Robotics 1

Wheeled Mobile Robots
Introduction and Kinematic Modeling

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Summary

- introduction
  - Wheeled Mobile Robot (WMR)
  - operating environments
  - basic motion problem
  - elementary tasks
  - block diagram of a mobile robot
- kinematic modeling
  - configuration space
  - wheel types
  - nonholonomic constraints (due to wheel rolling)
  - kinematic model of WMR
- examples of kinematic models
  - unicycle
  - car-like

Wheeled mobile robots

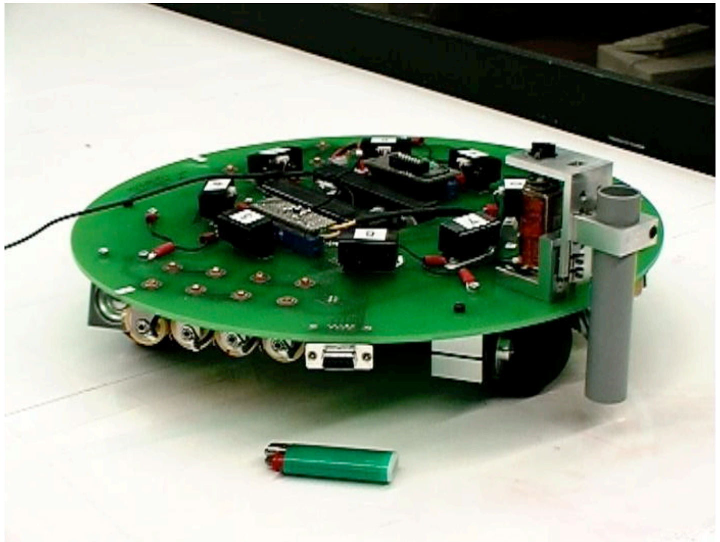
- locally restricted mobility ⟺ NONHOLONOMIC constraints

SuperMARIO & MagellanPro
(DIS, Roma)

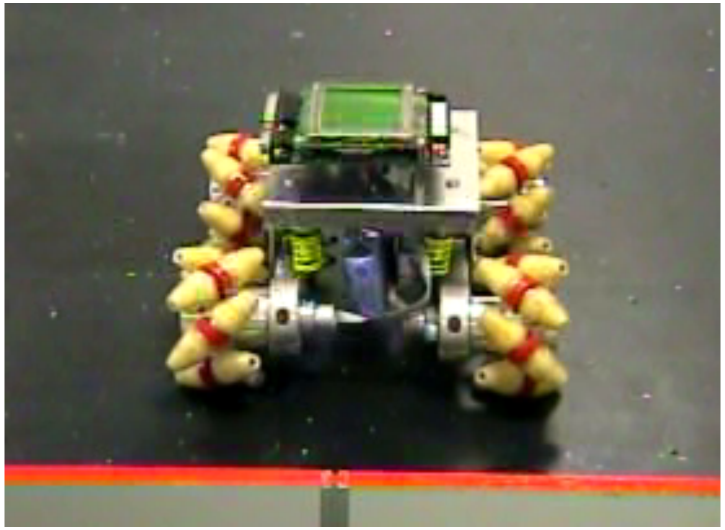
Hilare 2-Bis (LAAS, Toulouse)
with "off-hooked" trailer

