Model checking with Interval Temporal Logic: Results and Perspectives

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Model checking with interval temporal logics is emerging as a viable alternative to model checking with standard point-based temporal logics, such as LTL, CTL, CTL*, and the like. The behavior of the system is modelled by means of (finite) Kripke structures, as usual. However, while temporal logics which are interpreted "point-wise" describe how the system evolves state-by-state, and predicate properties of system states, those which are interpreted "interval-wise" express properties of computation stretches, spanning a sequence of states. A proposition letter is assumed to hold over a computation stretch (interval) if and only if it holds over each component state (homogeneity assumption).

The most well-known interval temporal logic is Halpern and Shoham's modal logic of time intervals HS (Halpern and Shoham 1991), which features one modality for each possible ordering relation between a pair of intervals, apart from equality (Allen 1983). In the first part of our contribution, we provide an overview of the main results on model checking with HS and its fragments under the homogeneity assumption. In particular, we show that the problem turns out to be non-elementarily decidable and EXPSPACE-hard for full HS, but it is often computationally much better for its fragments (Molinari et al. 2016; Molinari, Montanari, and Peron 2018; Bozzelli et al. 2018b; 2018a). We conclude this part with a short account of a recent generalization of the proposed model checking framework that allows one to use regular expressions to define the behavior of proposition letters over intervals in terms of the component states (Bozzelli et al. 2017b; 2017c).

When one attempts at properly locating the interval way to model checking in the general landscape of research on formal verification methods, a natural question arises: is there any advantage in replacing points by intervals as the primary temporal entities, or is it just a matter of taste?

In the second part of the contribution, we briefly analyse the expressiveness of HS in model checking, in comparison with those of LTL, CTL, and CTL* (Bozzelli et al. 2016). To this end, we consider three semantic variants of HS: the state-based one, that allows time to branch both in the past and in the future, the computation-tree-based one, that allows time to branch in the future only, and the trace-based variant, that disallows time to branch. These variants are compared among themselves and to the aforementioned standard logics, getting a complete picture. In particular, we show that HS with trace-based semantics is equivalent to LTL (but at least exponentially more succinct), HS with computation-tree-based semantics is equivalent to finitary CTL*, and HS with state-based semantics is incomparable with all of them (LTL, CTL, and CTL*).

In the last part of the contribution, we briefly illustrate ongoing work and possible future developments.

We first discuss the few existing gaps in the characterization of the computational complexity of model checking with HS fragments. We focus on the model checkig / satisfiability problems for the logic of prefixes and suffixes over finite linear orders, under the homogeneity assumption, that we only know to be EXPSPACE-hard (notice that the three semantic variants of HS coincide over it). We know that the model checking / satisfiability problems for the logic of subintervals over finite linear orders, under the homogeneity assumption, are PSPACE-complete (the subinterval modality can be easily expressed in terms of the modalities for prefixes and suffixes) (Bozzelli et al. 2017a). The proof benefits from a spatial encoding of the models for the logic and a suitable contraction technique. Unfortunately, there is no a natural way to generalize such a solution to the logic of prefixes and suffixes.

Then, we reason about possible replacements of finite Kripke structures by more expressive system models. There are at least two directions worth to be explored here. On the one hand, one may consider the replacement of finite Kripke structures by inherently interval-based models (no restriction on the evaluation of proposition letters), to allow one to directly describe systems on the basis of their interval behavior/properties, such as, e.g., those involving actions with duration, accomplishments, or temporal aggregations. Timeline-based (planning) systems, that model the behaviour of a system by a set of timelines governed by a set of transition functions (one for each timeline) and a set of synchronization rules (that constrain, among other things, the relationships among the various timelines), are a natural option (Cialdea Mayer, Orlandini, and Umbrico 2016). On the other hand, one may think of replacing finite Kripke stuctures by richer ones, such as, for instance, visibly pushdown systems (Alur and Madhusudan 2004), that make it possible to encode recursive programs and infinite state systems.

Last but not least, an investigation of the possible exploitation of model checking with interval temporal logic in the context of machine learning has been recently undertaken. More precisely, it has been shown that model checking a single interval model can be successfully used for temporal dataset evaluation (Della Monica et al. 2017).

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