Robotics I

Midterm classroom test – November 29, 2019

Exercise 1 [6 points]

The initial orientation of a rigid body with respect to a basis reference frame is given by the matrix

$$\boldsymbol{R}_i = \begin{pmatrix} \dfrac{\sqrt{2}}{2} & 0 & \dfrac{\sqrt{2}}{2} \\ 0 & -1 & 0 \\ \dfrac{\sqrt{2}}{2} & 0 & -\dfrac{\sqrt{2}}{2} \end{pmatrix}.$$

The final desired orientation $\boldsymbol{R}_f$ is expressed in terms of roll-pitch-yaw angles $(\alpha, \beta, \gamma) = (\pi/3, \pi/3, -\pi/2)$ in the sequence ZYX around the fixed axes associated to the initial orientation. Find a pair $(\boldsymbol{r}, \theta)$ such that the relative change of orientation of the body is represented by the axis-angle method associated to the unit vector $\boldsymbol{r}$ and angle θ. Comment on how the same result can be obtained when the unit vector $\boldsymbol{r}$ is expressed in terms of the basis reference frame, rather than in the frame associated to $\boldsymbol{R}_i$.

Exercise 2 [6 points]

Consider the 6R Universal Robots UR10 manipulator in Fig. 1, where a possible set of Denavit-Hartenberg (DH) frames has been defined.

- On the extra sheet #1 provided separately [*to be returned with your name*], complete the table of DH parameters. Enter in the table numerical values (expressed in [rad] or [mm]), including those of the joint variables $\boldsymbol{q} = \boldsymbol{\theta}$ in the configuration shown. In the drawing, all data are given already in mm.

- Provide the numerical value of the position of the origin O_6 of frame 6 in the shown configuration.
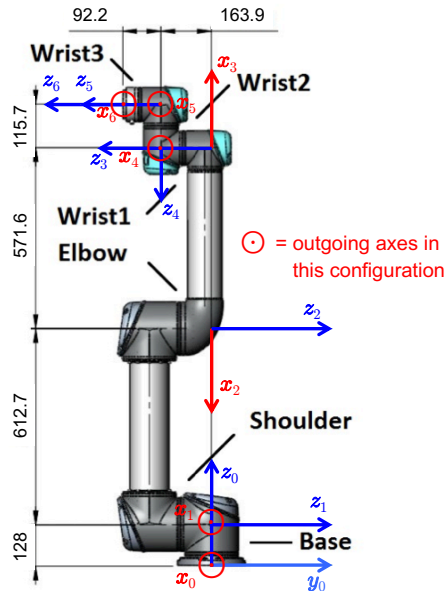

Figure 1: A possible assignment of DH frames for the UR10 robot.

Exercise 3 [6 points]

Consider the planar 2R robot in Fig. 2, with the numerical data $L = 0.4$, $A = 0.4$, and $B = 0.3$ [m]. An end-effector frame RF_e is attached at point P to the gripper, with the $\boldsymbol{z}_e$ axis along the approach direction.
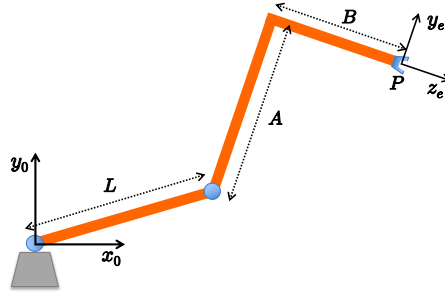

Figure 2: A planar 2R robot, with a L-shaped second link.

- Draw accurately the primary workspace of the robot. Which is its secondary workspace?
- Assign the link frames and define the joint variables $\boldsymbol{q} = (q_1, q_2)$ according to the Denavit-Hartenberg (DH) convention. Let the origin O_2 of the DH frame 2 be placed at point P. Complete the associated table of parameters.
- Determine the matrix ${}^2\boldsymbol{R}_e \in SO(3)$.
- Provide all solutions, if any, to each of the following three inverse kinematics problems, where the end-effector position $\boldsymbol{p}_e \in \mathbb{R}^2$ (i.e., reduced to the plane of motion) is given as input:

$$\boldsymbol{p}_{e,1} = \begin{pmatrix} 0 \\ -0.9 \end{pmatrix}; \qquad \boldsymbol{p}_{e,2} = \begin{pmatrix} -0.4 \\ 0.7 \end{pmatrix}; \qquad \boldsymbol{p}_{e,3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Exercise 4 [6 points]