Wheeled mobile robots

- full mobility ⟷ OMNIDIRECTIONAL robots



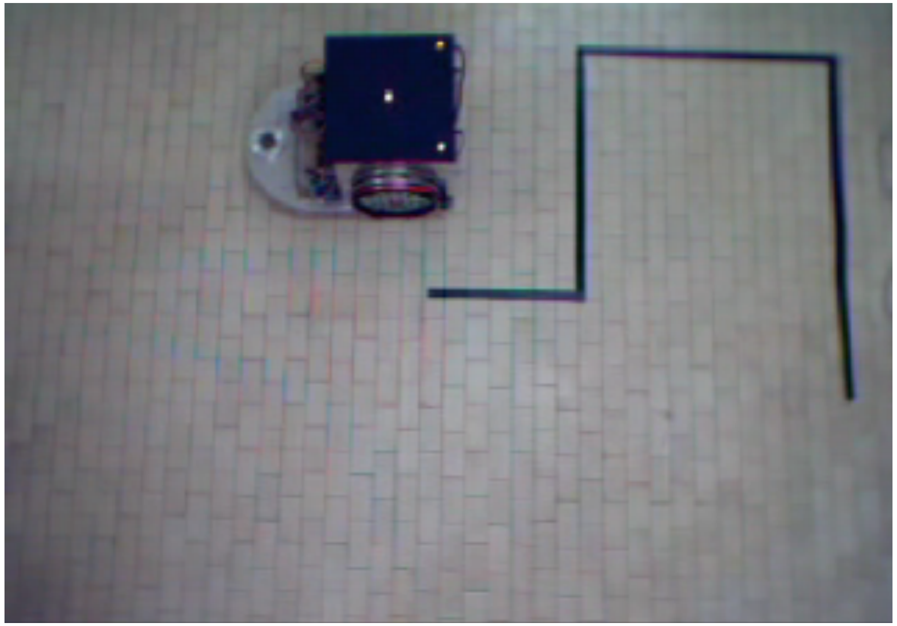
Tribolo

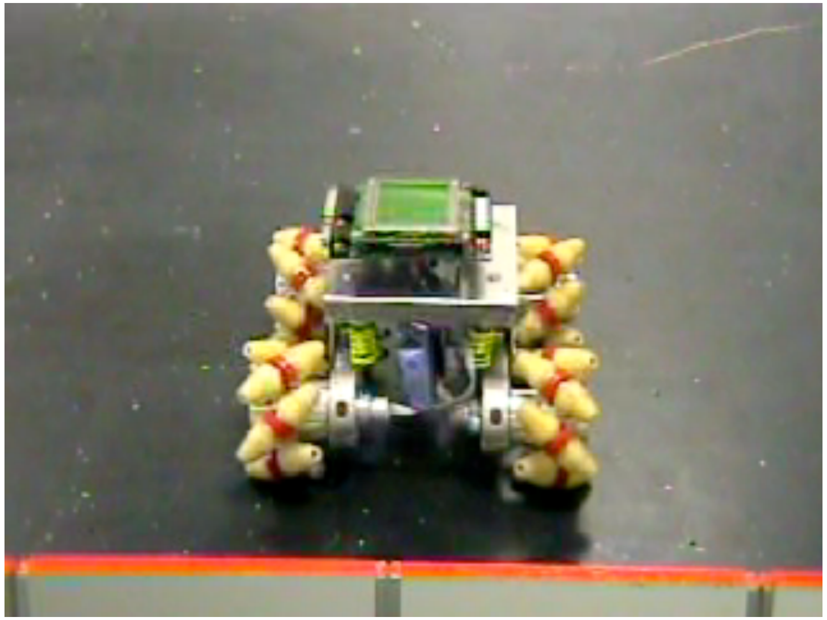


Omni-2

Video

- SuperMARIO
- Omni-2

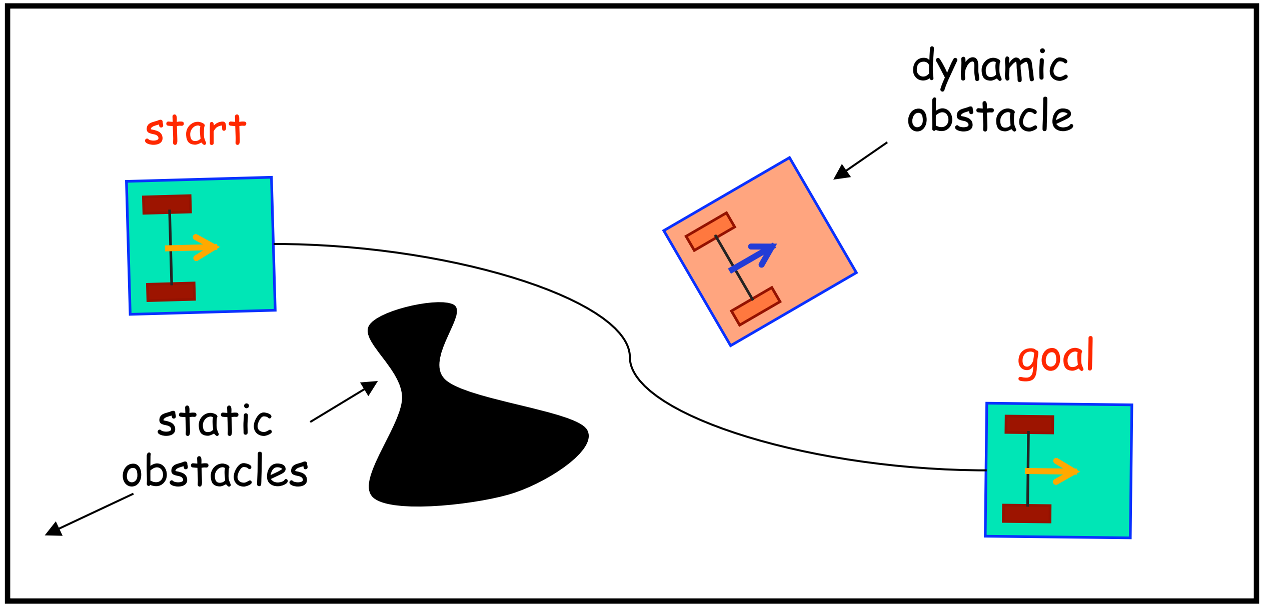




Operating environments

- external 3D
  - unstructured
    - natural vs. artificial landmarks
- internal 2D
  - known
    - availability of a map (possibly acquired by robot sensors in an exploratory phase)
  - unknown
    - with static or dynamic obstacles

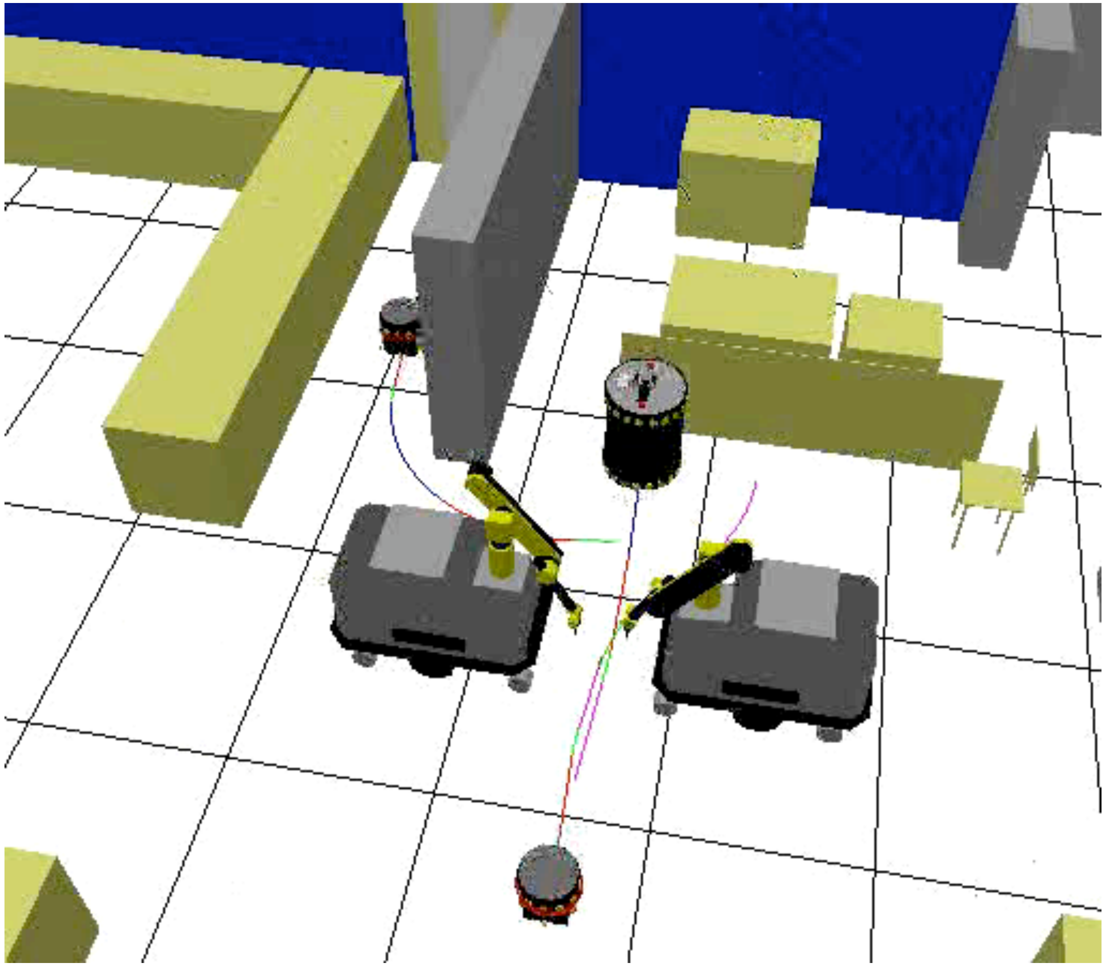
Basic motion problem



- high computational complexity of the planning problem
- dynamic environment (including multiple robots)
- restricted mobility of robotic vehicle

⇒ analysis of elementary tasks

Multi-robot environment



2 Pioneer
1 Nomad XR-400
2 Hilare with on-board manipulator arm

- 5 robots in simultaneous motion

Elementary motion tasks

- point-to-point motion
  - in the configuration space
- path following
- trajectory tracking
  - geometric path + timing law
- purely reactive (local) motion

⬇

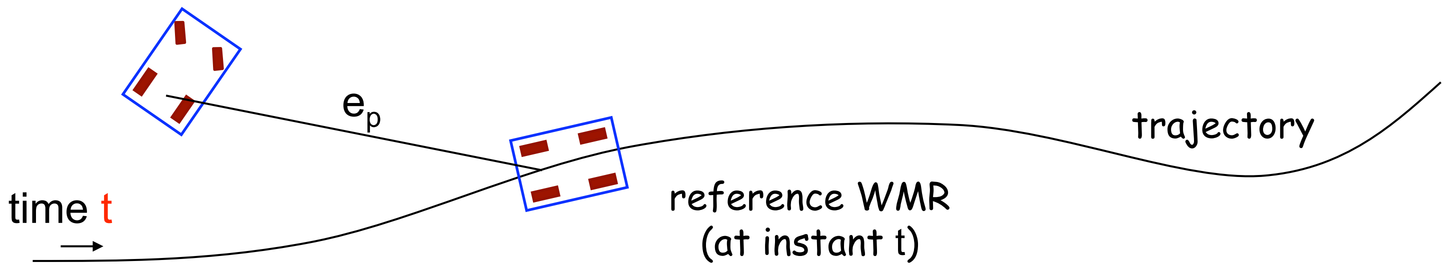
mixed situations of planning and control

