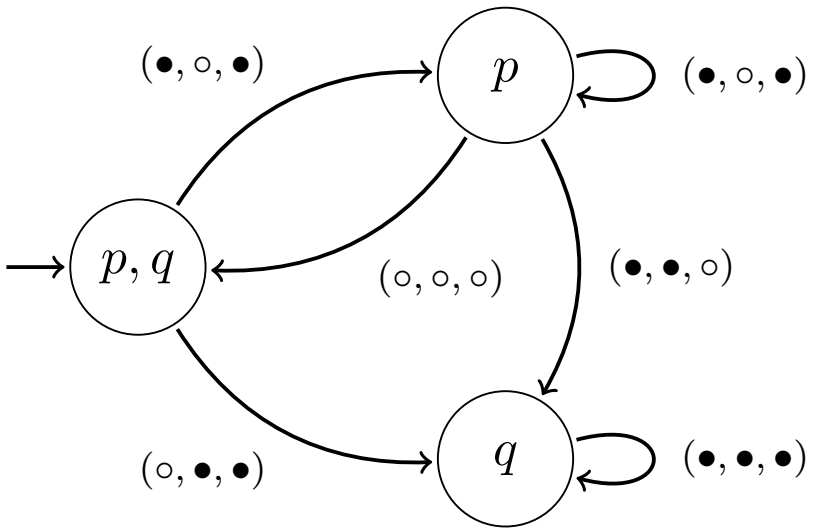
Reasoning about Knowledge and Strategies

Bastien Maubert and Aniello Murano

Università degli Studi di Napoli
Federico II

Distributed synthesis

Input: A concurrent game structure and a formula $\varphi \in$ LTL
Output: A distributed strategy to enforce φ



p, q are *atomic propositions*

$\circ, \bullet$ are *actions*

strategies σ : Histories $\rightarrow$ Actions

indistinguishability relations $\sim_a$ on states

Imperfect information

1. Strategies must be consistent with players' information

Constraint on strategies:

$$\text{If } h \sim_a h', \text{ then } \sigma_a(h) = \sigma_a(h').$$

2. Makes epistemic reasoning meaningful and useful

Example: opacity

A system is *opaque* for property P if a spy never knows whether the current execution is in P.

Classic definition:

$$\forall h, \exists h' \text{ s.t. } h \sim_{\text{spy}} h' \text{ and } h' \notin P$$

With epistemic temporal logic:

$$\mathbf{G}\neg K_{\text{spy}} P$$

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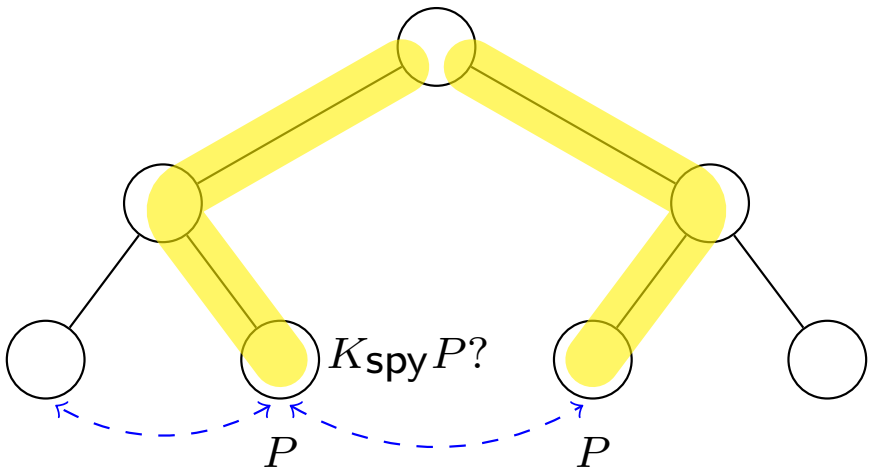
With epistemic temporal logic:

$$\exists \sigma (c, \sigma) \mathbf{G} \neg K_{\text{spy}} P$$

Semantics of knowledge when reasoning about strategies

Yellow subtree: controller's strategy
Blue arrows: spy's indistinguishability relation



Two possible semantics:

- Uninformed semantics: players ignore each other's strategy
  $\rightarrow K_{\mathsf{spy}}P$ does not hold
- Informed semantics: players know each other's strategy
  $\rightarrow K_{\mathsf{spy}}P$ holds

