Robotics I

Midterm classroom test – November 24, 2017

Exercise 1 [8 points]

Consider the 3-dof (RPR) planar robot in Fig. 1, where the joint coordinates $\boldsymbol{r} = \begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix}^T$ have been defined in a free, arbitrary way, with reference to a base frame RF_b.

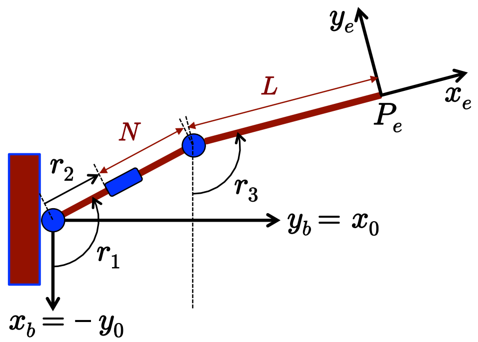


Figure 1: A RPR planar robot.

- Determine position and orientation of the end-effector frame RF_e in terms of the coordinates $\boldsymbol{r}$.
- Assign the Denavit-Hartenberg (DH) frames and define the joint coordinates $\boldsymbol{q} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}^T$ according to the standard DH convention, starting from the reference frame RF_0. Provide the DH table, the expression of the homogeneous transformation matrices between the successive frames that have been assigned. Determine position and orientation of the end-effector frame RF_e in terms of the coordinates $\boldsymbol{q}$.
- Find the transformation $\boldsymbol{q} = \boldsymbol{f}(\boldsymbol{r})$ between the two sets of coordinates so as to associate the same kinematic configurations of the robot. Does this mapping have singularities?

Exercise 2 [5 points]

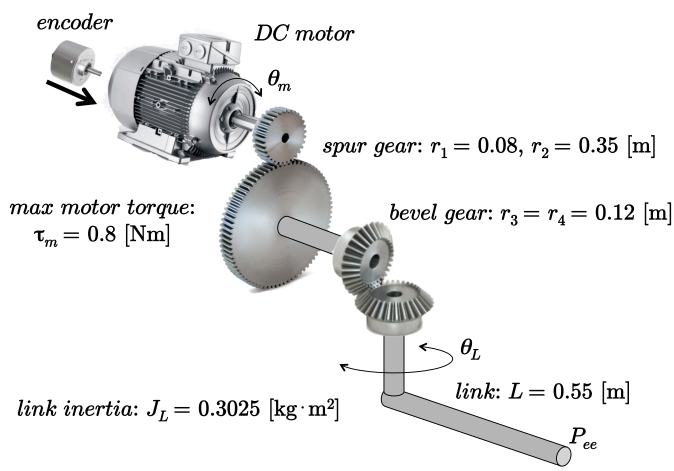
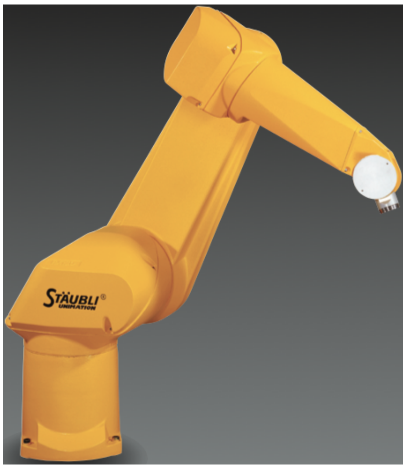


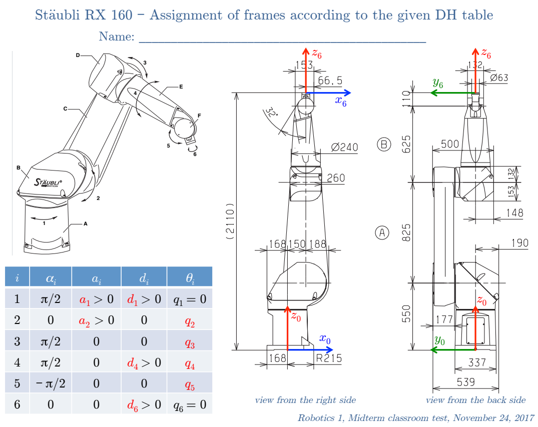
Figure 2: A DC servomotor that drives a robot link through transmissions.