Write a simple (pseudo-)code that solves the following inverse kinematics problem with the iterative Newton method. The task (direct) kinematics is

$$\boldsymbol{r} = \begin{pmatrix} L_1 \cos q_1 + L_2 \cos q_2 + L_3 \cos q_3 \\ L_1 \sin q_1 + L_2 \sin q_2 + L_3 \sin q_3 \\ q_3 - q_2 \end{pmatrix}.$$

Let $L_1 = 0.4$, $L_2 = 0.3$, and $L_3 = 0.2$ [m]. If the desired task value $\boldsymbol{r}_d$ and the initial guess $\boldsymbol{q}^{(0)}$ for the solution are, respectively,

$$\boldsymbol{r}_d = \begin{pmatrix} 0.7 \\ 0.5 \\ 0 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{q}^{(0)} = \begin{pmatrix} 0 \\ \pi/2 \\ \pi/2 \end{pmatrix},$$

what is the numerical value of the next guess $\boldsymbol{q}^{(1)}$ at the end of iteration 1? Do you think the sequence $\{\boldsymbol{q}^{(0)}, \boldsymbol{q}^{(1)}, \boldsymbol{q}^{(2)}, \dots\}$ generated by this method will converge to a solution of the problem? For the given value of $\boldsymbol{r}_d$, how many solutions do you expect to have for this inverse problem?

Exercise 5 [6 points]

A number of questions and statements are reported on the extra sheet #2. Fill in your answers and/or comments on the same sheet [*to be returned with your name*], providing also a *short* motivation/explanation for each item.

[180 minutes, open books]

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Solution of Midterm Test

November 29, 2019

Exercise 1

The orientation of a rigid body, as expressed by a ZYX sequence of roll-pitch-yaw angles (α, β, γ), i.e., with respect to a set of fixed axes, is represented by the rotation matrix

$$\boldsymbol{R}_{ZYX}^{RPY}(\alpha, \beta, \gamma) = \boldsymbol{R}_X(\gamma)\boldsymbol{R}_Y(\beta)\boldsymbol{R}_Z(\alpha)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta & -\sin\alpha\cos\beta & \sin\beta \\ \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma & \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & -\cos\beta\sin\gamma \\ \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma & \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma & \cos\beta\cos\gamma \end{pmatrix},$$

where the order in the product of elementary rotation matrices is reversed, as required. The final desired orientation of the body with respect to the frame with orientation $\boldsymbol{R}_i = {}^0\boldsymbol{R}_i$ is specified as

$$ {}^i\boldsymbol{R}_f = \boldsymbol{R}_{ZYX}^{RPY}\left(\frac{\pi}{3}, \frac{\pi}{3}, -\frac{\pi}{2}\right) = \begin{pmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{1}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} 0.25 & -0.433 & 0.866 \\ -0.433 & 0.75 & 0.5 \\ -0.866 & -0.5 & 0 \end{pmatrix}. \tag{1}$$

Operatively, to obtain this matrix one can either evaluate numerically the symbolic matrix $\boldsymbol{R}_{ZYX}^{RPY}$, or evaluate numerically the single elementary rotation matrices $\boldsymbol{R}_X$, $\boldsymbol{R}_Y$ and $\boldsymbol{R}_Z$ and then do their products.

To represent this change of orientation by the axis-angle method with a unit vector $\boldsymbol{r}$ and angle θ, we need to solve the equation[1]

$$\boldsymbol{R}(\boldsymbol{r}, \theta) = \boldsymbol{r}\boldsymbol{r}^T + \left(\boldsymbol{I} - \boldsymbol{r}\boldsymbol{r}^T\right)\cos\theta + \boldsymbol{S}(\boldsymbol{r})\sin\theta = {}^i\boldsymbol{R}_f.$$

