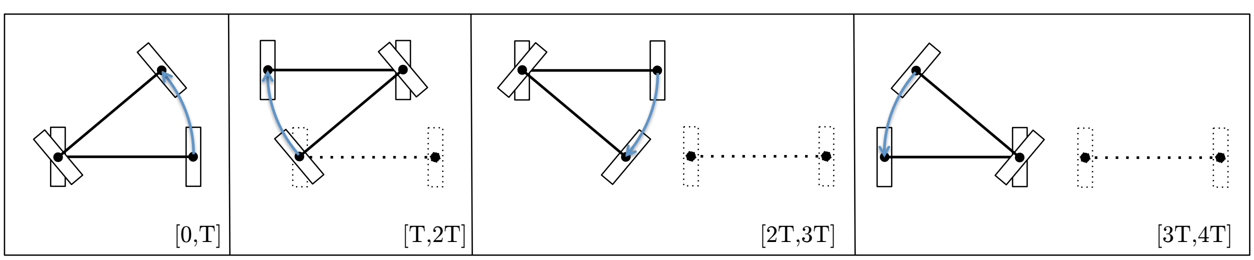
$$\boldsymbol{g}_3 = [\boldsymbol{g}_1, \boldsymbol{g}_2] = \frac{r^2}{d} \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix},$$

i.e., it is directed along the Zero Motion Line. A simple computation shows that

$$\det(\boldsymbol{g}_1\, \boldsymbol{g}_2\, \boldsymbol{g}_3) = r^4/d^2.$$

Hence, the accessibility rank condition is satisfied and the system is controllable.

The result of a Lie Bracket control maneuver is easily drawn as follows (the initial configuration is shown dashed):



Note that the achieved displacement is exactly in the direction of the Zero Motion Line; this means that for the considered system the $O(\epsilon^3)$ terms are identically zero.

Solution of Problem 2

(just the main ideas are sketched, other options are possible)
The potential field module is trivial (see slides on Motion Planning 3). The problem, however, is that a unicycle robot is not *free-flying* in its configuration space due to its nonholonomy; in particular, its representative point P (with coordinates $\boldsymbol{p}$) can only move instantaneously in the direction of the sagittal axis, whereas the artificial force field $\boldsymbol{f}$ at $\boldsymbol{p}$, which depends only on the obstacle and goal placement, may be oriented in any direction. As a consequence, setting $\dot{\boldsymbol{p}} = \boldsymbol{f}(\boldsymbol{p})$ is impossible in this case.

To transform the artificial force field $\boldsymbol{f}(\boldsymbol{p})$ in actual velocity inputs v, ω, one possibility is to assume that $\boldsymbol{f}$ acts on a point B (with coordinates $\boldsymbol{p}_B$) which is displaced from P of a certain distance b along the sagittal axis, as in input-output linearization control (see slides on Motion Control of WMRs: Trajectory Tracking). In fact, B can move in any direction, and the velocity inputs v, ω that realize a certain Cartesian velocity for B are easily computed by inverting the input-output map. The idea is then to set $\dot{\boldsymbol{p}}_B = \boldsymbol{f}(\boldsymbol{p}_B)$, and then compute v, ω that realize $\dot{\boldsymbol{p}}_B$.

A localization module will also be necessary for making (an estimate of) $\boldsymbol{q}$ available to the transformation module (the input-output matrix depends on $\boldsymbol{q}$, in particular on θ). From $\boldsymbol{q}$, it is straightforward to compute (an estimate of) $\boldsymbol{p}_B$ to be passed to the potential field module.

As for the effectiveness of the above navigation strategy, the potential field itself will be free of local minima because the robot and the obstacles are circular (*world of spheres)*. Isolated saddle points will be present; the total field is zero there but the robot can easily escape with a small perturbation. This means that a point robot subject to such field (including isolated perturbations) would always converge to the destination, provided that the latter is outside the range of infuence of all obstacles. For our unicycle robot, however, it will be point B that converges to the destination, while point P will actually lie on a circle of radius b centered at the destination. If b can be chosen small (this depends on the bounds on the input velocities), the final navigation error will be acceptable.

Solution of Problem 3

Since the position of the landmarks is unknown, this is a SLAM problem. Define the extended state vector to be estimated as $\boldsymbol{\chi} = (x \;\; y \;\; \theta \;\; x_{l,1} \;\; y_{l,1} \;\; \ldots \;\; x_{l,L} \;\; y_{l,L})^T$, where (x, y, θ) are the robot generalized coordinates and $(x_{l,i}, y_{l,i})$, for $i = 1, \ldots, L$, are the Cartesian coordinates of the landmarks. The nonlinear discrete-time model describing the motion of the extended robot+beacons system is then

$$\boldsymbol{\chi}_{k+1} = \boldsymbol{\chi}_k + \begin{pmatrix} v_k \cos\theta_k \\ v_k \sin\theta_k \\ \omega_k \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} v_{1,k} \\ v_{2,k} \\ v_{3,k} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

where $v_{i,k}$ is a white gaussian noise with zero mean and covariance $V_{i,k}$ $(i = 1, \dots, 3)$. Note how the landmarks being fixed reflects on the last $2L$ rows of the above equation.

As for the model of the measurement $\boldsymbol{y}$, we have L pairs of readings (range+bearing) coming from the robot sensor:

$$\boldsymbol{y}_k = \begin{pmatrix} \boldsymbol{h}_1(\boldsymbol{q}_k) \\ \vdots \\ \boldsymbol{h}_L(\boldsymbol{q}_k) \end{pmatrix} + \begin{pmatrix} \boldsymbol{w}_{1,k} \\ \vdots \\ \boldsymbol{w}_{L,k} \end{pmatrix}$$

with

$$\boldsymbol{h}_i(\boldsymbol{q}_k, i) = \begin{pmatrix} \sqrt{(x_k - x_{l,i})^2 + (y_k - y_{l,i})^2} \\ \text{atan2}(y_{l,i} - y_k, x_{l,i} - x_k) - \theta_k \end{pmatrix}$$

where $\boldsymbol{w}_i, k$ is a white gaussian noise with zero mean and covariance $\boldsymbol{W}_{i,k}$ $(i = 1, \dots, L)$. The rest of the problem is trivial: linearize the process and measurement models and then derive the EKF equations.

If the sensor cannot identify the landmark, an association map should be used, to be estimated on the basis of the innovation norm weighted by matrix S_{ij}^{-1} (Mahalanobis distance, see slides on Localization 2).