With reference to the servo-drive sketched in Fig. 2 and the data therein, we need to measure the position of the tip point P_{ee} of the link with a resolution of 0.01 [mm]. A suitable incremental encoder with quadrature detection is chosen to measure the angular position θ_m of the DC motor, which is capable of delivering a maximum torque τ_m. How many pulses per turn should generated by the optical disk on each of the channels A and B? How many bits should be used by the digital counter of the encoder? Neglecting dissipative effects, what should be the value of the motor inertia J_m in order to maximize the angular acceleration of the link for a given motor torque? With this choice, what is the maximum linear acceleration achievable by the tip of the link?

Exercise 3 [12 points]

Consider the 6-dof robot Stäubli RX 160 in Fig. 3. In the extra sheet provided separately, the Denavit-Hartenberg (DH) table of parameters is specified, in part numerically and in part symbolically. The two DH frames 0 and 6 are already drawn on the manipulator (in two views). In the shown 'straight upward' robot configuration, the first and last joint variables take the values $q_1 = q_6 = 0$. Draw directly on the extra sheet the remaining DH frames, according to the DH table. Provide all parameters labeled in red in the table, i.e., the missing numerical values of the constant parameters and of the joint variables q_2 to q_5 when the robot is in the 'straight upward' configuration. [Please, make clean drawings and return the sheet with your name written on it.]





i	α_i	a_i	d_i	θ_i
1	$\pi/2$	$a_1 > 0$	$d_1 > 0$	$q_1 = 0$
2	0	$a_2 > 0$	0	q_2
3	$\pi/2$	0	0	q_3
4	$\pi/2$	0	$d_4 > 0$	q_4
5	$-\pi/2$	0	0	q_5
6	0	0	$d_6 > 0$	$q_6 = 0$

Figure 3: The Stäubli RX 160 robot manipulator and the extra sheet (provided separately).

Exercise 4 [5 points]

The orientations of two right-handed frames RF_A and RF_B with respect to a third right-handed frame RF_0 (all having the same origin) are specified, respectively, by the rotation matrices

$$
{}^0\boldsymbol{R}_A = \begin{pmatrix} \frac{3}{4} & \sqrt{\frac{3}{8}} & -\frac{1}{4} \\ -\sqrt{\frac{3}{8}} & \frac{1}{2} & -\sqrt{\frac{3}{8}} \\ -\frac{1}{4} & \sqrt{\frac{3}{8}} & \frac{3}{4} \end{pmatrix} \qquad \text{and} \qquad {}^0\boldsymbol{R}_B = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}.
$$

Determine, if possible, a unit vector $\boldsymbol{r}$ and an angle $\theta < 0$ such that the axis-angle rotation matrix $\boldsymbol{R}(\boldsymbol{r}, \theta)$ provides the orientation of the frame RF_B with respect to the frame RF_A.

[180 minutes, open books but no computer or smartphone]

Solution of Midterm Test

November 24, 2017

Exercise 1

The direct kinematics in terms of the coordinates $\boldsymbol{r} = \begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix}^T$ is easily computed as

$$
{}^b\boldsymbol{p}_e(\boldsymbol{r}) = \begin{pmatrix} (r_2+N)\cos r_1 + L\cos r_3 \\ (r_2+N)\sin r_1 + L\sin r_3 \\ 0 \end{pmatrix}, \qquad {}^b\boldsymbol{R}_e(\boldsymbol{r}) = \begin{pmatrix} \cos r_3 & -\sin r_3 & 0 \\ \sin r_3 & \cos r_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
$$