Indeed, the unit vector computed in this way will naturally be expressed in the frame associated to ${}^0\boldsymbol{R}_i$, i.e., $\boldsymbol{r} = {}^i\boldsymbol{r}$. Let R_{ij} be the elements of matrix ${}^i\boldsymbol{R}_f$ in (1). Since

$$\sin\theta = \frac{1}{2}\sqrt{(R_{21} - R_{12})^2 + (R_{13} - R_{31})^2 + (R_{32} - R_{23})^2} = 1 \neq 0, \qquad \cos\theta = \frac{\text{trace}\left({}^i\boldsymbol{R}_f\right) - 1}{2} = 0;$$

the problem is regular and the two (specular) solution pairs $(\boldsymbol{r}, \theta)$ are given by

$$\boldsymbol{r}_1 = \frac{1}{2\sin\theta}\begin{pmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.866 \\ 0 \end{pmatrix}, \qquad \theta_1 = \text{ATAN2}\{\sin\theta, \cos\theta\} = \frac{\pi}{2}, \tag{2}$$

and $(\boldsymbol{r}_2, \theta_2) = (-\boldsymbol{r}_1, -\theta_1)$.

In order to obtain the same solution using the unit vector ${}^0\boldsymbol{r}_1$ (what follows apply also to ${}^0\boldsymbol{r}_2$), namely expressed in terms of the basis reference frame, we compute

$$ {}^0\boldsymbol{r}_1 = {}^0\boldsymbol{R}_i\, {}^i\boldsymbol{r}_1 = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & -1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} -0.5 \\ 0.866 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.3536 \\ -0.866 \\ -0.3536 \end{pmatrix}.$$

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[1]A completely different result would be obtained if the problem request was to use the axis-angle method $\boldsymbol{R}(\boldsymbol{r}', \theta')$ to represent the final orientation in terms of the basic reference frame, i.e., $\boldsymbol{R}(\boldsymbol{r}', \theta') = {}^0\boldsymbol{R}_f = {}^0\boldsymbol{R}_i\,{}^i\boldsymbol{R}_f$. In that case, the computed unit axis $\boldsymbol{r}'$ would have been expressed directly in the basic frame, i.e., $\boldsymbol{r}' = {}^0\boldsymbol{r}'$.

It is easy to verify (left as an exercise for the reader) that the following identity holds:

$$^0\boldsymbol{R}_f = \boldsymbol{R}(^0\boldsymbol{r}_1, \theta_1)\,^0\boldsymbol{R}_i = {}^0\boldsymbol{R}_i\,\boldsymbol{R}(^i\boldsymbol{r}_1, \theta_1) = {}^0\boldsymbol{R}_i\,^i\boldsymbol{R}_f. \tag{3}$$

In fact, the absolute orientation of the final frame w.r.t. the basis (zero) reference frame can be obtained either by two rotations defined both w.r.t. fixed axes (with the reverse order in the product of rotations, as in the first identity in (3)), or by two rotations, the second of which is defined w.r.t. the axes obtained after the first one (chain rule of products with moving axes, as in the second and third identities in (3)).

Exercise 2

The Denavit-Hartenberg parameters uniquely associated to the frames specified for the UR10 robot (see also Fig. 3) are given in Tab. 1. Note that this is NOT the frame assignment used for this robot in the DIAG Robotics Lab. The position of the origin O_6 in the shown configuration is found just by inspection as $\boldsymbol{p}_6 = \begin{pmatrix} 0 & -(163.9 + 92.2) & (128 + 612.7 + 571.6 + 115.7) \end{pmatrix}^T = \begin{pmatrix} 0 & -256.1 & 1428 \end{pmatrix}^T$ [mm] —there is no need to perform lengthy computations with the DH homogenous transformation matrices!
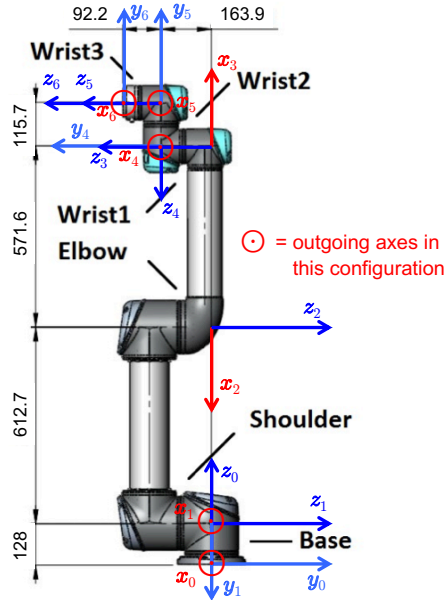

