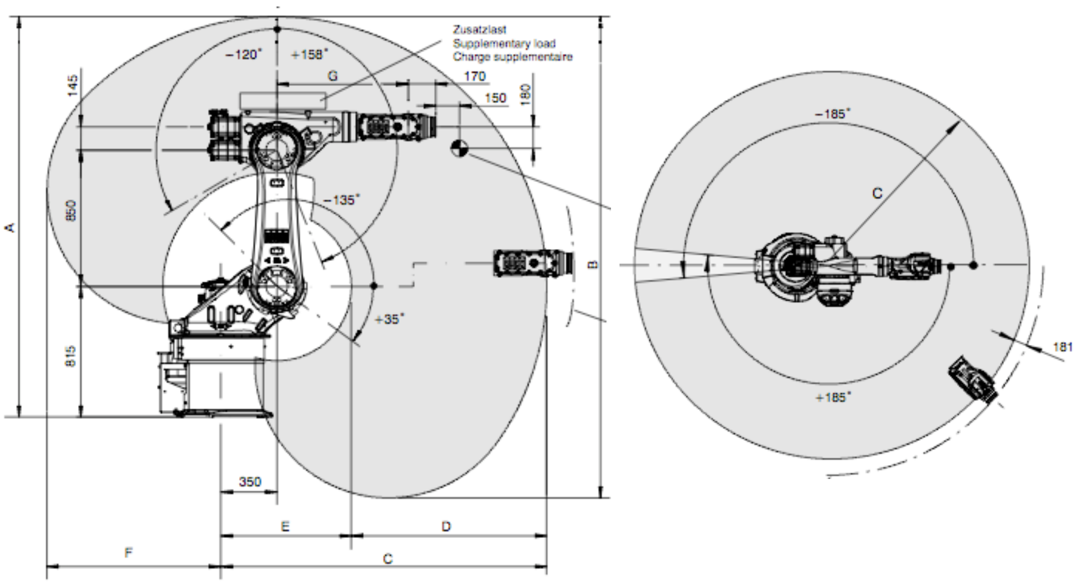
Robotics I

July 7, 2010

Based on the data sheet of the KUKA KR 30-3 robot with six revolute joints and spherical wrist:

i) assign the reference frames according to the Denavit-Hartenberg formalism, putting the origin of the reference frame RF_0 on the floor, and derive the associated table of parameters;

ii) using the numerical data (in mm), determine the position of the spherical wrist center with respect to RF_0 in the configuration $\boldsymbol{\theta} = (0, \pi/2, 0, \theta_4, \theta_5, \theta_6)$, for arbitrary θ_i, $i = 4, 5, 6$.



	A	B	C	D	E	F	G
KR 30-3	2498	3003	2033	1218	815	1084	820

[120 minutes; open books]

Solution

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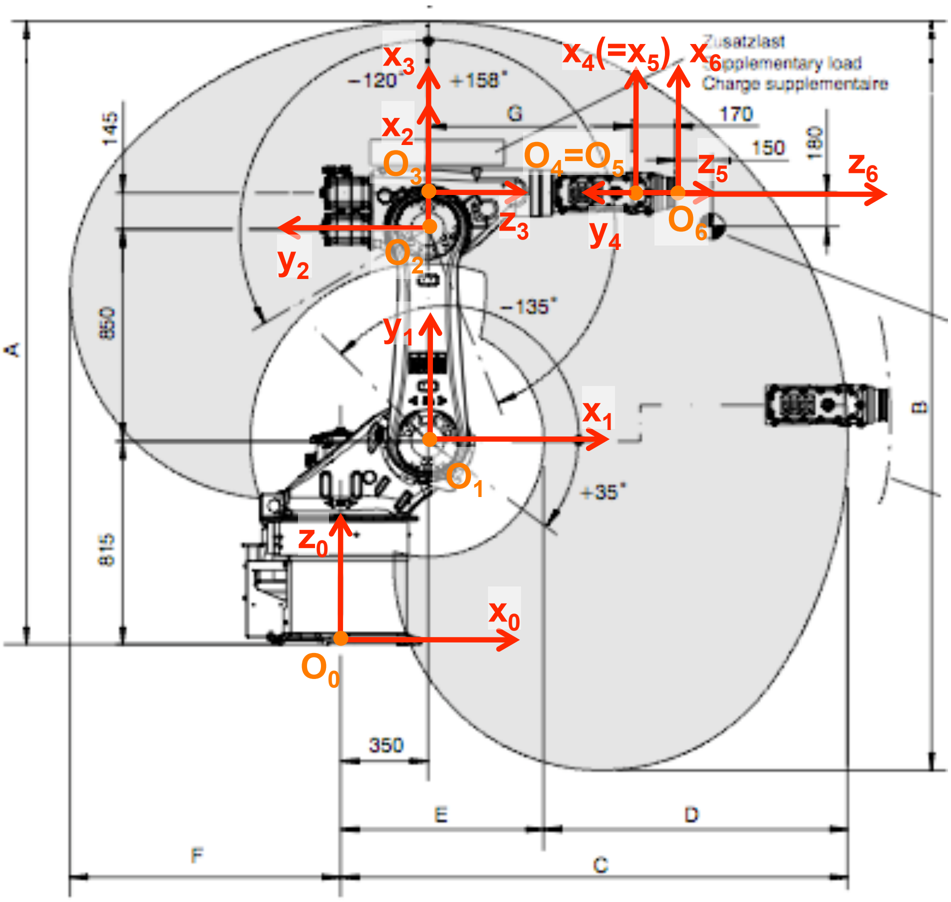