Indeed, if these quantities have to be expressed with respect to the (DH) frame 0 indicated in Fig. 1, we need to introduce a constant rotation. Since the two frames have the same origin, we obtain

$$
{}^0\boldsymbol{R}_b = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \Rightarrow \quad
\begin{aligned}
{}^0\boldsymbol{p}_e(\boldsymbol{r}) &= {}^0\boldsymbol{R}_b\,{}^b\boldsymbol{p}_e(\boldsymbol{r}) = \begin{pmatrix} (r_2+N)\sin r_1 + L\sin r_3 \\ -(r_2+N)\cos r_1 - L\cos r_3 \\ 0 \end{pmatrix}, \\
{}^0\boldsymbol{R}_e(\boldsymbol{r}) &= {}^0\boldsymbol{R}_b\,{}^b\boldsymbol{R}_e(\boldsymbol{r}) = \begin{pmatrix} \sin r_3 & \cos r_3 & 0 \\ -\cos r_3 & \sin r_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\end{aligned}
$$

Keeping into account the already specified assignment of frame 0 in Fig. 1, a feasible assignment of DH frames for the RPR robot is shown in Fig. 4, with associated parameters given in Tab. 1.

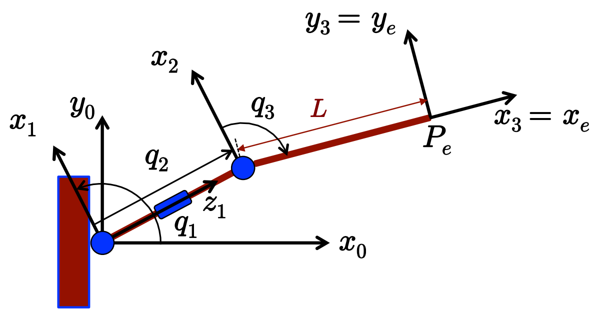


Figure 4: Assignment of DH frames for the RPR planar robot.

i	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	0	q_1
2	$-\pi/2$	0	q_2	0
3	0	L	0	q_3

Table 1: Parameters associated to the DH frames in Fig. 4.

The DH homogenous transformations take the following expressions:

$$
{}^0\boldsymbol{A}_1(q_1) = \begin{pmatrix} \cos q_1 & 0 & \sin q_1 & 0 \\ \sin q_1 & 0 & -\cos q_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad {}^1\boldsymbol{A}_2(q_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix},
$$

$$
{}^2\boldsymbol{A}_3(q_3) = \begin{pmatrix} \cos q_3 & -\sin q_3 & 0 & L\cos q_3 \\ \sin q_3 & \cos q_3 & 0 & L\sin q_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
$$

The direct kinematics in terms of the coordinates $\boldsymbol{q} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}^T$ is then

$$
{}^0\boldsymbol{T}_e(\boldsymbol{q}) = {}^0\boldsymbol{A}_1(q_1)\,{}^1\boldsymbol{A}_2(q_2)\,{}^2\boldsymbol{A}_3(q_3) = \begin{pmatrix} {}^0\boldsymbol{R}_e(\boldsymbol{q}) & {}^0\boldsymbol{p}_e(\boldsymbol{q}) \\ 0\ 0\ 0 & 1 \end{pmatrix},
$$

with

$$
{}^0\boldsymbol{p}_e(\boldsymbol{q}) = \begin{pmatrix} q_2 \sin q_1 + L\cos(q_1+q_3) \\ -q_2\cos q_1 + L\sin(q_1+q_3) \\ 0 \end{pmatrix}, \quad {}^0\boldsymbol{R}_e(\boldsymbol{q}) = \begin{pmatrix} \cos(q_1+q_3) & -\sin(q_1+q_3) & 0 \\ \sin(q_1+q_3) & \cos(q_1+q_3) & 0 \\ 0 & 0 & 1 \end{pmatrix}.
$$

The transformation that matches the robot configurations using the different sets of coordinates is

$$
\boldsymbol{q} = \boldsymbol{f}(\boldsymbol{r}) = \begin{pmatrix} r_1 \\ r_2 + N \\ r_3 - r_1 - \dfrac{\pi}{2} \end{pmatrix} \qquad \Longleftrightarrow \qquad \boldsymbol{r} = \boldsymbol{f}^{-1}(\boldsymbol{q}) = \begin{pmatrix} q_1 \\ q_2 - N \\ q_1 + q_3 + \dfrac{\pi}{2} \end{pmatrix},
$$

which is invertible and without singularities. It is easy to check that, e.g.,

$$
{}^0\boldsymbol{p}_e(\boldsymbol{q})|_{\boldsymbol{q}=\boldsymbol{f}(\boldsymbol{r})} = {}^0\boldsymbol{p}_e(\boldsymbol{r}), \qquad {}^0\boldsymbol{R}_e(\boldsymbol{q})|_{\boldsymbol{q}=\boldsymbol{f}(\boldsymbol{r})} = {}^0\boldsymbol{R}_e(\boldsymbol{r}).
$$

