Knowing-How Under Uncertainty

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In this paper[1] we study an interplay between knowledge, strategies, and uncertainty in multiagent systems. Consider an example of a traffic situation depicted in Figure 1, where a self-driving truck t is approaching an intersection at the same time as a regular car c. Although there is a stop sign instructing the car to yield to the truck, the car's driver does not notice the sign and does not slow down. This is detected by the radar on the self-driving truck t. The truck has two strategies that potentially can prevent a collision with the car: to accelerate or to break. How effective each of these strategies is depends on the speed of the car c. If the speed of the car is slow, the truck must accelerate to avoid being hit by the car in the rear half. If the speed is high, the truck must brake to avoid being hit in the front half. Suppose, see Figure 2, that the truck will avoid the collision by accelerating if the speed of car c is at most 58 miles per hour (mph) and that the truck will avoid the collision by breaking if the speed of the car is at least 56mph. In the interval between 56 and 58mph both strategies would allow the truck to avoid a collision.

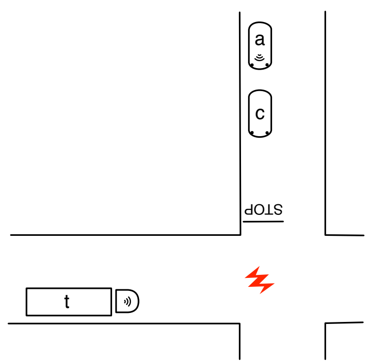

Figure 1: Road situation.

Let us further assume that the actual speed of the car is 55mph, but the truck's radar can only detect the speed of the car with a precision of $\pm$6mph. Thus, The truck only knows that the speed of the car is somewhere in the interval between 49 and 61mph, see Figure 2. Thus, truck t does not know which of the two strategies would allow it to prevent a collision. Note that in this situation truck t has a strategy to avoid collision, but it does not know what this strategy is. If an agent t has a strategy to achieve goal φ, she knows that she has such a strategy, and she knows what this strategy is, then we say that she has a *know-how* strategy and denote this by $\mathsf{H}_t\varphi$. In this paper we study the existence of know-how strategies depending on the degree of uncertainty of the information available to the agent. We represent the degree of uncertainty by the superscript of the modality. For example, we write $\neg\mathsf{H}^6_t$("Collision is avoided.") to say that truck t does *not* have a know-how strategy to avoid a collision if it determines the speed of car c with a precision of $\pm$6mph. However, if the truck is able to determine the speed of the car with a precision of $\pm$2mph, then truck t has a know-how strategy to prevent the collision: H^2_t("Collision is avoided.").

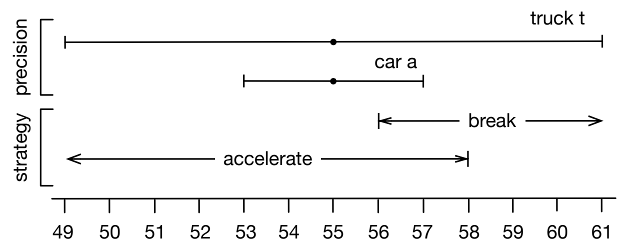

Figure 2: Strategies and precision.

Now suppose that an autonomous car a is driving right behind car c. From this position car a can measure the speed of car c with precision $\pm$2mph. Thus, car a knows that the speed of car c is between 53 and 57mph. Assuming that car a is aware of truck's radar precision, it can see that no matter where within the interval between 53 and 57mph the speed of car c is, truck t does *not* have a know-how strategy to avoid collision. We write this as $\mathsf{K}^2_a\neg\mathsf{H}^6_t$("Collision is avoided."), where modality K^2_a denotes the knowledge of car a when it is able to determine the speed of car c with a precision of $\pm$2mph. As another example, although statement H^2_t("Collision is avoided.") is true as we have seen earlier, car a does not know about this: $\neg\mathsf{K}^2_a\mathsf{H}^2_a$("Collision is avoided."). Indeed, due to the precision of car a's equipment, as far as car a is concerned, the speed of the car c is between 53 and 57mph. If it is 56.5mph, then statement H^2_t("Collision is avoided.") would

[1]This is a joint work with Jia Tao.

not be true. A similar setting appears in many real world examples. For example, a robot might be facing a challenge of navigating with different uncertainty factors (Ferguson and Stentz 2004; Brafman et al. 1997). A pilot might be facing a challenge of landing an aircraft safely using imprecise data about wind speed as well as the plane's position, speed, and load. A medical doctor needs to prescribe medicine relying on imprecise data from a patient's subjective description of symptoms and not absolutely precise lab test results. Governments often introduce new economic policies based on incomplete statistical data about the economy.

The interplay between knowledge modality K_a and know-how modality H_a, both without a degree of uncertainty, has been recently studied by (Ågotnes and Alechina 2016; Wang 2016; Fervari et al. 2017; Naumov and Tao 2017b; 2017a; 2018b; 2018a). In this paper we study the interplay between modalities K_a^c and H_a^c, where degree c of uncertainty refers to the precision with which agent a can position herself in an arbitrary metric space. A sound and complete logical system that describes properties of modality K_a^c was introduced in (Naumov and Tao 2015). The current paper extends that system to include modality H_a^c.

In addition to propositional tautologies in language Φ, our logical system has the following five axioms:

1. Zero Confidence: $\varphi \to \mathsf{K}_a^0\varphi$,
2. Truth: $\mathsf{K}_a^c\varphi \to \varphi$,
3. Negative Introspection: $\neg\mathsf{K}_a^c\varphi \to \mathsf{K}_a^d\neg\mathsf{K}_a^{c+d}\varphi$,
4. Distributivity: $\mathsf{K}_a^c(\varphi \to \psi) \to (\mathsf{K}_a^c\varphi \to \mathsf{K}_a^c\psi)$,
5. Strategic Positive Introspection: $\mathsf{H}_a^{c+d}\varphi \to \mathsf{K}_a^c\mathsf{H}_a^d\varphi$.

The first four of these axioms come from (Naumov and Tao 2015). The Strategic Positive Introspection axiom without a degree of uncertainty first appeared in (Ågotnes and Alechina 2016) and is also present in (Wang 2016; Fervari et al. 2017; Naumov and Tao 2017b; 2017a; 2018b; 2018a). The metric-space based semantics of the Strategic Positive Introspection axiom in our settings is significantly different from the indistinguishability relation based semantics of the similar axiom without uncertainty superscript in the know-how literature. Blending the know-how and the degree of uncertainty lines of research into one logical system that captures non-trivial interplay between the two notions is the main contribution of this paper.

We write $\vdash \varphi$ if formula $\varphi \in \Phi$ is provable from the above axioms using the Monotonicity, K-Necessitation, H-Necessitation, and Modus Ponens inference rules:

$$\frac{\varphi \to \psi}{\mathsf{H}_a^c\varphi \to \mathsf{H}_a^c\psi}, \quad \frac{\varphi}{\mathsf{K}_a^c\varphi}, \quad \frac{\varphi}{\mathsf{H}_a^c\varphi}, \quad \frac{\varphi, \varphi \to \psi}{\psi}.$$

We write $X \vdash \varphi$ if formula $\varphi \in \Phi$ is provable from the theorems of our logical system and an additional set of axioms X using only the Modus Ponens inference rule.

In this paper we combine knowledge under uncertainty modality K_a^c and strategic know-how modality H_a^c, and prove the strong completeness of the obtained system with respect to one class of semantics and the weak completeness with respect to another.

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