Elementary motion tasks (cont'd)

- point-to-point motion (e.g., parking)

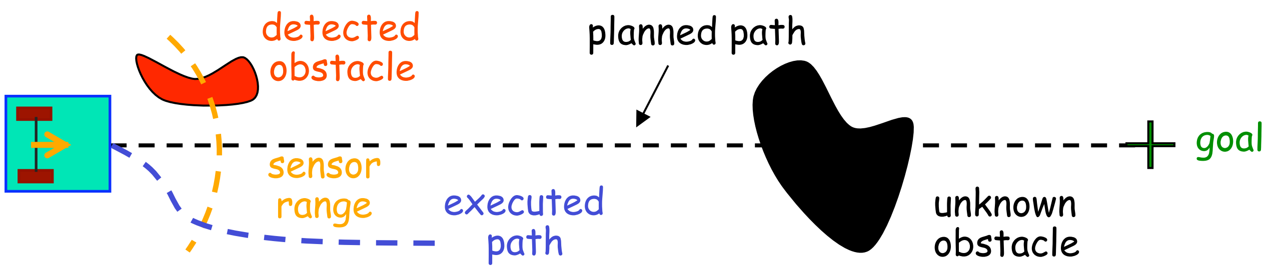
- path following

- trajectory tracking

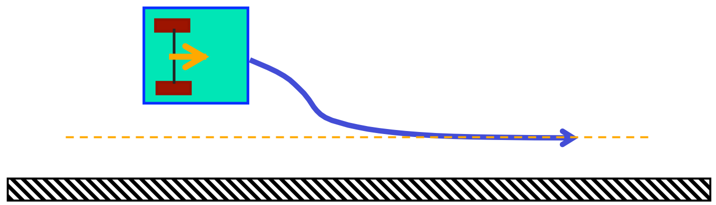


Elementary motion tasks (cont'd)

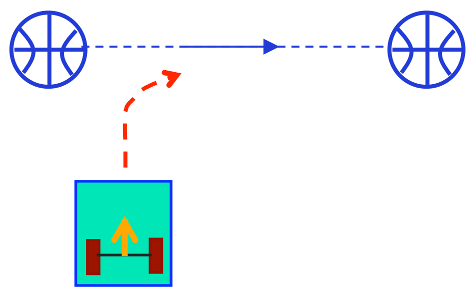
- examples of reactive motion
  - on-line obstacle avoidance



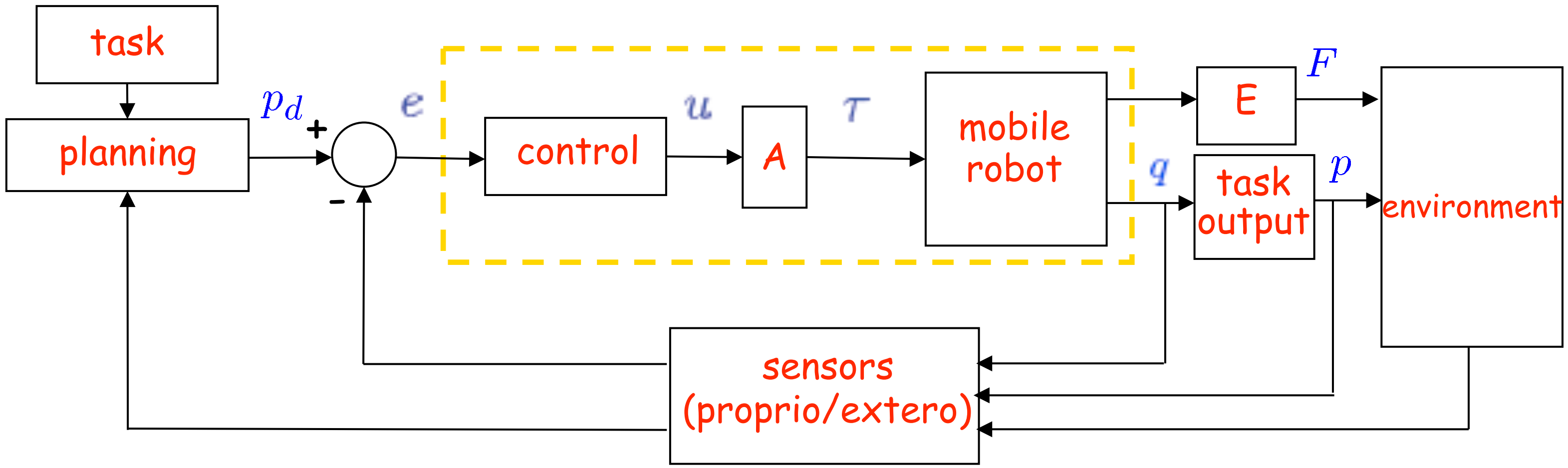
  - wall following



  - target tracking



Block diagram of a mobile robot



actuators (A) DC motors with reduction

task output (even identity, i.e., q)

effectors (E) on-board manipulator, gripper, …

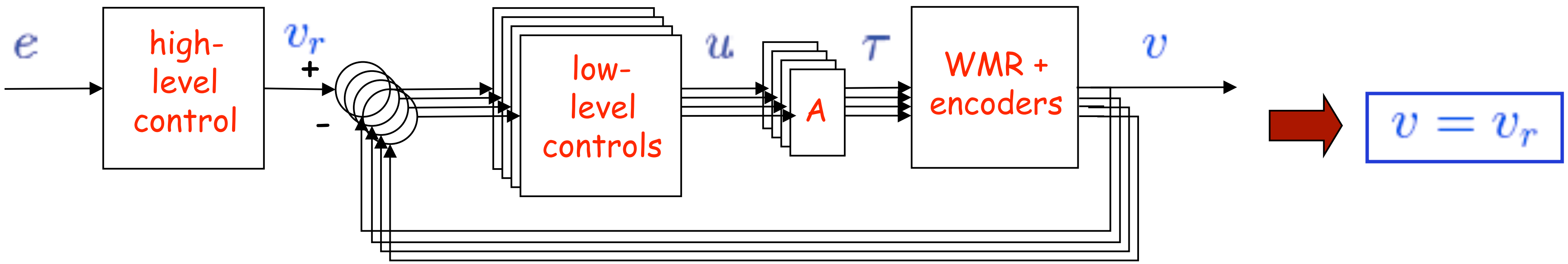
sensors

- proprioceptive: encoders, gyroscope, …
- exteroceptive: bumpers, rangefinders (IR = infrared, US = ultrasound), structured light (laser+CCD), vision (mono, stereo, color, …)

