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New Problems and Innovative Methods in Nonlinear Optimization

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ABSTRACTS

of the

INVITED LECTURES

Recent developments in copositive programming

Immanuel M. Bomze

University of Vienna

A symmetric matrix is called copositive, if it generates a quadratic form taking no negative values over the positive orthant. Contrasting to positive-semidefiniteness, checking copositivity is NP-hard. In a copositive program, we have to minimize a linear function of a symmetric matrix over the copositive cone subject to linear constraints. This program is convex, so there are no non-global local solutions. On the other hand, there are several hard non-convex programs which can be formulated as copositive programs. By using the copositive formulation, the complexity of solving these non-convex programs is shifted towards the complexity of sheer feasibility. As a consequence, duality theory is slightly more complex than in the linear programming case. For instance, there can be a positive duality gap, or the dual program may not attain the optimal value although it is feasible and bounded. This talk addresses basic theory, algorithms and applications, among them combinatorial problems like max-clique and max-cut, or supervised (SVM) and unsupervised learning (clustering).

Mutual Impact of Numerical Linear Algebra and Large-Scale Optimization

Marco D'Apuzzo

Second University of Naples

The solution of linear systems for computing search directions is ubiquitous in optimization algorithms, being often the most computationally expensive task. Thus, effective numerical linear algebra is crucial for effective numerical optimization. Many optimization codes have benefited, in terms of both efficiency and robustness, from advances in numerical linear algebra algorithms and software. On the other hand, much progress in numerical linear algebra has been spurred by the need of solving linear systems with special features in the context of optimization. This talk addresses some critical aspects of this mutual impact. The focus is on the large and sparse case and considerable attention is given to the use of iterative approaches for solving linear systems within the interior point framework.

Co-authors: S. Cafieri, V. De Simone, D. di Serafino, G. Toraldo.

Deterministic Global Optimization: Advances in Convex Underestimation Methods and Applications

Christodoulos A. Floudas

Princeton University

In this presentation, we will provide an overview of the research progress in global optimization. The focus will be on important contributions during the last five years, and will provide a perspective for future research opportunities. The overview will cover the areas of (a) twice continuously differentiable constrained nonlinear optimization, (b) mixed-integer nonlinear optimization, and (c) optimization with differential-algebraic models. Subsequently, we will present our recent fundamental advances in (i) convex envelope results for multi-linear functions, (ii) a piecewise quadratic convex underestimator for twice continuously differentiable functions, (iii) the generalized alpha-BB framework, (iv) new results on functional forms for convex underestimators of twice continuously differentiable functions, and (v) our recently improved convex underestimation techniques for univariate and multivariate functions. Computational studies will illustrate the potential of these advances.

Multilevel Quadratic Programming Techniques for Graph Partitioning

William Hager

University of Florida

In earlier work, we showed that the NP complete graph partitioning problem could be reformulated as a continuous quadratic programming problem. We now embed this quadratic programming problem into a multilevel algorithm. The subproblems obtained by the multilevel coarsening of the initial graph are approximately solved using a series of continuous quadratic programming techniques, including gradient projection methods for computing local minimizers, sphere constrained global minimization methods for computing solutions over approximations to the feasible set, and a block exchange quadratic programming technique for escaping from local minimizers. Numerical comparisons are given with the code METIS, a multilevel algorithm based on the Fiduccia/Mattheyses exchange techniques for treating the subproblems.

Solution Methods for Multi-valued Variational Inequalities

IGOR V. KONNOV

Kazan University

Variational inequalities represent very useful and powerful tools for investigation and solution of many equilibrium type problems arising in Economics, Engineering, Operations Research and Mathematical Physics. The talk is devoted to several rather new approaches from [1]–[5] to construction of solution methods for variational inequalities with multi-valued cost mappings. We recall that the problem is to find a point $x^* \in X$ such that

$$\exists g^* \in G(x^*), \quad \langle g^*, x - x^* \rangle \geq 0 \quad \forall x \in X,$$

where X is a nonempty, closed and convex subset of a finite-dimensional space E, and $G : X \to \Pi(E)$ is a multi-valued mapping. These problems involve in particular multi-valued inclusions, complementarity and fixed point problems, non-smooth optimization problems, and mixed variational inequalities.

It is well-known that the multi-valuedness creates certain difficulties for providing convergence of many iterative methods, which are applied successfully to single-valued problems, especially, without integrability or strengthened monotonicity properties, which leads to necessity of construction of new solution methods. Another problem is to attain fast convergence similar to that in the single-valued case.