Exercise 2

The requested resolution $\Delta = 10^{-5}$ [m] on the linear motion at the tip of the link of length $L = 0.55$ [m] needs to be transformed into an angular one $\delta = \Delta/L = 1.8182 \cdot 10^{-5}$ [rad] at the base of the link and then, via the two transmission gears with reduction ratios $N_{\text{bevel}} = r_4/r_3 = 1$ and $N_{\text{spur}} = r_2/r_1 = 0.35/0.08 = 4.375$ respectively, into an angular resolution on the motor axis

$$
\delta_m = N_{\text{spur}} \cdot N_{\text{bevel}} \cdot \frac{\Delta}{L} = 7.9545 \cdot 10^{-5}\ [\text{rad}] = (4.56 \cdot 10^{-3})^\circ.
$$

Taking into account the factor 4 introduced by the quadrature electronics, the number of pulses per turn N_{ppt} of the optical disk should be at least

$$
N_{\text{ppt}} = \left\lceil \frac{2\pi}{4 \cdot \delta_m} \right\rceil = 19748.
$$

Accordingly, the digital counter should have at least a number of bits

$$
N_{\text{bit}} = \lceil \log_2 19748 \rceil = 15.
$$

Being the link inertia (around its axis of rotation) $J_L = 0.3025$ [kg·m^2] and the reduction ratio of the transmissions $N = N_{\rm spur} \cdot N_{\rm bevel} = 4.375$, the optimal value of the motor inertia according to the requested criterion is

$$J_m = \frac{J_L}{N^2} = 0.0158\,[\text{kg·m}^2].$$

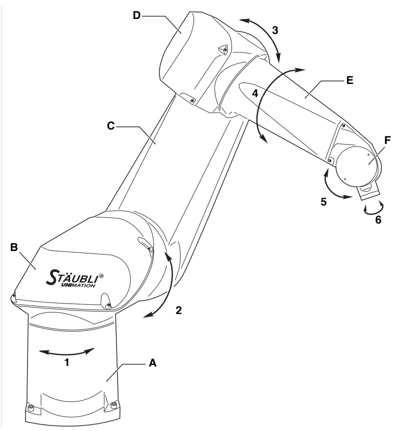
Accordingly, with a maximum available motor torque $\tau_m = 0.8$ [Nm] on the motor axis, the maximum torque at the link base is $\tau_{L,\max} = N \cdot \tau_m = 3.5$ [Nm]. Since $N^2 = J_L/J_m$, the balance of torques on the link side provides

$$\tau_L = J_L\ddot{\theta}_L + N(J_m\ddot{\theta}_m) = (J_L + J_m N^2)\ddot{\theta}_L = 2J_L\ddot{\theta}_L.$$

The maximum tip acceleration $a_{\max}$ of the link tip will then be

$$a_{\max} = L\,\ddot{\theta}_{L,\max} = L\,\frac{\tau_{L,\max}}{2\,J_L} = 0.55\,[\text{m}] \cdot 5.7851\,[\text{rad·s}^{-2}] = 3.1818\,[\text{m·s}^{-2}].$$

Exercise 3



i	α_i	a_i	d_i	θ_i
1	$\pi/2$	150	550	$q_1 = 0$
2	0	825	0	$q_2 = \pi/2$
3	$\pi/2$	0	0	$q_3 = \pi/2$
4	$\pi/2$	0	625	$q_4 = \pi$
5	$-\pi/2$	0	0	$q_5 = 0$
6	0	0	110	$q_6 = 0$