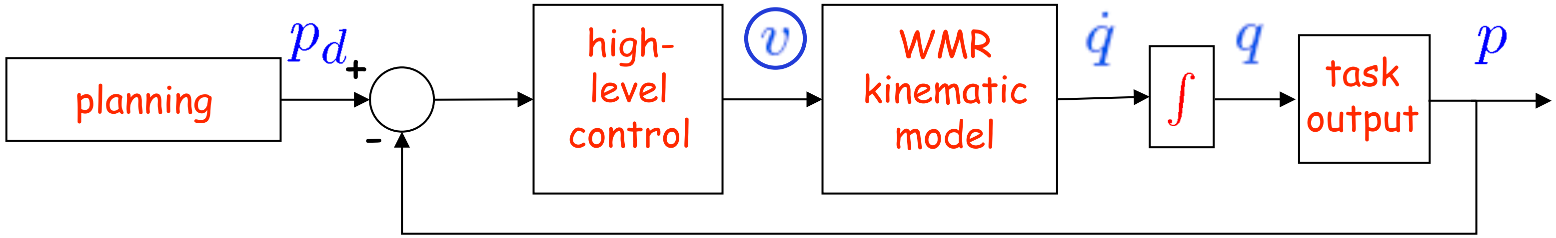
control

- high- / low-level
- feedforward (from planning) / feedback

Block diagram of a mobile robot (cont'd)



low-level control: analog velocity PI(D) loop with high gain (or digital, at high frequency)



high-level control: purely kinematics-based, with velocity commands

Configuration space

for wheeled mobile robots

- rigid body (one, or many interconnected)

pose of one body is given by a set of INDEPENDENT variables

  # total of descriptive variables (including all bodies)
- # total of HOLONOMIC (positional) constraints

  # generalized coordinates

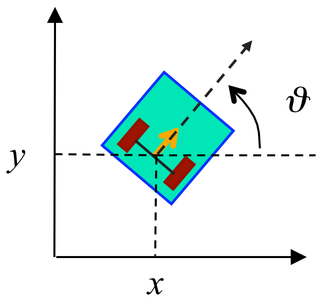
- wheels (of different types) in contact with the ground

(possibly) additional INTERNAL variables

configuration space $\mathcal{C}$

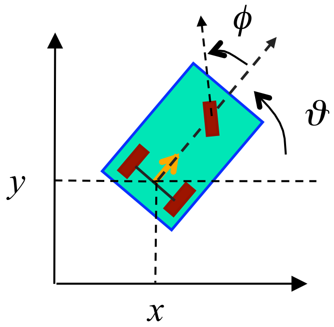
- parameterized through q
- dim $\mathcal{C}$ = n

Examples of configuration spaces



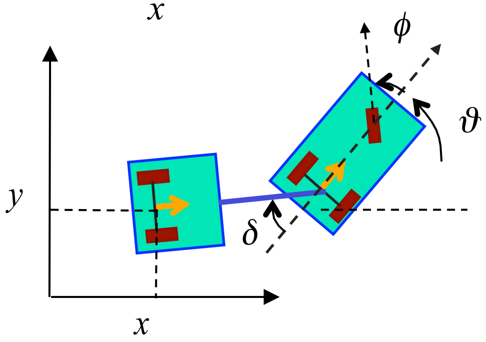
$$q = \begin{bmatrix} x \\ y \\ \vartheta \end{bmatrix}$$

dim $\mathcal{C}$ = 3



$$q = \begin{bmatrix} x \\ y \\ \vartheta \\ \phi \end{bmatrix}$$

dim $\mathcal{C}$ = 4

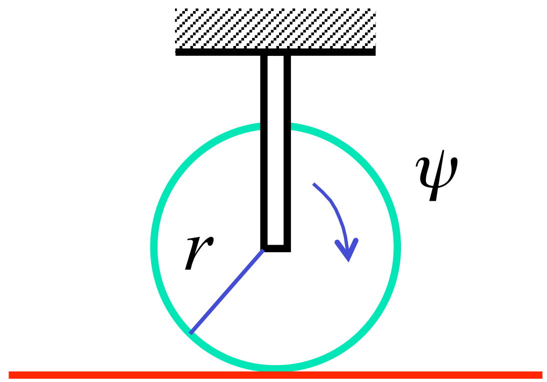


$$q = \begin{bmatrix} x \\ y \\ \vartheta \\ \phi \\ \delta \end{bmatrix}$$

dim $\mathcal{C}$ = 5

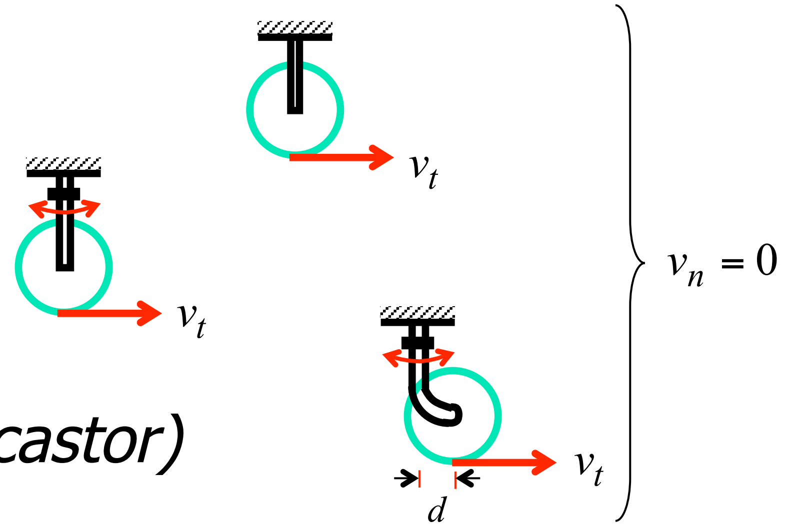
Additional configuration variables

in all previous cases, one can add in the parameterization of $\mathcal{C}$ also the rolling angle ψ of each wheel

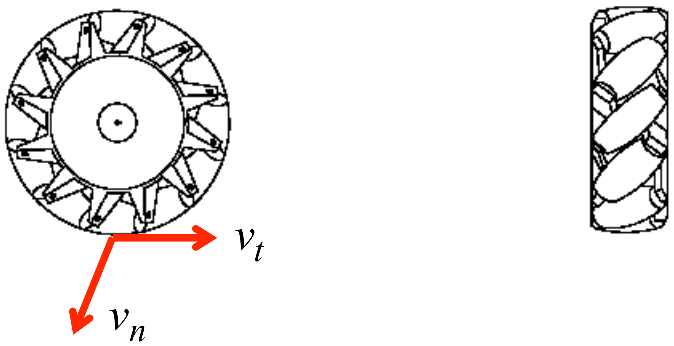


Types of wheels

- conventional
  - fixed
  - centered steering
  - off-centered steering *(castor)*



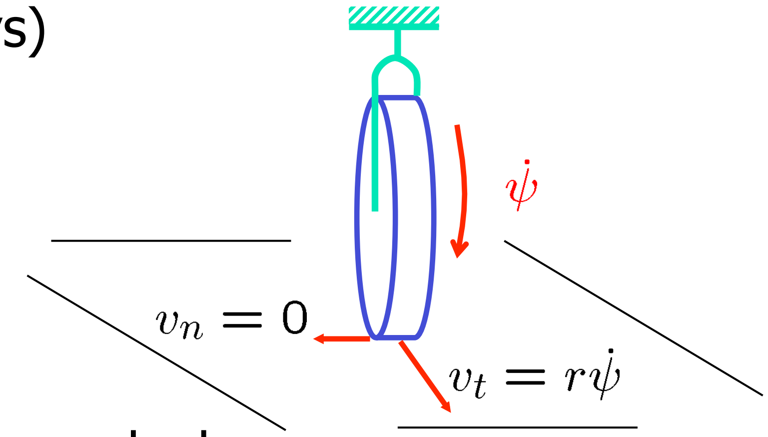
- omni-directional *(Mecanum/Swedish wheels)*