Figure 3: The assignment of DH frames as in Fig. 1, with all the $\boldsymbol{y}_i$ axes shown.

i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	$d_1 = 128$	$\theta_1 = 0$
2	0	$a_2 = -612.7$	0	$\theta_2 = \pi/2$
3	π	$a_3 = 571.6$	0	$\theta_3 = \pi$
4	$\pi/2$	0	$d_4 = 163.9$	$\theta_4 = -\pi/2$
5	$-\pi/2$	0	$d_5 = -115.7$	$\theta_5 = 0$
6	0	0	$d_6 = 92.2$	$\theta_6 = 0$

Table 1: DH parameters (in mm or rad), with the value of $\boldsymbol{\theta} \in \mathbb{R}^6$ in the shown 'home' configuration.

Exercise 3

The primary workspace WS_1 of the planar 2R robot with the L-shaped second link of Fig. 2 is shown in Fig. 4, where the numerical data about link geometry have been taken into account. This workspace is that of a standard planar 2R robot having the first link of length $L = 0.4$ [m] and the second of length $D = \sqrt{A^2 + B^2} = \sqrt{0.16 + 0.09} = 0.5$ [m]. Thus, it is a circular annulus with external radius $R = L + D = 0.9$ [m] and internal radius $r = |L - D| = 0.1$ [m]. The secondary workspace WS_2 is empty.
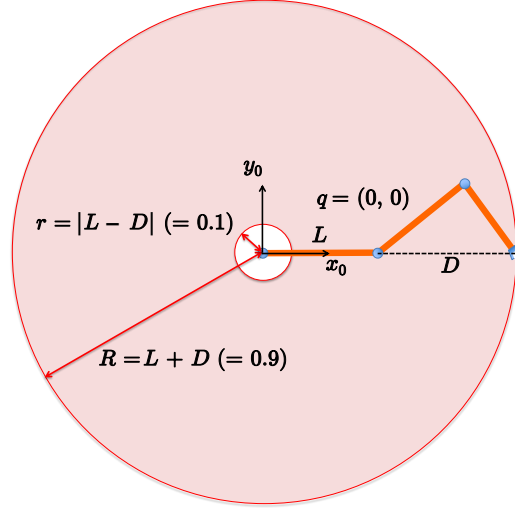

Figure 4: The primary workspace of the planar 2R robot with L-shaped second link of Fig. 2.

The Denavit-Hartenberg frames and the associated table of parameters for this robot are reported in Fig. 5. Note that $\boldsymbol{x}_2$, as required, is incident and orthogonal to the last defined joint axis (i.e., $\boldsymbol{z}_1$ at joint 2). Figure 4 shows also the robot in the configuration $\boldsymbol{q} = \boldsymbol{0}$. The constant rotation matrix ${}^2\boldsymbol{R}_e \in SO(3)$ from the DH frame RF_2 to the end-effector frame RF_e is given by

$$
{}^2\boldsymbol{R}_e = \begin{pmatrix} 0 & \sin\beta & \cos\beta \\ 0 & \cos\beta & -\sin\beta \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0.8 & 0.6 \\ 0 & 0.6 & -0.8 \\ -1 & 0 & 0 \end{pmatrix},
$$

with $\beta = \arctan(A/B) = \arctan 1.3333 = 53.13° = 0.9273$ [rad].

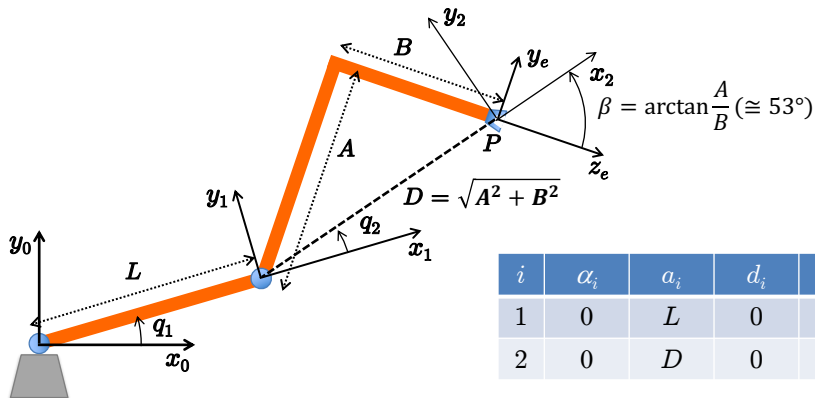| i	α_i	a_i	d_i	θ_i
1	0	L	0	q_1
2	0	D	0	q_2

