Self assessment - 00B

1 Exercise

Consider the Mass-Spring-Damper system with parameters m, μ and k, find analytically the natural modes for the special case $\mu = 2\sqrt{k\,m}$.

2 Exercise

Given the system

$$A = \begin{pmatrix} 2 & -1.5 \\ 2 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

- Find a "sensor", that is the C matrix, such that the unstable mode will never result in the output free response.
- What is the corresponding impulse response?
- Is the system asymptotically stable?

3 Exercise

Given the system

$$A = \begin{pmatrix} 0 & -0.5 \\ -2 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

- Compute the system eigenvalues and corresponding eigenspaces. Draw a phase plane plot of the typical qualitative state free evolutions (starting from different initial conditions that you choose and motivate).
- Compute the state impulse response (assuming zero initial state).
- Is the previous state impulse response diverging? Interpret the result in terms instantaneous state transfer and eigenspaces.
- Denote by λ_2 the resulting positive eigenvalue and assume the input does not contain $e^{\lambda_2 t}$, will the diverging exponential $e^{\lambda_2 t}$ appear in any forced output response?

4 Exercise

Consider the system matrix

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1+j & 0 \\ 0 & 0 & -1-j \end{pmatrix}$$

Find the particular change off coordinates T (which may have elements with complex numbers) that makes the system matrix become

$$TAT^{-1} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

5 Exercise

Given the dynamic matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

- Determine the eigenvalues and their multiplicities (algebraic and geometric).
- Is the corresponding system asymptotically stable, marginally stable or unstable?

6 Exercise

Given the dynamic matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Compute the matrix exponential e^{At}.
- Is there a particular choice of the input matrix B which will not lead to a diverging state impulse response?
- For a generic input matrix B, is there a particular choice of the output matrix C which will not lead to a diverging impulse response?

7 Exercise

Let the dynamic matrix be

$$A = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$$

- Find the spectral decomposition of A and compute the exponential e^{At}
- Draw some illustrative phase plane trajectories and verify on Matlab.

8 Exercise

Consider the Mass-Spring-Damper system (MSD).

- Choose the parameters such that the eigenvalues are real and distinct. Compute the maximum extension of the mass when a force impulse is applied.
- Same problem with a different choice of the parameters leading to a complex pair of eigenvalues.

9 Exercise

Consider the chemical reaction between two components described by the equations given in the slides.

- Find the change of coordinates that diagonalizes the dynamic matrix and interpret the result (conservation of some quantity relative to the 0 eigenvalue).
- Draw the phase plane plots highlighting the two eigenspaces.
- The Mass-Spring-Damper system with no spring ($K = 0$) has a similar dynamic behavior; what quantity is conserved in this case?

10 Exercise

Consider the electrical circuit in Fig. 1. Find the dynamic model and discuss its behavior when the two capacitors have an initial charge, i.e. when we have initial condition $v_{C1}(0)$ and $v_{C2}(0)$ and no input voltage v_i is applied.

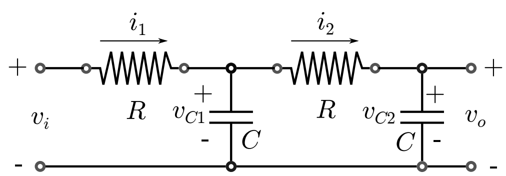

Figure 1: Electrical circuit exercise 09

11 Exercise

Consider the electrical circuit in Fig. 2.

- Find the dynamic model and discuss its behavior.
- Compare this system with the Mass-Damper system (i.e. MSD with no elastic spring).

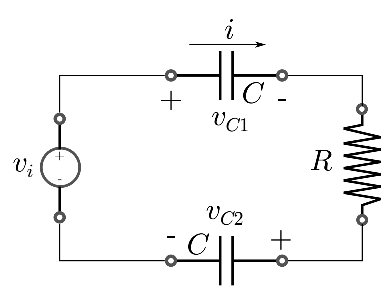

Figure 2: Electrical circuit