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Combining Real and Virtual Sensors for Measuring Interaction Forces and Moments Acting on a Robot

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Abstract— **We address the problem of estimating an external wrench acting along the structure of a robot manipulator, together with the contact position where the external force is being applied. For this, we consider the combined use of a force/torque sensor mounted at the robot base and of a model-based virtual sensor. The virtual sensor is provided by the residual vector commonly used for collision detection and isolation in human-robot interaction. Integrating the two types of measurement tools provides an efficient way to estimate all unknown quantities, using also the recursive Newton-Euler algorithm for dynamic computations. Different operative conditions are considered, including the special cases of point-wise interaction (pure contact force), known contact location, and of a base sensor measuring only forces. We highlight also the conditions for a correct estimation to be fully virtual, i.e., without resorting to a force/torque sensor. Realistic simulations assess the estimation performance for a 7R robot in motion, subject to an unknown external force applied to an unknown location.**

I. Introduction

When a robot physically interacts with its environment, and the task requires an accurate exchange of forces and moments at the tool/end-effector level, classical control solutions rely on the use of a 6D Force/Torque (F/T) sensor mounted on the robot final flange [1]. However, such a sensor does not measure external wrenches acting on the manipulator body, which is instead an increasingly relevant requirement in safe physical human-robot interaction. Indeed, if we could cover the surface of the robot links with a tactile skin [2], we would have a direct measure of the force vector at the contact.

A more practical alternative for measuring the contact force, or the full external wrench (i.e., force and moment), is to resort to joint torque sensors [3], [4]. For the purpose of wrench estimation, quasi-static operation is assumed, as well as the knowledge of the link involved in the interaction and of the contact point where the external force is being applied. For robots with rigid joints (and thus with no joint torque sensors), a novel possibility has been offered by the use of a model-based computation of the residual vector [5], [6], which allows to detect generic collisions and isolate the colliding link as well. When complemented by the use of external sensing, such as an RGB-D sensor like the Kinect [7], to localize the contact area on the robot, this arrangement can be used as a virtual sensor for estimating at least the contact forces [8], and also for controlling them [9].

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Nonetheless, contact forces that do not produce motion will not be detected, and thus neither estimated, in this way. Similarly, contact moments may not be easily estimated.

In this paper we explore the possibility of combining the use of the above virtual force sensing approach with a F/T sensor placed at the robot base (see Fig. 1). Intuitively speaking, using a base F/T sensor should allow capturing the effects of physical interactions occurring at any level along the robot structure, in particular detecting also external forces that do not perform work on robot motion. We would like thus to provide a few answers to a number of interesting questions such as: can we estimate with the base F/T sensor a full external wrench, without knowledge of the contact point or of the link involved? To what extent the virtual sensor machinery given by the residuals is able to resolve the need of geometric information about the contact? Since contacts tipically occur while the robot in motion, is the residual useful within the recursive Newton-Euler algorithm used for dynamic computations? What happens if the base F/T sensor is replaced by three load cells measuring only forces along three axes at the robot base? What if we remove completely also the real force sensor?

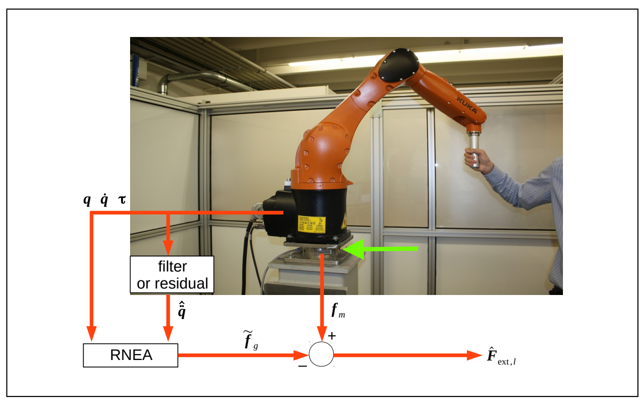

Fig. 1. Obtaining the external force using a base force sensor (indicated by the green arrow). Picture courtesy of Matthias Kurze (KUKA Roboter GmbH), within the SAPHARI project [10].

We note that the mounting of a F/T sensor at the robot base is not a novelty per se. They have been used, e.g., for measuring the reaction wrench needed to emulate the motion of a manipulator floating in space [11], for compensating joint torque friction [12], for estimating inertial robot parameters [13], or for realizing Cartesian impedance controllers [14]. However, the proposed application in combination with a virtual sensing approach appears to be new.