Figure 5: DH frames and table of parameters for the robot of Fig. 2.

The solutions of two of the assigned inverse kinematics problems are straightforward. For the first problem with $\boldsymbol{p}_{e,1} = \begin{pmatrix} 0 & -0.9 \end{pmatrix}^T$, since $\|\boldsymbol{p}_{e,1}\| = 0.9 = R$, the desired end-effector position is on the external boundary of the workspace on the negative $\boldsymbol{y}_0$ axis: therefore, the only solution is $\boldsymbol{q} = \begin{pmatrix} -\pi/2 & 0 \end{pmatrix}^T$ [rad]. For the third problem, since $\boldsymbol{p}_{e,3} = \boldsymbol{0}$ does not belong to the robot workspace, there will be no solution. Finally, for the second problem with $\boldsymbol{p}_{e,2} = \begin{pmatrix} -0.4 & 0.7 \end{pmatrix}^T$, it is $r = 0.1 < \|\boldsymbol{p}_{e,2}\| = 0.8062 < 0.9 = R$. We are thus in a regular situation, and the known formulas for the inverse kinematics of a planar 2R robot can be applied, using as link lengths $l_1 = L = 0.4$ and $l_2 = D = 0.5$. The two solutions are:

$$\boldsymbol{q}^{[a]} = \begin{pmatrix} 90° \\ 53.13° \end{pmatrix} = \begin{pmatrix} \pi/2 \\ 0.9273 \end{pmatrix} \text{[rad]}, \qquad \boldsymbol{q}^{[b]} = \begin{pmatrix} 149.49° \\ -53.13° \end{pmatrix} = \begin{pmatrix} 2.6091 \\ -0.9273 \end{pmatrix} \text{[rad]}, \qquad (4)$$

where, as usual, $q_2^{[b]} = -q_2^{[a]}$. We note that the first solution could have been found also with a simple geometric reasoning about the data of the problem (see Fig. 6).
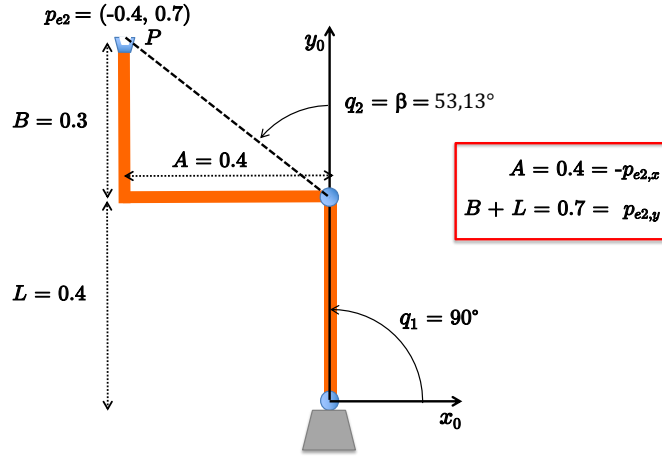

Figure 6: The geometric reasoning for obtaining the inverse kinematics solution $\boldsymbol{q}^{[a]}$ in (4).

Exercise 4

The given mapping

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} L_1 \cos q_1 + L_2 \cos q_2 + L_3 \cos q_3 \\ L_1 \sin q_1 + L_2 \sin q_2 + L_3 \sin q_3 \\ q_3 - q_2 \end{pmatrix} = \boldsymbol{r}(\boldsymbol{q}), \qquad (5)$$

with $\boldsymbol{q} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}^T$, has indeed a robotic interpretation. The task vector $\boldsymbol{r}$ contains the components of the end-effector position ($r_1 = p_x$, $r_2 = p_y$) and the relative (DH) angle between second and third link ($r_3 = \theta_3$) of a planar 3R robot, when the *absolute* angles q_i ($i = 1, 2, 3$) of the links w.r.t. the $\boldsymbol{x}_0$ axis are used as coordinates (see Fig. 7).

