## Robotics 1

## October 19, 2021

## Exercise \#1

Consider the 3-dof PPR robot in Fig. 1, with a jaw gripper mounted on the end effector.


Figure 1: A 3-dof PPR robot.
a) Assign and draw the robot frames according to the Denavit-Hartenberg (DH) convention. Place the origin of frame 0 on the floor and the origin of the last frame at the center of the gripper. Compile the associated table of DH parameters.
b) Check whether the last DH frame assigned coincides in orientation with the definition of the standard frame $(\boldsymbol{n}, \boldsymbol{s}, \boldsymbol{a})$ attached to a jaw gripper. If not, determine the rotation matrix ${ }^{3} \boldsymbol{R}_{g}$ needed to align the two frames.
c) Provide the expression of the direct kinematics $\boldsymbol{p}=\boldsymbol{f}(\boldsymbol{q})$ between $\boldsymbol{q}=\left(q_{1}, q_{2}, q_{3}\right)$ and the position $\boldsymbol{p}=\left(p_{x}, p_{y}, p_{z}\right)$ of the center of the gripper.
d) Derive the $3 \times 3$ Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ relating $\dot{\boldsymbol{q}}$ to the linear velocity $\boldsymbol{v}=\dot{\boldsymbol{p}}$ in two different ways, as part of the geometric Jacobian of the robot and using differentiation w.r.t. time.
e) Find all the singular configurations of matrix $\boldsymbol{J}(\boldsymbol{q})$. In one of such configurations $\boldsymbol{q}_{s}$, characterize which Cartesian directions are instantaneously accessible by the robot gripper and which not.

## Exercise \#2

For the robot in Fig. 1, using the associated symbolic DH parameters, determine a smooth and coordinated rest-to-rest joint trajectory that will move in $T$ seconds the robot gripper from the initial position $\boldsymbol{p}_{i}=\left(a_{1}+a_{3}, 0,0\right)$ to the final position $\boldsymbol{p}_{f}=\left(a_{1},-\delta, 0\right)$, with $\delta>0$. Sketch a plot of the obtained joint trajectory $\boldsymbol{q}_{d}(t)=\left(q_{1 d}(t), q_{2 d}(t), q_{3 d}(t)\right)$. What will be the maximum value of the norm of the joint velocity $\left\|\dot{\boldsymbol{q}}_{d}(t)\right\|$ during the interval $[0, T]$ ?