Differential constraints

- pure rolling constraints

  each wheel rolls on the ground without slipping (longitudinally) nor skidding (sideways)



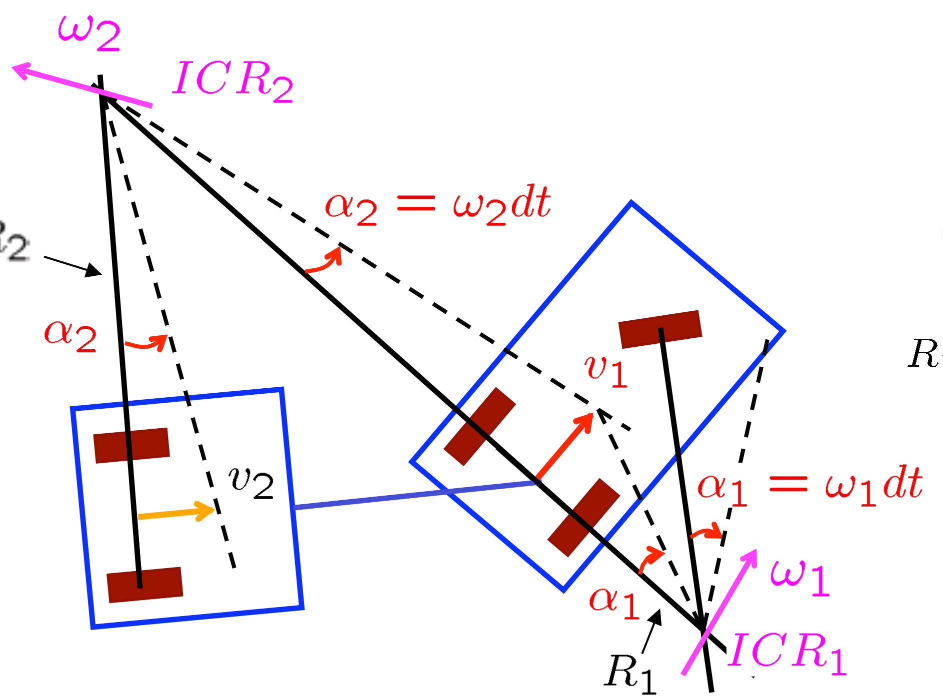
  - continuous contact
  - used in dead-reckoning (odometry)

- geometric consequence

  there is always an Instantaneous Center of Rotation (=ICR) where all wheel axes intercept: one ICR for each chassis (= rigid body) constituting the WMR

Instantaneous Center of Rotation

ICR: a graphical construction



input

$$\omega_1 = \frac{v_1}{R_1}$$

$$R_1 = |\overrightarrow{ICR_1 - P_1}|$$

$$\omega_2 = \frac{v_2}{R_2}$$

$$R_2 = |\overrightarrow{ICR_2 - P_2}|$$

computing in sequence (with some trigonometry): $v_1 \rightarrow \omega_1$, $v_1 \rightarrow \omega_2 \rightarrow v_2$

Nonholonomy

from constraints ...

- for each wheel, condition $v_n = 0$ can be written in terms of generalized coordinates and their derivatives

$$a(q)\dot{q} = 0$$

- for N wheels, in matrix form

$$A(q)\dot{q} = 0$$

- N differential constraints (in Pfaffian form = linear in velocity)

partially or completely integrable into

$$h_i(q) = 0 \quad i = 1, \ldots, k$$

⇩

reduction of $\mathcal{C}$

(dim n - k)

not integrable

⇩

NONHOLONOMY

$q \in \mathcal{C}$

but $\dot{q} \in \ker(A)$

Nonholonomy (cont'd)

... to feasible motion

$$A(q)\dot{q} = 0$$ nonintegrable (nonholonomic)

ALL feasible motion directions can be generated as

$$\dot{q} \in \ker A(q) \rightarrow \dot{q} = G(q)v$$

being

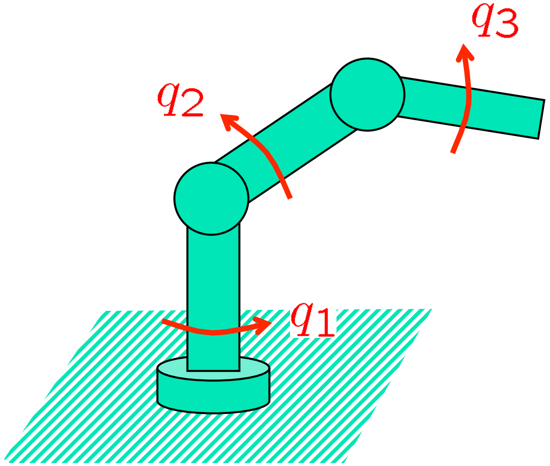
$$\operatorname{Im} G(q) = \ker A(q) \quad \forall q \in \mathcal{C}$$

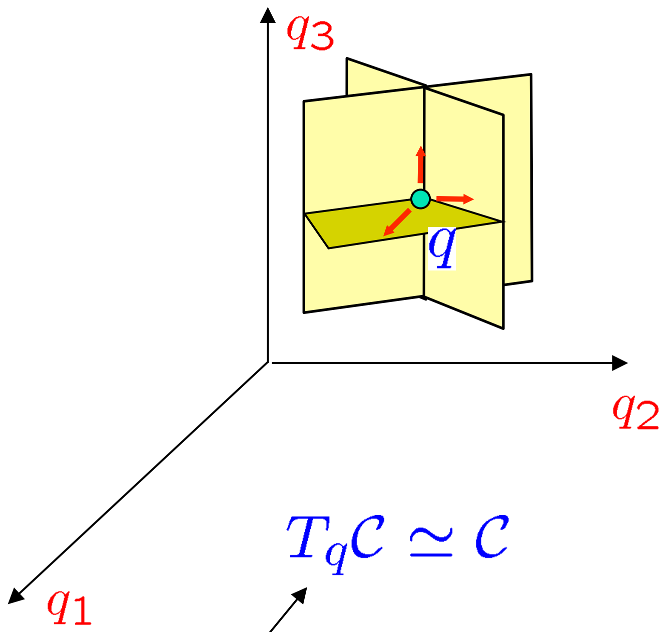
" the image of the columns of matrix *G*
coincides with the kernel of matrix *A* "