In order to solve the inverse kinematics problem

$$\boldsymbol{r}(\boldsymbol{q}) = \boldsymbol{r}_d = \begin{pmatrix} 0.7 \\ 0.5 \\ 0 \end{pmatrix}, \qquad (6)$$

we would like to use the Newton method with the iterative formula for $k = 0, 1, 2, \ldots$

$$\boldsymbol{q}^{(k+1)} = \boldsymbol{q}^{(k)} + \boldsymbol{J}^{-1}\left(\boldsymbol{q}^{(k)}\right)\left(\boldsymbol{r}_d - \boldsymbol{r}(\boldsymbol{q}^{(k)})\right), \qquad (7)$$
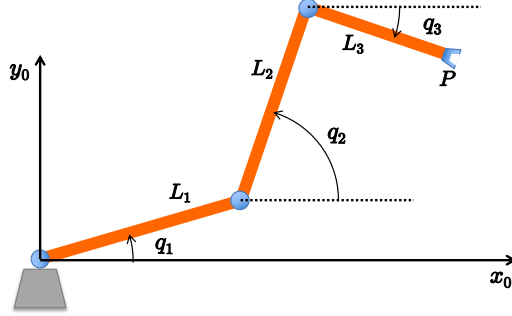
Figure 7: A planar 3R robot, with the definition of the used absolute coordinates.

where $\boldsymbol{J}(\boldsymbol{q})$ is the (analytical) Jacobian of the task kinematics. A pseudo-code should be written to this purpose (and eventually used as actual code in a chosen programming language, in order to compute a solution). This step is left to the reader.

Differentiating (5) with respect to $\boldsymbol{q}$ yields the 3×3 Jacobian matrix

$$\boldsymbol{J}(\boldsymbol{q}) = \frac{\partial \boldsymbol{r}(\boldsymbol{q})}{\partial \boldsymbol{q}} = \begin{pmatrix} -L_1 \sin q_1 & -L_2 \sin q_2 & -L_3 \sin q_3 \\ L_1 \cos q_1 & L_2 \cos q_2 & L_3 \cos q_3 \\ 0 & -1 & 1 \end{pmatrix}, \tag{8}$$

which is nonsingular unless $\det \boldsymbol{J}(\boldsymbol{q}) = L_1 \left(L_2 \sin(q_2 - q_1) + L_3 \sin(q_3 - q_1) \right) = 0$.

At the initial guess $\boldsymbol{q}^{(0)} = \begin{pmatrix} 0 & \pi/2 & \pi/2 \end{pmatrix}^T$, using also the robot length data, we have a task error

$$\boldsymbol{e}^{(0)} = \boldsymbol{r}_d - \boldsymbol{r}\left(\boldsymbol{q}^{(0)}\right) = \begin{pmatrix} 0.7 \\ 0.5 \\ 0 \end{pmatrix} - \begin{pmatrix} 0.4 \\ 0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0 \\ 0 \end{pmatrix} \neq \boldsymbol{0} \quad \Rightarrow \quad \left\| \boldsymbol{e}^{(0)} \right\| = 0.3.$$

Moreover, since

$$\boldsymbol{J}\left(\boldsymbol{q}^{(0)}\right) = \begin{pmatrix} 0 & -0.3 & -0.2 \\ 0.4 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \qquad \det \boldsymbol{J}\left(\boldsymbol{q}^{(0)}\right) = 0.2,$$

we can safely compute the first iteration of (7) for $k = 0$:

$$\boldsymbol{q}^{(1)} = \boldsymbol{q}^{(0)} + \boldsymbol{J}^{-1}\left(\boldsymbol{q}^{(k)}\right) \boldsymbol{e}^{(0)} = \begin{pmatrix} 0 \\ \pi/2 \\ \pi/2 \end{pmatrix} + \begin{pmatrix} 0 \\ -0.6 \\ -0.6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.9708 \\ 0.9708 \end{pmatrix}.$$

