## Robotics 1

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## Exercise 1

Consider the spatial 6-dof robot in Fig. 1.


Figure 1: A spatial 6-dof robot, with three prismatic joints followed by three revolute joints.

- Assign the frames according to the standard Denavit-Hartenberg (DH) convention and provide the corresponding table of parameters. The origin of the last DH frame should coincide with point $P$. Specify the signs of the linear DH parameters that are constant and non-zero, as well as the signs of the joint variables $q_{i}, i=1, \ldots, 6$, in the shown configuration.
- Determine the symbolic expression of all elements in the $6 \times 6$ geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$ of this robot and check that $\boldsymbol{q}_{0}=(1,1,1,-\pi / 2,-\pi / 2,-\pi / 2)$ is a nonsingular configuration.
- At $\boldsymbol{q}_{0}$, find the position of point $P$. Moreover, compute a joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^{6}$ that produces the velocity ${ }^{0} \boldsymbol{v}=(0.5,2,-2)[\mathrm{m} / \mathrm{s}]$ of $P$, while the end-effector has an angular velocity ${ }^{0} \boldsymbol{\omega}=(0,3,0)[\mathrm{rad} / \mathrm{s}]$.


## Exercise 2

Consider the planar 2P (Cartesian) robot in Fig. 2, where $m_{1}$ and $m_{2}$ are the masses of the two links in the serial chain. Each input force $F_{i}$ is bounded in absolute value by $F_{i, \max }>0$, for $i=1,2$. Find the expression of the minimum feasible time $T$ for a rest-to-rest robot motion from a start configuration $\boldsymbol{q}_{s}$ to a goal configuration $\boldsymbol{q}_{g}$. Compute the numerical value of $T$ with the following data: $m_{1}=5, m_{2}=2[\mathrm{~kg}]$; $F_{1, \text { max }}=10, F_{2, \text { max }}=5[\mathrm{~N}] ; \boldsymbol{q}_{s}=(0.3,-0.3), \boldsymbol{q}_{g}=(-0.3,0.3)[\mathrm{m}]$. Plot the evolutions of $F_{i}(t), \dot{q}_{i}(t)$, and $q_{i}(t)$, for $i=1,2$. In your solution, does the mass $m_{2}$ trace a linear path during the time-optimal motion?


Figure 2: A planar Cartesian robot.