Nonholonomy (cont'd)

a comparison …

dim $\mathcal{C}$ = 3





fixed-base manipulator

$$\dot{q} = v \quad (G(q) = I)$$

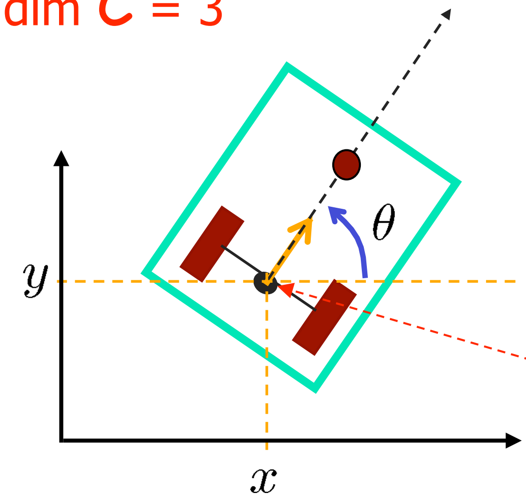
same number of commands
and generalized velocities

$$T_q\mathcal{C} \simeq \mathcal{C}$$

the space of feasible velocities has dimension **3**
and **coincides** with the tangent space
to the robot configuration space

Nonholonomy (cont'd)

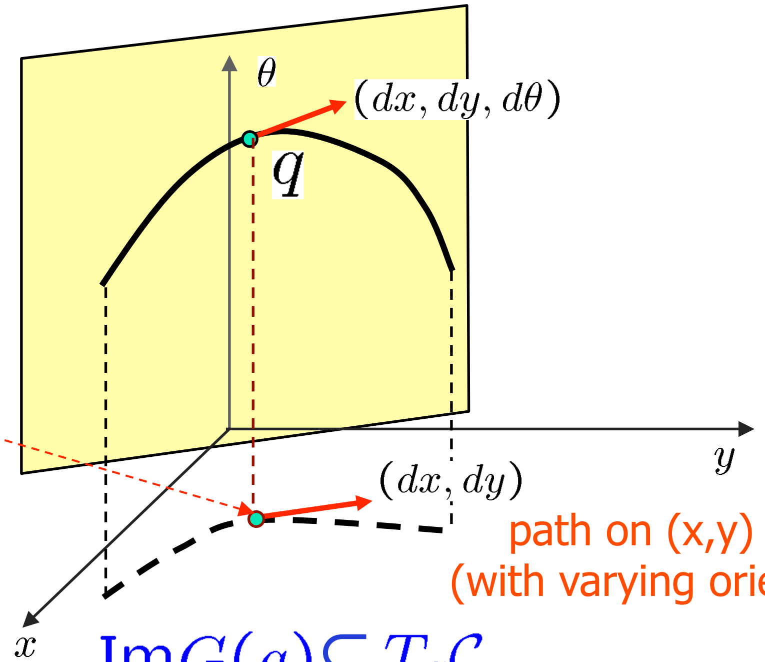
dim $\mathcal{C}$ = 3



wheeled mobile robot

$$\dot{q} = G(q)v$$

less number of commands
than generalized velocities!



path on (x,y) plane
(with varying orientation)

$$\mathrm{Im}\, G(q) \subset T_q\mathcal{C}$$

the space of feasible velocities has here dimension **2**
(a **subspace** of the tangent space)

Kinematic model of WMR

- provides all feasible directions of instantaneous motion
- describes the relation between the velocity input commands and the derivatives of generalized coordinates (a differential model!)

$$\dot{q} = G(q)v$$

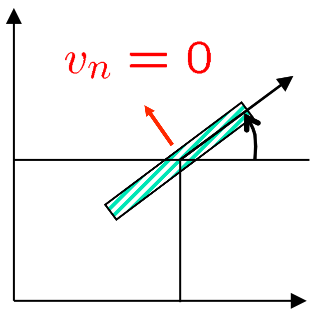
$q \in \mathcal{C}$ $\quad \text{dim}\mathcal{C} = n$ configuration space

$v \in \mathcal{V}$ $\quad \text{dim}\mathcal{V} = m$ (input) command space

with $m < n$

- needed for
  - studying the accessibility of $\mathcal{C}$ (i.e., the system "controllability")
  - planning of feasible paths/trajectories
  - design of motion control algorithms
  - incremental WMR localization (odometry)
  - simulation …

Unicycle (ideal)



$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$\frac{dy}{dx} = \tan\theta \quad \rightarrow \quad \dot{y}\cos\theta - \dot{x}\sin\theta = 0$$

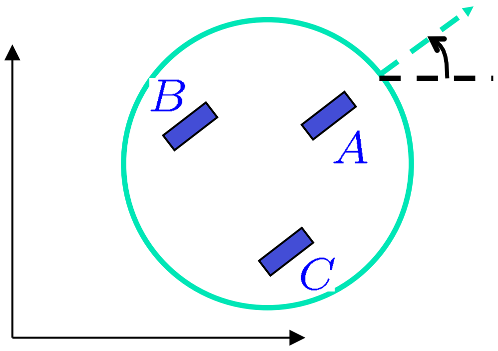
$$A(q) = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \end{bmatrix}$$

- the choice of a base $G(q)$ in the kernel of $A(q)$ can be made according to physical considerations on the real system

Unicycle (real)

a) three centered steering wheels [Nomad 200]

synchro-drive
(2 motors)



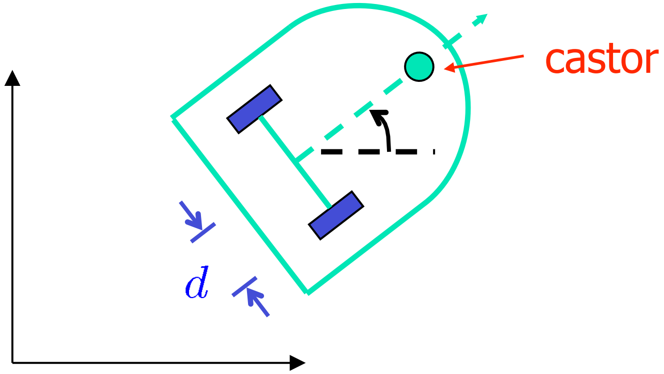
$$\dot{q} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

1 = linear speed
2 = angular speed
of the robot

$$\dot{\psi}_i = v_1/r \quad i \in \{A, B, C\}$$
$$\dot{\beta}_i = v_2$$

Unicycle (real)

b) two fixed wheels + castor [SuperMARIO, MagellanPro]



$$\dot{q} = \begin{bmatrix} \frac{\cos\theta}{2} & \frac{\cos\theta}{2} \\ \frac{\sin\theta}{2} & \frac{\sin\theta}{2} \\ \frac{1}{2d} & -\frac{1}{2d} \end{bmatrix} \begin{bmatrix} v_R \\ v_L \end{bmatrix}$$

linear speed of the two fixed wheels on the ground (R = right, L = left)

$$\dot{\psi}_R = v_R/r \qquad \dot{\psi}_L = v_L/r$$

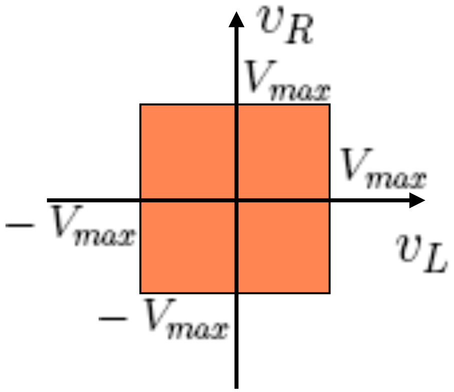
note: *d* is here the half-axis length (in textbook, it is the *entire* distance between the two fixed wheels!!)