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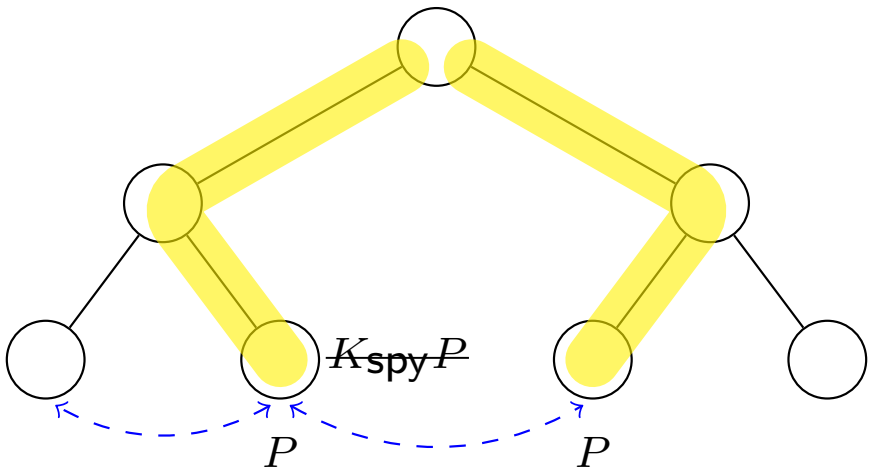

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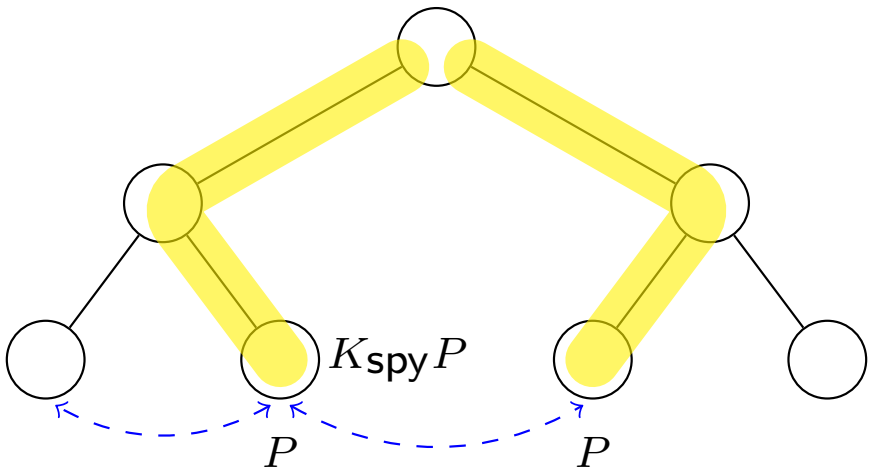

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Overview

Peterson and Reif (1979), Pnueli and Rosner (1990)

Distributed synthesis for reachability objective is undecidable.

Peterson and Reif (1979), Pnueli and Rosner (1990)

Decidable for LTL objectives when information is hierarchical.

For epistemic temporal objectives,

Distributed synthesis with hierarchical information is

- Undecidable for informed semantics

  [van der Meyden and Wilke, 2005]

- Decidable for uninformed semantics

  [Puchala, 2010]

SL with imperfect information and knowledge

SL (Chatterjee et al. 2010, Mogavero et al. 2014)

LTL $+$

- $\exists \sigma \, \varphi$

  "there exists a strategy σ such that φ"

- $(a, \sigma)\varphi$

  "when player a plays strategy σ, φ"

SL with imperfect information and knowledge

$\mathsf{SL}_{\mathsf{ii}}$ (Berthon et al. 2017)

LTL $+$

- $\exists^{o} \sigma \, \varphi$

  "there exists a strategy σ with observation o such that φ"

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SL with imperfect information and knowledge

ESL (M. and Murano, 2018)

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Distributed synthesis for opacity:

$$\exists^{o_1} \sigma_1 \, (c_1, \sigma_1) \exists^{o_2} \sigma_2 \, (c_2, \sigma_2) \mathbf{G} \neg K_s P$$

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Existence of Nash equilibria:

$$\exists^{o_1} \sigma_1 \, (a_1, \sigma_1) \exists^{o_2} \sigma_2 \, (a_2, \sigma_2) \left(\bigwedge_i \exists^{o_i} \sigma' \, (a_i, \sigma') \; \mathsf{Win}_i \rightarrow \mathsf{Win}_i \right)$$

Main result

Hierarchical instances

An ESL formula Φ is *hierarchical* if:

- innermost strategies observe better than outermost ones
- epistemic subformulas do not talk about current strategies

Considering the uninformed semantics of knowledge:

Theorem

Model-checking hierarchical instances of ESL is decidable.

Corollaries:

On systems with hierarchical information,
for epistemic temporal specifications,

We can solve

- distributed synthesis,
- module checking,
- existence of Nash equilibria,
- rational synthesis,
- ...

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Thank you!