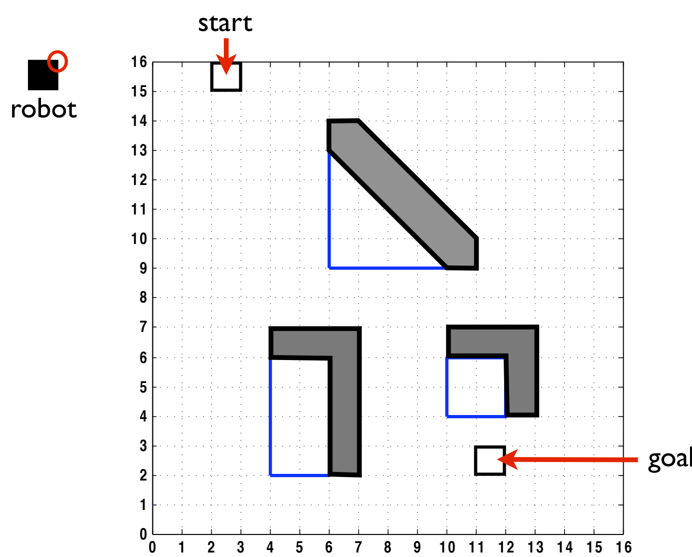
Autonomous and Mobile Robotics
Solution of Class Test no. 2

Solution of Problem 1

To build the $\mathcal{C}$-obstacles, it is necessary to choose a representative point for the robot. For example, choose the upper right vertex of the square robot (shown by a red circle in the following figures). The $\mathcal{C}$-obstacle boundaries are obtained by sliding the square robot along the obstacle boundaries, and keeping track of the associated motion of the representative point. The result is shown below, with the "increased" portions of the $\mathcal{C}$-obstacles shown in dark gray.

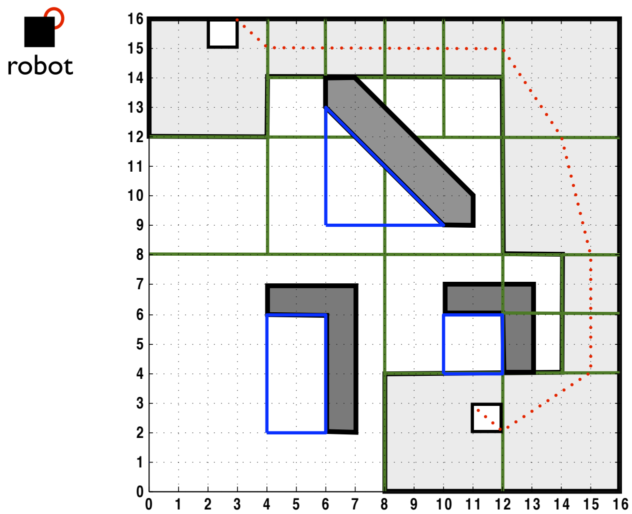


Approximate cell decomposition begins by (1) dividing the square configuration space into four square cells (2) perform collision checking to classify them as free, occupied or mixed (3) removing occupied cells (4) building a connectivity graph that contains free and mixed cells as nodes (5) searching the graph for a path (a channel of cells) from the cell containing the start configuration to the cell containing the goal configuration. If such a channel exists, its mixed cells are further partitioned, and the above cycle is repeated until a free channel is found (if possible).

In the present case, one possible solution is shown in the following figure. Here, 1-adjacency was used for building the connectivity graph[1]. The final robot path, shown in

[1]Since the robot can freely translate, the same is true for its representative point; therefore, the latter

dotted red line, was obtained from the free channel in light gray by joining the start to the goal configuration through a broken line going through the midpoints of common boundaries between consecutive cells.



Note that, even if 1-adjacency was used for building connectivity graphs and channels, the robot does move diagonally in the solution path.

Solution of Problem 2

The kinematic model of a rear-wheel drive car-like robot is

$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= v\tan\phi/\ell \\
\dot{\phi} &= \omega
\end{aligned}
$$

where x, y are the Cartesian coordinates of the rear wheel axle midpoint, θ is the orientation of the vehicle w.r.t. the x axis, ϕ is the steering angle, ℓ is the axle-to-axle distance, v is the driving velocity and ω is the steering velocity. Using Euler integration, a discrete-time nonlinear process model is derived as

$$
\begin{aligned}
x_{k+1} &= x_k + v_k T_s \cos\theta_k + v_{1,k} \\
y_{k+1} &= y_k + v_k T_s \sin\theta_k + v_{2,k} \\
\theta_{k+1} &= \theta_k + v_k T_s \tan\phi_k/\ell + v_{3,k} \\
\phi_{k+1} &= \phi_k + \omega_k T_s + v_{4,k}
\end{aligned}
$$

can in principle travel between two cells even if they share only a vertex (as in 2-adjacency). This means that 2-adjacency could have also been used.