Figure 1: D-H frame assignment for the KUKA KR 30-3 robot

With reference to Fig. 1, the table of Denavit-Hartenberg parameters is the following:

i	α_i	d_i	a_i	θ_i
1	$\pi/2$	$d_1 = 815$	$a_1 = 350$	θ_1
2	0	0	$a_2 = 850$	θ_2
3	$\pi/2$	0	$a_3 = 145$	θ_3
4	$-\pi/2$	$d_4 = 820$	0	θ_4
5	$\pi/2$	0	0	θ_5
6	0	$d_6 = 170$	0	θ_6

The numerical values (in mm) assigned to the constant parameters in the table are taken from the data sheet. The robot configuration shown corresponds to $\boldsymbol{\theta} = (0\ \frac{\pi}{2}\ 0\ 0\ 0\ 0)^T$. Only the first three homogeneous transformation matrices are needed for determining the position of the center $W = 0_4 = 0_5$ of the spherical wrist:

$$
{}^0\boldsymbol{A}_1(\theta_1) = \begin{pmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^1\boldsymbol{A}_2(\theta_2) = \begin{pmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2\boldsymbol{A}_3(\theta_3) = \begin{pmatrix} c_3 & 0 & s_3 & a_3c_3 \\ s_3 & 0 & -c_3 & a_3s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
$$

In fact, we may compute the position ${}^0\boldsymbol{p}_{04}$ of O_4 as

$$
{}^0\boldsymbol{p}_{04,hom} = \begin{pmatrix} {}^0\boldsymbol{p}_{04} \\ 1 \end{pmatrix} = {}^0\boldsymbol{A}_1(\theta_1)\,{}^1\boldsymbol{A}_2(\theta_2)\,{}^2\boldsymbol{A}_3(\theta_3)\,{}^3\boldsymbol{A}_4(\theta_4) \begin{pmatrix} \boldsymbol{0} \\ 1 \end{pmatrix}
$$

but also as

$$
{}^0\boldsymbol{p}_{04,hom} = {}^0\boldsymbol{A}_1(\theta_1)\,{}^1\boldsymbol{A}_2(\theta_2)\,{}^2\boldsymbol{A}_3(\theta_3) \begin{pmatrix} {}^3\boldsymbol{p}_{34} \\ 1 \end{pmatrix} \qquad \text{with} \quad {}^3\boldsymbol{p}_{34} = \begin{pmatrix} 0 \\ 0 \\ d_4 \end{pmatrix}.
$$

The most efficient way is to perform computations as matrix/vector products:

$$
{}^2\boldsymbol{p}_{24,hom} = {}^2\boldsymbol{A}_3(\theta_3)\,{}^3\boldsymbol{p}_{34,hom} = \begin{pmatrix} a_3c_3 + d_4s_3 \\ a_3s_3 - d_4c_3 \\ 0 \\ 1 \end{pmatrix},
$$

$$
{}^1\boldsymbol{p}_{14,hom} = {}^1\boldsymbol{A}_2(\theta_2)\,{}^2\boldsymbol{p}_{24,hom} = \begin{pmatrix} (a_2 + a_3c_3 + d_4s_3)c_2 - (a_3s_3 - d_4c_3)s_2 \\ (a_2 + a_3c_3 + d_4s_3)s_2 + (a_3s_3 - d_4c_3)c_2 \\ 0 \\ 1 \end{pmatrix},
$$

$$
{}^0\boldsymbol{p}_{04,hom} = {}^0\boldsymbol{A}_1(\theta_1)\,{}^1\boldsymbol{p}_{14,hom} = \begin{pmatrix} (a_1 + (a_2 + a_3c_3 + d_4s_3)c_2 - (a_3s_3 - d_4c_3)s_2)c_1 \\ (a_1 + (a_2 + a_3c_3 + d_4s_3)c_2 - (a_3s_3 - d_4c_3)s_2)s_1 \\ d_1 + (a_2 + a_3c_3 + d_4s_3)s_2 + (a_3s_3 - d_4c_3)c_2 \\ 1 \end{pmatrix}.
$$

At the desired configuration (for the first three joints), substituting the numerical values, yields

$$
{}^0\boldsymbol{p}_{04}(0, \frac{\pi}{2}, 0) = \begin{pmatrix} a_1 + d_4 \\ 0 \\ d_1 + a_2 + a_3 \end{pmatrix} = \begin{pmatrix} 1170 \\ 0 \\ 1810 \end{pmatrix} \text{ [mm]}.
$$

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