The new guess leads to

$$\boldsymbol{r}\left(\boldsymbol{q}^{(1)}\right) = \begin{pmatrix} 0.6823 \\ 0.4127 \\ 0 \end{pmatrix} \quad \Rightarrow \quad \boldsymbol{e}^{(1)} = \boldsymbol{r}_d - \boldsymbol{r}\left(\boldsymbol{q}^{(1)}\right) = \begin{pmatrix} 0.0177 \\ 0.0873 \\ 0 \end{pmatrix} \quad \Rightarrow \quad \left\| \boldsymbol{e}^{(1)} \right\| = 0.0891,$$

showing a substantial progress toward a solution. As a matter of fact, thanks to the quadratic convergence rate of Newton method when near to a solution, we obtain (with our Matlab code) convergence of the sequence to a solution in only 4 iterations, with an accuracy on the error norm of less than $\epsilon = 10^{-5}$. The obtained numerical results are summarized in Tab. 2.

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k	$\boldsymbol{q}^{(k)}$	$\boldsymbol{r}^{(k)}$
0	$\begin{pmatrix} 0 & 1.5708 & 1.5708 \end{pmatrix}$	$\begin{pmatrix} 0.4 & 0.5 & 0 \end{pmatrix}$
1	$\begin{pmatrix} 0 & 0.9708 & 0.9708 \end{pmatrix}$	$\begin{pmatrix} 0.68232 & 0.41267 & 0 \end{pmatrix}$
2	$\begin{pmatrix} 0.24857 & 0.92796 & 0.92796 \end{pmatrix}$	$\begin{pmatrix} 0.68744 & 0.4986 & 0 \end{pmatrix}$
3	$\begin{pmatrix} 0.28297 & 0.88812 & 0.88812 \end{pmatrix}$	$\begin{pmatrix} 0.69953 & 0.49962 & 0 \end{pmatrix}$
4	$\begin{pmatrix} 0.28555 & 0.88616 & 0.88616 \end{pmatrix}$	$\begin{pmatrix} 0.7 & 0.5 & 0 \end{pmatrix}$

k	$\boldsymbol{e}^{(k)}$	$\left\| \boldsymbol{e}^{(k)} \right\|$
0	$\begin{pmatrix} 0.3 & 0 & 0 \end{pmatrix}$	0.3
1	$\begin{pmatrix} 0.017679 & 0.087332 & 0 \end{pmatrix}$	0.089104
2	$\begin{pmatrix} 0.012558 & 0.0013956 & 0 \end{pmatrix}$	0.012635
3	$\begin{pmatrix} 0.00047072 & 0.00037521 & 0 \end{pmatrix}$	0.00060197
4	$\begin{pmatrix} 1.8871 \cdot 10^{-6} & 1.1168 \cdot 10^{-6} & 0 \end{pmatrix}$	$2.1928 \cdot 10^{-6}$

Table 2: Convergence with the Newton method (7) in solving the nonlinear system (6): Joint configurations and associated task values (top), and task errors and their norms (bottom) during the first four iterations.

The following remarks are in order.

- The first solution guess $\boldsymbol{q}^{(0)}$ already satisfies the constraint of a common absolute orientation for the second and third link, namely $q_3 - q_2 = 0$. The Newton method will keep this constraint satisfied over the iterations ($q_2^{(k)} = q_3^{(k)}$, $\forall k$).

- As a result, the third component of the task error, i.e., $e_3^{(k)}$, the only one with angular units, will remain identically zero. Therefore, the norm reported in the last column of Tab. 2 will have no problem of unit inconsistency, being always made just by the two components e_x and e_y of the position error.

- Enforcing $q_2 = q_3$ through the specific choice of $\boldsymbol{r}_d$ allows us to draw a simple conclusion on the number of solutions to the given inversion problem. Under such premise, this is equivalent to solving an inverse kinematics problem for a planar 2R robot having the first link of length $L_1 = 0.4$ [m] and the second of length $L_2' = L_2 + L_3 = 0.5$ [m]. Since $\|\boldsymbol{r}_d\| = 0.8602$, and this value is provided only by the positional task components, the desired end-effector position will be strictly inside the workspace of the equivalent planar 2R robot (using the notation of Exercise 3, $r = |L_2' - L_1| = 0.1 < 0.8602 < 0.9 = L_1 + L_2' = R$). It follows that the original inversion problem will have exactly two solutions —one of which has been found already with the iterative Newton method ($\boldsymbol{q}^* = \boldsymbol{q}^{(4)}$).