where T_s is the sampling interval and $\boldsymbol{v}_k^T = (v_{1,k} \ \ldots \ v_{4,k})$ is a white gaussian process noise with zero mean and covariance matrix $\boldsymbol{V}_k$. This model is in the general form

$$\boldsymbol{q}_{k+1} = \boldsymbol{f}(\boldsymbol{q}_k, \boldsymbol{u}_k) + \boldsymbol{v}_k$$

At the k-th step, the Cartesian coordinates $\boldsymbol{p}_{s,k} = (x_{s,k}, y_{s,k})$ of the sensor are a function of the robot configuration $\boldsymbol{q}_k$; in particular, one has $(x_{s,k}, y_{s,k}) = (x_k + \ell \cos\theta_k/2, y_k + \ell \sin\theta_k/2)$. The output equation, which provides a measure of the distance between the sensor and the single landmark placed at the origin, is therefore

$$z_k = \sqrt{x_{s,k}^2 + y_{s,k}^2} + w_k = h(\boldsymbol{q}_k) + w_k$$

where w_k is a white gaussian measurement noise with zero mean and (co)variance W_k. Note that z_k, $h(\boldsymbol{q}_k)$, w_k and W_k are all scalars.

The linearization of the process and output equations, respectively evaluated at the previous estimate $\hat{\boldsymbol{q}}_k$ and at the prediction $\hat{\boldsymbol{q}}_{k+1|k}$, gives

$$\boldsymbol{F}_k = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}_k}\right|_{\boldsymbol{q}_k = \hat{\boldsymbol{q}}_k} = \begin{pmatrix} 1 & 0 & -v_k T_s \sin\hat{\theta}_k & 0 \\ 0 & 1 & v_k T_s \cos\hat{\theta}_k & 0 \\ 0 & 0 & 1 & \dfrac{v_k T_s}{\ell \cos^2 \hat{\phi}_k} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$\boldsymbol{H}_{k+1} = \left.\frac{\partial h}{\partial \boldsymbol{q}_k}\right|_{\boldsymbol{q}_k = \hat{\boldsymbol{q}}_{k+1|k}} = \left.\frac{\partial h}{\partial \boldsymbol{p}_{s,k}} \frac{\boldsymbol{p}_{s,k}}{\partial \boldsymbol{q}_k}\right|_{\boldsymbol{q} = \hat{\boldsymbol{q}}_{k+1|k}}$$

$$= \frac{1}{\sqrt{\hat{x}_{s,k+1|k}^2 + \hat{y}_{s,k+1|k}^2}} \begin{pmatrix} \hat{x}_{s,k+1|k} & \hat{y}_{s,k+1|k} & \dfrac{\ell}{2}(\hat{y}_{s,k+1|k} \cos\hat{\theta}_{k+1|k} - \hat{x}_{s,k+1|k} \sin\hat{\theta}_{k+1|k}) & 0 \end{pmatrix}$$

Here, $\hat{x}_{s,k+1|k}, \hat{y}_{s,k+1|k}$ are simply the sensor coordinates at the predicted configuration $\hat{\boldsymbol{q}}_{k+1|k}$.

The EKF equations are therefore obtained as follows.

1. State and covariance prediction:

$$\begin{aligned} \hat{\boldsymbol{q}}_{k+1|k} &= \boldsymbol{f}(\hat{\boldsymbol{q}}_k, \boldsymbol{u}_k) \\ \boldsymbol{P}_{k+1|k} &= \boldsymbol{F}_k \boldsymbol{P}_k \boldsymbol{F}_k^T + \boldsymbol{V}_k \end{aligned}$$

2. Correction:

$$\begin{aligned} \hat{\boldsymbol{q}}_{k+1} &= \hat{\boldsymbol{q}}_{k+1|k} + \boldsymbol{R}_{k+1} \nu_{k+1} \\ \boldsymbol{P}_{k+1} &= \boldsymbol{P}_{k+1|k} - \boldsymbol{R}_{k+1} \boldsymbol{H}_{k+1} \boldsymbol{P}_{k+1|k} \end{aligned}$$

where the innovation

$$\nu_{k+1} = z_{k+1} - \sqrt{\hat{x}_{s,k+1}^2 + \hat{y}_{s,k+1}^2}$$

is a scalar quantity and the Kalman gain matrix

$$\boldsymbol{R}_{k+1} = \boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T(\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + W_{k+1})^{-1}$$

is a 4×1 matrix (note that $\boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1|k}\boldsymbol{H}_{k+1}^T + W_{k+1}$ is actually a scalar, so no matrix inverse computation is required).

In these equations, $\boldsymbol{P}_k$ obviously denotes the covariance of the estimate, which will be initialized at a certain value reflecting the uncertainty on the initial estimate $\hat{q}_0$.