The paper is organized as follows. In Sec. II, we formulate the dynamic equations in the presence of external wrenches in the Lagrange and Newton-Euler settings, and recall the

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estimation of the joint external torque by means of the residual. Section III provides a number of estimation methods of the unknown quantities in the physical interaction, under various assumptions and combining the use of the real and virtual sensors in different ways. Finally, the results of the numerical simulations performed on a 7R robot with two selected methods are reported in Sec. IV.

II. Interaction Forces and Moments

Consider a robot as an open kinematic chain of n rigid links connected through rigid joints. We will analyze the effect of external forces and moments acting on the robot, first from the Lagrangian point of view, and then in Newton-Euler mechanics. Both approaches are used in our method.

A. Lagrangian dynamic model

The Lagrangian dynamic model takes the form

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{ext}, \quad (1)$$

where $\boldsymbol{q} \in \mathbb{R}^n$ are the generalized (joint) coordinates, $\boldsymbol{\tau} \in \mathbb{R}^n$ are the commanded motor torques, $\boldsymbol{B}(\boldsymbol{q})$ is the positive definite robot inertia matrix, $\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q})$ contains centrifugal, Coriolis and gravity terms, and $\boldsymbol{\tau}_{ext}$ is the *external joint torque*, i.e., the joint torque generated by external forces and moments. If the robot is subject to a single external wrench $\boldsymbol{W}_{ext} = (\boldsymbol{F}_{ext}^T \ \boldsymbol{M}_{ext}^T)^T \in \mathbb{R}^6$, with the force acting at a contact point $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}) \in \mathbb{R}^3$ along its structure, then

$$\boldsymbol{\tau}_{ext} = \boldsymbol{J}_{\boldsymbol{p}}^T(\boldsymbol{q})\boldsymbol{W}_{ext}, \quad (2)$$

where $\boldsymbol{J}_{\boldsymbol{p}}(\boldsymbol{q}) \in \mathbb{R}^{6\times n}$ is the *contact Jacobian*, such that

$$\begin{pmatrix} \boldsymbol{v}_{\boldsymbol{p}} \\ \boldsymbol{\omega}_{\boldsymbol{p}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_{\boldsymbol{v}_{\boldsymbol{p}}}(\boldsymbol{q}) \\ \boldsymbol{J}_{\boldsymbol{\omega}_{\boldsymbol{p}}}(\boldsymbol{q}) \end{pmatrix} \dot{\boldsymbol{q}} = \boldsymbol{J}_{\boldsymbol{p}}(\boldsymbol{q})\dot{\boldsymbol{q}}, \quad (3)$$

with $\boldsymbol{v}_{\boldsymbol{p}} = \dot{\boldsymbol{p}}$ being the linear velocity of point $\boldsymbol{p}$, and $\boldsymbol{\omega}_{\boldsymbol{p}}$ the angular velocity of the link which this point belongs to. Two remarks are in place.

Remark 1. If point $\boldsymbol{p}$ is on link l, $l \in \{1, \ldots, n\}$, the linear and angular velocities $\boldsymbol{v}_{\boldsymbol{p}}$ and $\boldsymbol{\omega}_{\boldsymbol{p}}$ will not be affected by $\dot{q}_i$, for $i \in \{l+1, \ldots, n\}$. Thus, only the first l columns of $\boldsymbol{J}_{\boldsymbol{p}}$ can be different from zero, while the last $n-l$ are identically zero. As a result, from (2) it is easy to see that an external wrench acting on link l will only affect the first l components of $\boldsymbol{\tau}_{ext}$.

Remark 2. All points on link l have the same angular velocity $\boldsymbol{\omega}_l$. Thus, $\boldsymbol{J}_{\boldsymbol{\omega}_{\boldsymbol{p}}}$ is independent from the position of $\boldsymbol{p}$ on link l. The opposite is true for the linear velocity and $\boldsymbol{J}_{\boldsymbol{v}_{\boldsymbol{p}}}$. Therefore, from (2) and (3) we see that the effect of $\boldsymbol{F}_{ext}$ on $\boldsymbol{\tau}_{ext}$ depends on $\boldsymbol{p}$, while that of $\boldsymbol{M}_{ext}$ does not.

In presence of multiple external wrenches acting on the robot, $\boldsymbol{\tau}_{ext}$ will simply be the sum of the single external contributions. In the following, we mainly focus on the single contact case.