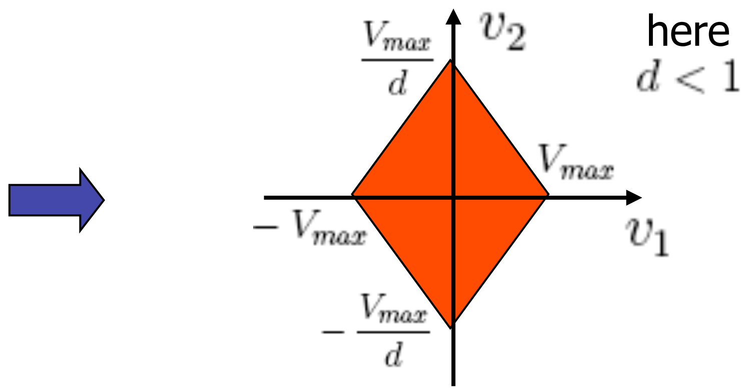
Equivalence of the two models

a) ⇔ b) by means of a transformation (invertible and constant) between inputs

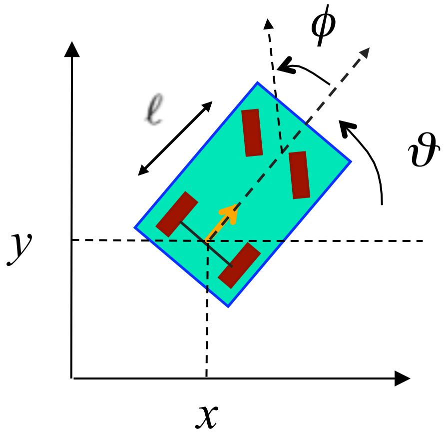
$$\begin{cases} v_1 = \dfrac{v_R + v_L}{2} \\ v_2 = \dfrac{v_R - v_L}{2d} \end{cases} \Leftrightarrow \begin{cases} v_R = v_1 + d v_2 \\ v_L = v_1 - d v_2 \end{cases}$$

…however, pay attention to how possible (equal) bounds on maximum speed of the two wheels are transformed!



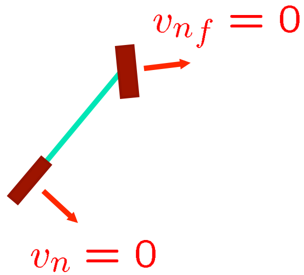


Car-like

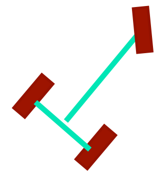


$$q = \begin{bmatrix} x \\ y \\ \vartheta \\ \phi \end{bmatrix}$$

ideal ("telescopic" view)



tricycle



with differential gear
on rear wheels

$$\begin{cases} \dot{y}\cos\theta - \dot{x}\sin\theta = 0 \\ \dot{y}_f\cos(\theta+\phi) - \dot{x}_f\sin(\theta+\phi) = 0 \end{cases}$$

$$x_f = x + \ell\cos\theta \qquad y_f = y + \ell\sin\theta$$

$$A(q)\dot{q} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\sin(\theta+\phi) & \cos(\theta+\phi) & \ell\cos\phi & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$$

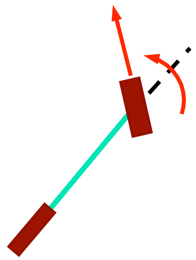
Car-like (continued)

- FD = Front wheel Drive

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & 0 \\ \sin\theta\cos\phi & 0 \\ (1/\ell)\sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1f} \\ v_2 \end{bmatrix}$$

linear and angular speed of front wheel

$\simeq$ kinematic model of unicycle with trailer (e.g., Hilare 2-bis)



Car-like (continued)

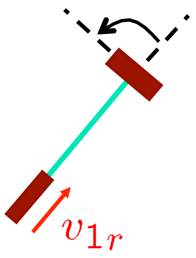
- RD = Rear wheel Drive

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ (1/\ell)\tan\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1r} \\ v_2 \end{bmatrix}$$

← linear speed of rear wheel (medium point of rear-axis)

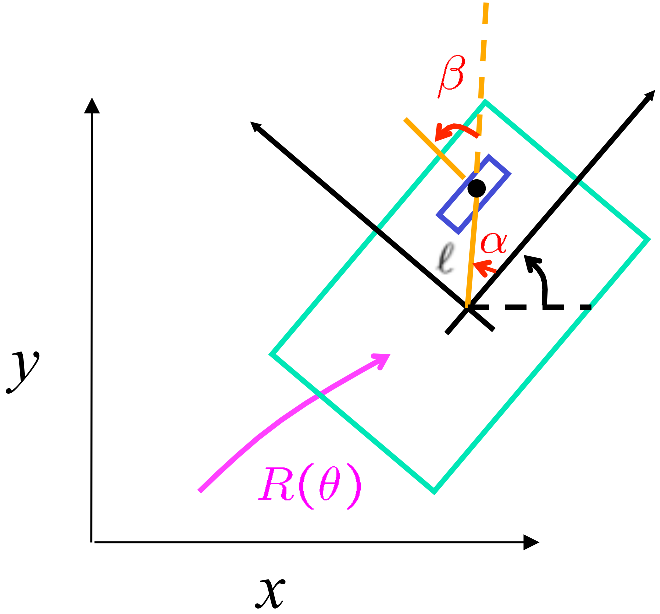
singularity at $\phi = \pm\frac{\pi}{2}$

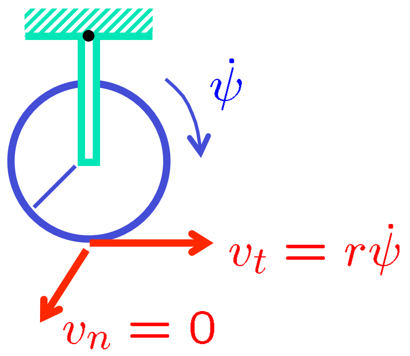
(the model is no longer valid)



General constraint form by wheel type

a) *f* = *fixed* or centered *s* = *steerable*



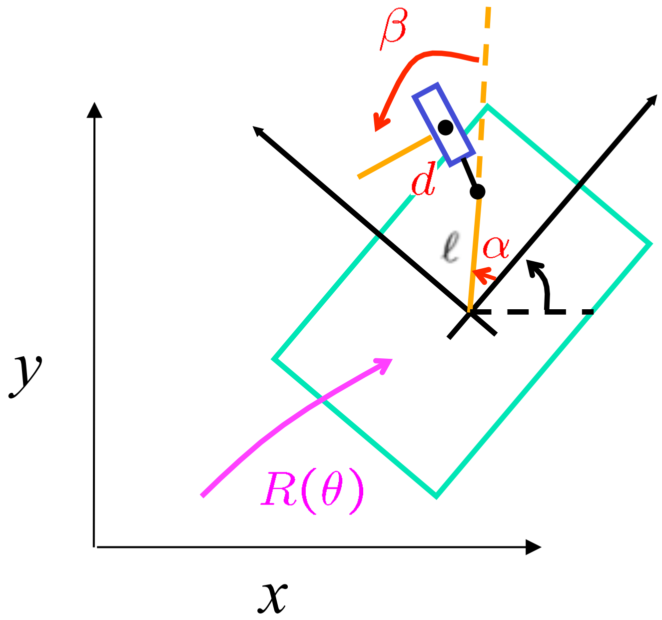


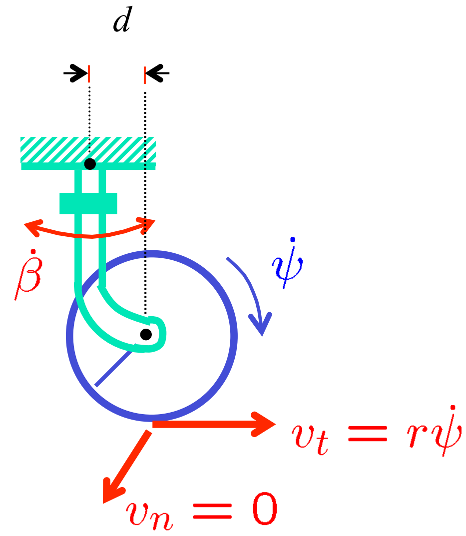
$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & \ell \sin\beta \end{bmatrix} R^T(\theta) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$$