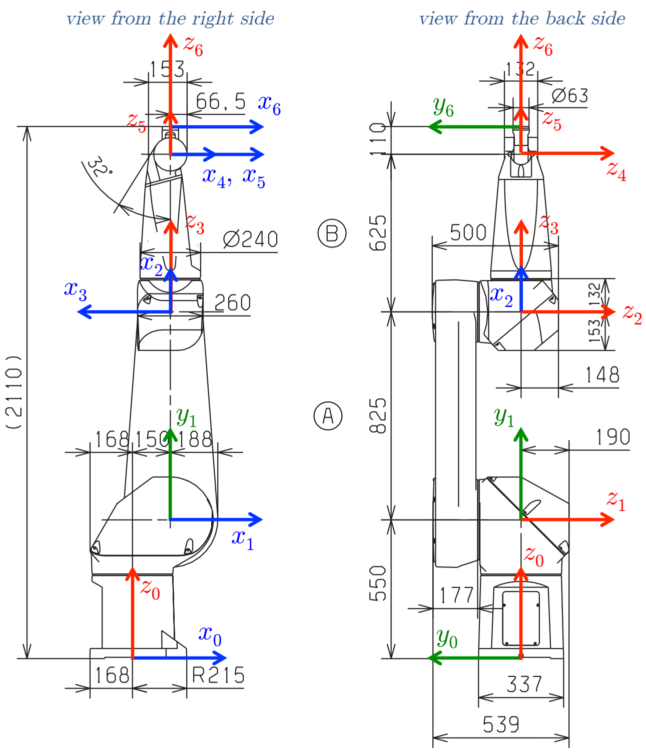


Figure 5: The DH frames and the complete table of parameters for the Stäubli RX 160 robot manipulator. The numerical values of $\boldsymbol{q} = \boldsymbol{\theta}$ refer to the shown 'straight upward' configuration.

The assignment of Denavit-Hartenberg frames for the Stäubli robot RX 160 according to the given table is shown in Fig. 5. The numerical values of all symbolic parameters are reported, with the values of the joint variables $\boldsymbol{q} = \boldsymbol{\theta}$ when the robot is in the 'straight upward' configuration.

Exercise 4

The relative orientation of frame RF_B with respect to frame RF_A is expressed by the rotation matrix

$$
{}^A\boldsymbol{R}_B = {}^0\boldsymbol{R}_A^T \cdot {}^0\boldsymbol{R}_B = \begin{pmatrix} \frac{1}{2\sqrt{2}} & -\sqrt{\frac{3}{8}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2\sqrt{2}} & -\sqrt{\frac{3}{8}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0.3536 & -0.6124 & -0.7071 \\ 0.8660 & 0.5 & 0 \\ 0.3536 & -0.6124 & 0.7071 \end{pmatrix},
$$

whose elements will be denoted by R_{ij}. Therefore, the equation $\boldsymbol{R}(\boldsymbol{r},\theta) = {}^A\boldsymbol{R}_B$ should be solved for $\boldsymbol{r}$ and θ, using the inverse mapping of the axis-angle representation. Since

$$
\sin\theta = \pm\frac{1}{2}\sqrt{(R_{12}-R_{21})^2 + (R_{13}-R_{31})^2 + (R_{23}-R_{32})^2} = \pm\, 0.9599 \neq 0, \tag{1}
$$

the problem at hand is regular, and two distinct solutions can be found depending on the choice of the $+$ or $-$ sign in the expression of $\sin\theta$. From

$$
\cos\theta = \frac{1}{2}\left(R_{11} + R_{22} + R_{33} - 1\right) = 0.2803,
$$

taking the $-$ sign in (1) will yield a solution angle $\theta < 0$, as requested. Thus

$$
\theta = \text{ATAN2}\left\{-0.9599, 0.2803\right\} = -1.2867 \text{ [rad]} = -73.72^\circ
$$

and

$$
\boldsymbol{r} = \frac{1}{2\sin\theta}\begin{pmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{pmatrix} = \begin{pmatrix} 0.3190 \\ 0.5525 \\ -0.7701 \end{pmatrix}.
$$

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