Exercise 5

Answer to the questions or comment/complete the statements, providing also a *short* motivation/explanation (within the given lines of text) for each of the 7 items.

1. Are there 3-dof robots with just a single inverse kinematics solution in their primary workspace? If so, which ones? If not, why?

A: *Yes, indeed. These are all PPP (Cartesian and gantry type) robots, no matter which is the sequence of prismatic joints —and also for non-perpendicular joint axes (twist angles $\alpha_i \neq \pm \pi/2$)!*

2. In order to measure the joint velocities of a robot, extra dedicated sensors may not be needed since ...

A: *... a digital position encoder, especially with high resolution and stability at high speed, can be used. Joint velocity is then estimated online by numerical differentiation of position measures (with various BDF = Backward Differentiation Formulas). Use of (kinematic) Kalman filters can also be considered.*

3. A large reduction ratio for a robot joint transmission is good because ..., and is bad because ...

A: *It is good because it amplifies the torque available beyond the transmission for accelerating loads with larger inertia. It is bad because larger reduction ratios are usually accompanied by higher energy dissipation due to friction and possible backlash, both effects reducing efficiency. On the other hand, speed reduction per se is not a major problem.*

4. Use of link acceleration measurements to generate torques that move the robot may be critical. Why?

A: *Assume no delays and no extra flexibility effects in a torque command loop based on the acceleration $\ddot{x}$ measured on a rotating link, at a distance d from its joint axis. Since we have $\tau = J\ddot{\theta} = (J/d)\ddot{x}$, torque and acceleration are at the same differential level. If we let τ depend on $\ddot{x}$, there would be an algebraic loop and thus problems with causality (and stability) of such feedback law.*

5. Compare an incremental encoder with $N = 900$ pulses per turn and quadrature electronics, mounted on a motor connected to the link with a reduction ratio $n_r = 40$, with a 16-bit absolute encoder mounted directly on the link side of the transmission. Which is better in terms of link position resolution?

A: *The resolution in the first case is $r_1 = 360°/(4N \cdot n_r) = 360°/(4 \cdot 900 \cdot 40) = 2.5 \cdot 10^{-3}$ [deg]. In the second case, the number of tracks $N_t = 16$ equals the bits used, so $r_2 = 360°/2^{N_t} = 360°/65536 = 5.5 \cdot 10^{-3}$ [deg]. Since $r_1 < r_2$, the considered incremental encoder arrangement gives a slightly better resolution (more than twice better, although still less than an order of magnitude).*

6. An installed 6-dof industrial robot has repeatability $\rho = 0.1$ [mm] and accuracy $\delta = 0.6$ [mm] in a certain region of its workspace. Which of these two parameters can be improved, and how?

A: *Repeatability depends on quality of the components (which cannot be changed on an installed robot). Robot accuracy can be improved instead by calibration (δ more than halved). Such procedures (software routines, with some extra sensing) can be used even if the robot is already installed on the factory floor.*

7. An object of mass $m = 5$ [kg] is hanging statically to a 6D F/T sensor, whose only non-zero outputs are $f_z = -49.05$ [N], $\mu_x = 7.3575$ [Nm]. Where is the object center of mass located in the sensor frame?

A: *A frame is placed at the center of the symmetric cylindric body of the F/T sensor, with its z-axis going up. In this sensor frame, the center of mass of the object is located on a vertical line passing through point $(x,y) = (0, -0.15)$ [m]. In fact, if the gravity force $f_z = -mg_0 = -5g_0 = -49.05$ [N] (in the opposite direction of z) has a vertical line of action crossing the negative y-axis at a distance $d = 0.15$ [m] from the origin, it will produce a positive (ccw) momentum around the x-axis of the sensor equal to $\mu_x = f_z \cdot (-d) = (-49.05) \cdot (-0.15) = 7.3575$ [Nm]. The object is certainly below the F/T sensor (it hangs), but nothing more can be said on the z-component of its position from this single measure.*

$* * * * *$