## Robotics I

April 2, 2014

## Exercise 1

Consider a robot with four revolute joints, having the Denavit-Hartenberg parameters of Table 1.

| $i$ | $\alpha_{i}$ | $a_{i}$ | $d_{i}$ | $\theta_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\pi / 2$ | $a_{1}>0$ | $d_{1}>0$ | $q_{1}$ |
| 2 | $\pi / 2$ | $a_{2}>0$ | 0 | $q_{2}$ |
| 3 | $-\pi / 2$ | 0 | $d_{3}>0$ | $q_{3}$ |
| 4 | 0 | $a_{4}>0$ | 0 | $q_{4}$ |

Table 1: Denavit-Hartenberg parameters of a 4-dof robot

- Sketch the robot and the associated Denavit-Hartenberg frames in two different configurations: i) $\boldsymbol{q}_{A}=\mathbf{0}$, and ii) $\boldsymbol{q}_{B}=\left(\begin{array}{llll}0 & \pi / 2 & -\pi / 2 & \pi / 2\end{array}\right)^{T}$.
- Provide the symbolic expression of the direct kinematics map $\boldsymbol{p}=\boldsymbol{f}(\boldsymbol{q}) \in \mathbb{R}^{3}$ for the position $\boldsymbol{p}$ of the origin of frame 4.


## Exercise 2

For the robot of Exercise 1, find the joint torque $\boldsymbol{\tau} \in \mathbb{R}^{4}$ that balances a force ${ }^{0} \boldsymbol{F}=\left(\begin{array}{lll}0 & 10 & 0\end{array}\right)^{T}[\mathrm{~N}]$ applied to the origin of frame 4 , when the robot is in the configuration $\boldsymbol{q}=\boldsymbol{q}_{B}$. Keep the symbolic dependence on parameters that are not specified numerically.

## Exercise 3

Plan a smooth minimum time trajectory $q_{d}(t)$ for a robot joint that provides rest-to-rest motion from $q_{i n}=90^{\circ}$ to $q_{f i n}=-90^{\circ}$, with velocity and acceleration equal to zero at the initial and final instants and satisfying the bounds $\left|\dot{q}_{d}(t)\right| \leq 90^{\circ} / \mathrm{s}$ and $\left|\ddot{q}_{d}(t)\right| \leq 90^{\circ} / \mathrm{s}^{2}$. Give the final expression of $q_{d}(t)$ and plot approximately this solution trajectory and its first and second time derivatives. Provide also the minimum feasible time $T$ and the maximum absolute value attained by the velocity and by the acceleration.
[180 minutes; open books]