constant *(f)* or variable *(s)*

General constraint form by wheel type

b) *o = steerable with off-set (off-centered)*





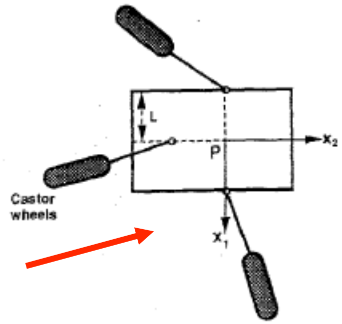
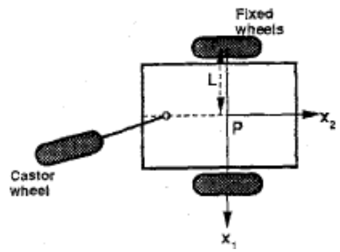
$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & d+\ell\sin\beta \end{bmatrix} R^T(\theta) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} + d\dot{\beta} = 0$$

variable

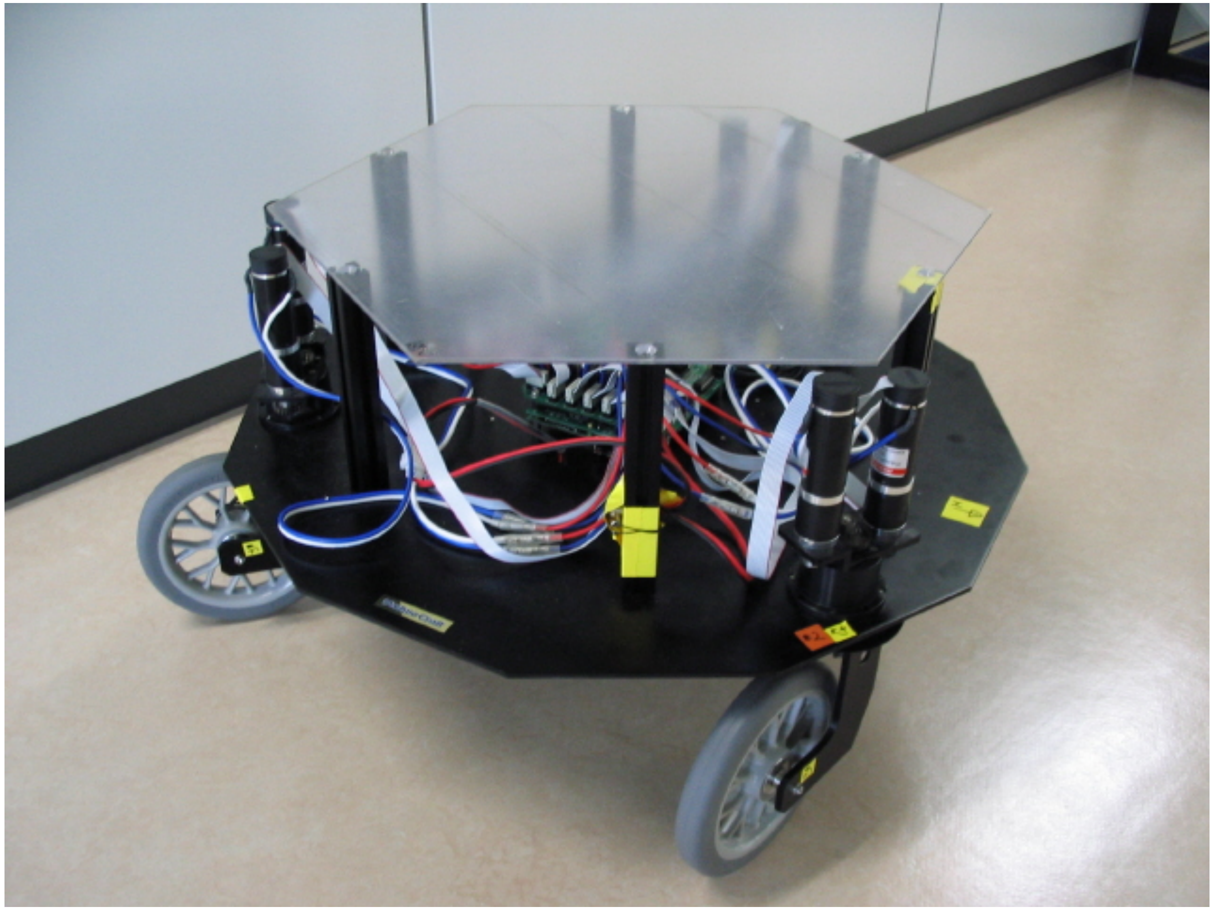
Possible kinematic "classes"

5 possible classes for the WMR kinematic model (single chassis)

$$N = N_f + N_s + N_o = \text{number of wheels}$$

class	description	example (N = 3)
I	$N_f = N_s = 0$ *is an omnidirectional WMR!*	 $N_o = 3$
II	$N_s = 0$, $N_f \geq 1$ on same axis	 $N_f = 2$, $N_o = 1$

Example of class I WMR
(omnidirectional)



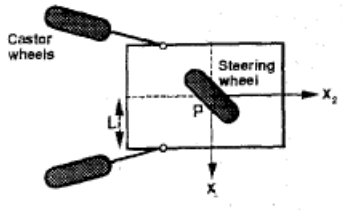
with three conventional off-centered wheels,
independently actuated

Possible kinematic "classes" (cont'd)

III

$N_f = 0$

$N_s \geq 1$ synchronized if > 1

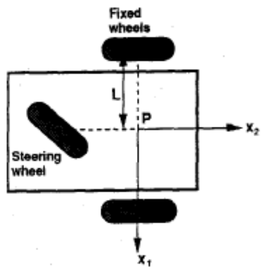


$N_s = 1$
$N_o = 2$

IV

$N_f \geq 1$ on same axis

$N_s \geq 1$ at least one out of the common axis of the two fixed wheels

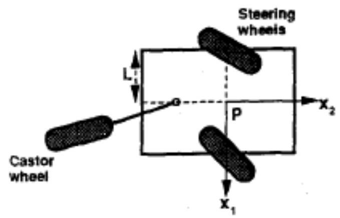


$N_f = 2$
$N_s = 1$

V

$N_f = 0$

$N_s \geq 2$ synchronized if > 2



$N_s = 2$
$N_o = 1$

- WMRs in same class are characterized by same "maneuverability"
- previous models of WMRs fit indeed in this classification: SuperMARIO (class II), Nomad 200 (class III), car-like (class IV)