In this talk, we first describe iterative solution methods for multi-valued complementarity problems under order monotonicity type properties. Next, we present iterative methods for monotone mixed variational inequality problems, which involve either general convex or max-type convex functions, which attain a linear rate of convergence. We also describe iterative methods for general multi-valued variational inequalities and multi-valued inclusions, which converge to a solution under mild monotonicity type assumptions and establish their complexity estimates which are equivalent to a linear rate of convergence.

References

[1] I.V. Konnov (1993), On combined relaxation methods' convergence rates, *Russian Math. (Iz. VUZ)* **37**, no.12, 89–92.

[2] I.V. Konnov (1996), A general approach to finding stationary points and the solution of related problems, *Comput. Maths and Math. Phys.* **36**, 585–593.

[3] I.V. Konnov (2001), Combined Relaxation Methods for Variational Inequalities (Lecture Notes in Economics and Mathematical Systems, Vol.495), Springer-Verlag, Berlin, Heidelberg, New York.

[4] I.V. Konnov (2002), A combined relaxation method for a class of nonlinear variational inequalities, *Optimization* **51**, 127–143.

[5] I.V. Konnov (2007), An extension of the Jacobi algorithm for multi-valued mixed complementarity problems, *Optimization* **56**, 399–416.

New DIRECT-type algorithms for global optimization problems

STEFANO LUCIDI

Sapienza Università di Roma

In this talk we propose new deterministic methods for global optimization. All these methods draw their inspiration from the DIRECT-algorithm. In fact their approach is to partition the feasible set into a growing number of hyperintervals, to evaluate the objective function on the center of every hyperintervals and, at each iteration, to select the *most promising* hyperintervals to be further partitioned.

All these methods have the common feature of using the same partitioning technique of the DIRECT algorithm. This partitioning technique produces at most $2n$ new hyperintervals and, hence, it is not much expensive. Nevertheless it is able to adapt itself to the particular selection rule used. In fact, a less restrictive selection rules leads to a more uniform partition of the feasible set, while a more restrictive one leads the algorithm to generate points clustered in particular regions of the feasible set.

The new algorithms try to improve the efficiency of the previous versions of the DIRECT algorithm by following and, possibly, combining the following ideas:

- to select the *promising* hyperintervals by using new criteria able to exploit further information on the objective function, if available;
- to incorporate the use of suitable local minimizations starting from *the most promising* points among those generated by the DIRECT-type algorithm.

Numerical results are reported which show the interest of the proposed algorithms.

Co-authors: G. Liuzzi, V. Piccialli.

Practical Augmented Lagrangian Methods

J. M. Martínez

State Universitry of Campinas, Brazil

We consider the minimization of a scalar function subject to two types of constraints. Optimization subject to second-level constraints is supposed to be easy. First-level constraints are incorporated to the Augmented Lagrangian function. Each outer iteration of the Augmented Lagrangian Method consists in the approximate minimization of the Augmented Lagrangian subject to second-level constraints. Penalty parameters and Lagrange multipliers are updated after each outer iteration.Global minimizers are obtained in the limit if global minimization is used in the subproblems. Under relaxed KKT conditions in the subproblems, feasibility and global convergence is obtained using the CPLD constraint qualification. Under additional constraint qualifications (yet weaker than regularity) the method converges to second-order stationary points, provided that a second-order method is used in the subproblems. Practical aspects of the implementation will be commented. This includes the choice of the algorithm that solves subproblems and acceleration-refinement issues. Applications will be discussed.

On the global solution of linear programs with linear complementarity constraints

Jong-Shi Pang

Rensselaer Polytechnic Institute, NY

This paper presents a parameter-free integer-programming based algorithm for the global resolution of a linear program with linear complementarity constraints (LPEC). The cornerstone of the algorithm is a minimax integer program formulation that characterizes and provides certificates for the three outcomes—infeasibility, unboundedness, or solvability—of an LPEC. An extreme point/ray generation scheme in the spirit of Benders decomposition is developed, from which valid inequalities in the form of satisfiability constraints are obtained. The feasibility problem of these inequalities and the carefully guided linear programming relaxations of the LPEC are the workhorse of the algorithm, which also employs a specialized procedure for the sparsification of the satifiability cuts. We establish the finite termination of the algorithm and report computational results using the algorithm for solving randomly generated LPECs of reasonable sizes. The results establish that the algorithm can handle infeasible, unbounded, and solvable LPECs effectively.

This is joint work with Jing Hu, John Mitchell, Kristin Bennett, and Gautam Kunapuli.

Equilibrium problems of variational type: models, methods and algorithms

Massimo Pappalardo

University of Pisa

The study of a wide class of problems regarding both the project and the management of flow networks has lead to the formulation of mathematical models very similar each to others. A common characteristic that comes from the analysis of the above problems lies in the variational feature of the characterizing equilibrium conditions. It has been stated, in fact, that the equilibrium concept can be expressed by means of a variational model and shows a strict connection to the real problem. This is shown, in particular, in the traffic network where variational inequalities permit us to establish the well-known Wardrop principle in a mathematical setting and in economic framework where variational inequalities permit us to establish the Nash equilibrium theory. But while the classical differentiable VI in finite dimensional spaces has been largely developed in literature both from the theoretical point of view and the applicative one, we observe in the last period that more refined variational models are much considered. So, we will analyze the case in which the operator is non differentiable, the case of generalized variational inequalities where the operator is point-to-set and the case of quasi variational inequalities in which the domain depends on the solution. We will study these models from two viewpoints: the former consists in studying the existence and uniqueness of solutions, the latter is devoted to solution methods. We will show the importance of the concept of gap function which allows us to transform the problems in a global optimization problem. The algorithms mainly developed in literature for these classes of models are linked to monotonicity assumptions on the operator. Our target is to check if we can weaken the monotonicity assumptions.

Co-authors: Barbara Panicucci and Mauro Passacantando

Analysis of Greedy Approximation with Non-submodular Potential Functions

Panos M. Pardalos

University of Florida

In recent work of Du, Graham, Wan, Wu and Zhao, we introduced a new method which can analyze a large class of greedy approximations with non-submodular potential functions, including some long-standing heuristics for Steiner trees, connected dominating sets, and power-assignment in wireless networks. There exist many greedy approximations for various combinatorial optimization problems, such as set covering, Steiner tree, subset-interconnection designs, etc.. There are also many methods to analyze these in the literature. However, all of the previously known methods are suitable only for those greedy approximations with submodular potential functions.

Revisiting convergence of SAA/SPO methods for two stage stochastic programs

Daniel Ralph

University of Cambridge, UK

Stochastic programs can be approximated by replacing expectations over random variables by sums over a finite samples of random variables. This general idea has had great success as demonstrated in the convergence analysis of sample average approximation (SAA) methods by Shapiro and others, and of sample path optimization (SPO) by Robinson and others. The practical efficiency of these methods is somewhat explained by the exponential rates of convergence that have been shown in many instances of SAA. We examine exponential convergence again to broaden its application to certain two stage stochastic programs.

Co-author: Huifu Xu, University of Southampton, UK.

Large Scale Global Optimization in practice

Fabio Schoen

Università di Firenze

In this talk we will introduce some widely studied problems which can be formulated as large scale global optimization ones; all these problems display a huge number of local optima. Despite their inherent complexity, these optimization problem lend themselves to solution in quite reasonable computational times by means of a suitable combination of a few basic tools:

- local optimization, by means of standard methods
- neighborhood exploration, by means of suitably defined random perturbations
- diversity enforcement and cooperation among concurrent runs of a global optimization algorithm
- "seeding" by means of suitably chosen starting points

These simple ideas have been succesfully applied to many different problem classes, including atomic cluster conformation problems (under Lennard-Jones or Morse potential), molecular conformation problems (Binary Lennard-Jones and water TIP4P clusters), circle and sphere packing in containers, distance geometry problems and sensor location.

In this talk we will present the common ideas which led to the implementation of successful methods for these problems as well as the specialized tools which were needed in order to exploit each problem's structure.

The Role of Metric Projectors in Nonlinear Conic Optimization Problems

Defeng Sun

National University of Singapore

The non-polyhedral nature of the closed convex cones presented in nonlinear conic optimization problems complicates our understanding on these problems both theoretically and numerically. In this talk, we emphasize that variational analysis on the metric projection operator over the closed convex cones plays a key role in studying these problems. We demonstrate this, in particular by using the metric projection operator over the cone of symmetric and positive semidefinite matrices, in several aspects including perturbation analysis, convergence analysis of algorithms, and numerical computations.

A Convex Optimization Approach for a Class of Nonconvex Quadratic Estimation Problems

Marc Teboulle

Tel-Aviv University

Many problems in data fitting and estimation which arise in a variety of scientific disciplines such as: signal processing, statistics, physics, economy and medicine, require to solve an over-determined system of linear equations where the data (matrix and right hand side) is contaminated by noise.

A common methodology to model this situation and get a stabilized solution, is via the so-called Regularized Total Least Squares (RTLS) approach which leads to formulate the estimation problem as one of minimizing the ratio of two convex quadratic functions subject to a quadratic constraint. The specific form of the (RTLS) problem involving the ratio of two special convex functions does not appear to play any advantageous role toward its solution, and the key difficulty, namely its *nonconvexity* remains. As a result, several well known methods devised to solve it are not guaranteed to converge to a global optimum, and at best are proven to converge to a point satisfying first order necessary optimality conditions.

This talk first briefly reviews such class of estimation problems and then focuses on the more general nonconvex problem of minimizing the ratio of two (possibly indefinite) quadratic functions (RQ) subject to a degenerate ellipsoid. We prove that under a certain mild assumption on the problem's data, problem (RQ) admits an exact semidefinite programming relaxation, and an optimal solution of the original problem (RQ) can be extracted from the optimal solution of this semidefinite formulation. Building on this hidden convexity property of problem (RQ), we then study a simple and efficient iterative procedure for solving problem (RQ) which is proven to globally converge superlinearly. Moreover, it is shown that this algorithm produces an ε-global optimal solution of (RQ) in no more than $O(\kappa\sqrt{\log \varepsilon^{-1}})$ iterations, for some positive constant κ given in terms of the problem's data. This result also provides a theoretical justification to some recent successful computational results reported in the literature in the special context of (RTLS). Numerical experiments will illustrate the attractiveness of the algorithm.

This talk is based on a joint work with Amir Beck.

A globally convergent primal-dual interior-point filter method for nonlinear programming (ipfilter): new filter optimality measures and computational results

Luis N. Vicente

University of Coimbra

This talk describes the development of an optimization solver (ipfilter) for large scale nonlinear programming problems. The underlying algorithm is based on the primal-dual interior-point filter framework developed in the paper by M. Ulbrich, S. Ulbrich, and L. N. Vicente, A globally convergent primal-dual interior-point filter method for nonlinear programming, Mathematical Programming, 100 (2004) 379-410.

The method is based on the application of the filter technique to the globalization of the primal-dual interior-point algorithm, avoiding the use of merit functions and the updating of penalty parameters. The algorithm decomposes the primal-dual step obtained from the perturbed first-order necessary conditions into a normal and a tangential step. Each entry in the filter is a pair of coordinates: one resulting from feasibility and centrality (associated with the normal step); the other resulting from optimality (complementarity and duality) and related with the tangential step. The method possesses global convergence to first-order critical points.

We will describe the extension of the algorithm to new filter optimality entries which are, theoretically, better tailored to the purpose of minimization.

We will show some of the features of ipfilter, its current capabilities and limitations. We will present numerical results for large-scale problems.

This is joint work with Renata Silva (Univ. Coimbra), Michael Ulbrich (Tech. Univ. Munich), and Stefan Ulbrich (Tech. Univ. Darmstadt).

Strong Duality and Stability in Conic Convex Optimization

Henry Wolkowicz

University of Waterloo

For nonlinear convex optimization problems, in the absence of a constraint qualification strong duality need not hold. Moreover, constraint qualifications are closely tied to numerical stability and well posedness. The Extended Lagrange Slater dual of Ramana is a Semidefinite Programming, SDP, dual for which strong duality holds without a constraint qualification. We explain when an extension of Ramana's algorithm to general conic convex problems efficiently computes a strong dual. We also give other strong duals that do not require a constraint qualification, and that are defined solely in terms of the data of the given problem.

We then present an algorithm that solves in an efficient and stable way feasible conic convex optimization problems, including those for which the Slater constraint qualification fails. Necessary and sufficient conditions for a finite nonzero duality gap are given, and it is shown how these can be used to generate instances satisfying this property. Numerical tests are included.

Co-authors: Simon Schurr and Levent Tuncel (Department of Combinatorics and Optimization, University of Waterloo)

Semidefinite Programming Relaxation Model for Graph Realization and Sensor Network Localization

Yinyu Ye

Stanford University, CA

We present semidefinite programming (SDP) based approaches for the position estimation problem in Euclidean distance geometry such as graph realization and sensor network localization. We develop an SDP relaxation model and use the duality theory to derive necessary and/or sufficient conditions for whether a network is "realizable or localizable" or not, when the distance measures are accurate. We also present error analyses of the SDP solution when the distance measures are noisy. Furthermore, we develop a further relaxation such that large-scale problems can be solved efficiently, and demonstrate computational effectiveness of the SDP relaxation model.

Subspace Techniques for Nonlinear Optimization

Ya-xiang Yuan

Chinese Academy of Sciences

In this talk, we review various subspace techniques that are used in constructing of numerical methods for nonlinear optimization. The subspace techniques are getting more and more important as the optimization problems we have to solve are getting larger and larger in scale. The applications of subspace techniques have the advantage of reducing both computation cost and memory size. Actually in many standard optimization methods (such as conjugate gradient method, limited memory quasi-Newton method, projected gradient method, and null space method) there are ideas or techniques that can be viewed as subspace techniques. The essential part of a subspace method is how to choose the subspace in which the trial step or the trust region should belong. Model subspace algorithms for unconstrained optimization and constrained optimization are given respectively, and different proposals are made on how to choose the subspaces. As an example, we also present an interior point method based on subspace techniques.