B. Estimating the external joint torque

Assume to measure[1] the robot state $(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and to have the commanded torque $\boldsymbol{\tau}$ at disposal. Knowing the kinematic and dynamic parameters of the robot, the external joint torque $\boldsymbol{\tau}_{ext}$ can be estimated *without* the use of any extra sensor (e.g., joint torque or F/T sensor), in two different ways.

[1] Joint velocity is typically obtained by numerical differentiation of encoder measurements.

The first way to estimate $\boldsymbol{\tau}_{ext}$ is by computing the so-called *residual vector*, which, at time t, is defined as

$$\boldsymbol{r}(t) = \boldsymbol{K}\left(\boldsymbol{m} - \int_0^t \left(\boldsymbol{\tau} + \boldsymbol{C}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{r}\right) \mathrm{d}t'\right), \quad (4)$$

where $\boldsymbol{m} = \boldsymbol{B}(\boldsymbol{q})\dot{\boldsymbol{q}}$ is the robot generalized momentum, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is a factorization matrix for $\boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ such that $\dot{\boldsymbol{B}} = \boldsymbol{C} + \boldsymbol{C}^T$, and $\boldsymbol{K} = \mathrm{diag}\{K_i\} > 0$ is a diagonal gain matrix. Typically, $\boldsymbol{C}$ is defined in terms of Christoffel symbols of the second kind [1], but it can also be defined differently and computed with a recursive numerical algorithm as in [15]. It can be shown from (1) and (4) that

$$\dot{\boldsymbol{r}} = \boldsymbol{K}(\boldsymbol{\tau}_{ext} - \boldsymbol{r}), \quad (5)$$

i.e., $\boldsymbol{r}$ is a filtered version of $\boldsymbol{\tau}_{ext}$. The larger the elements in $\boldsymbol{K}$ are, the higher the bandwidths of the scalar filters, and the faster the convergence of $\boldsymbol{r}$ to $\boldsymbol{\tau}_{ext}$. It has been shown in several different experiments [5], [6], [8], [9] that, by choosing sufficiently large gains, one can assume

$$\hat{\boldsymbol{\tau}}_{ext} = \boldsymbol{r} \approx \boldsymbol{\tau}_{ext}, \quad (6)$$

where the hat denotes an estimate. Thanks to Remark 1, the index l of the link in contact can be identified from the estimate (6) of $\boldsymbol{\tau}_{ext}$ as the largest i such that $|\hat{\tau}_{ext,i}| > \tau_{min}$, where τ_{min} is a small threshold chosen a priori. However, it should be noted that if the non-zero part of the contact Jacobian is a 'tall' matrix (has more rows than columns) and/or is not of full rank (in a kinematic singularity), then part of the contact information will be lost in the null space of the transposed Jacobian, potentially causing false negatives in the collision detection.

The second way to estimate $\boldsymbol{\tau}_{ext}$ is by extracting $\hat{\ddot{\boldsymbol{q}}}$ numerically from $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$, and then computing $\hat{\boldsymbol{\tau}}_{ext}$ from (1) with $\ddot{\boldsymbol{q}} = \hat{\ddot{\boldsymbol{q}}}$. On the other hand, the same equation could be used to obtain $\hat{\ddot{\boldsymbol{q}}}$ from $\hat{\boldsymbol{\tau}}_{ext}$ (by inversion of the robot inertia matrix), if the latter has already been obtained from the residual (6). In this sense, the estimates $\hat{\ddot{\boldsymbol{q}}}$ and $\hat{\boldsymbol{\tau}}_{ext}$, respectively of $\ddot{\boldsymbol{q}}$ and $\boldsymbol{\tau}_{ext}$, are equivalent to each other. Indeed, the choice of whether to compute one quantity and obtain in turn the other or vice versa, will depend on numerical accuracy of the available robot data, on the noise level of primary measurements, and on which quantity is more useful for other purposes as well.

Remark 3. In passing, we note that the discussion so far and all what will follow can be extended to robots with flexible joints in a straightforward way, by simply substituting the commanded torque $\boldsymbol{\tau}$ with the elastic torque $\boldsymbol{\tau}_e$ exerted on the links through the flexible transmissions. This torque is provided by a joint torque sensor, or by two encoders measuring the position on the motor and link sides of the flexible joint [5], assuming in addition a known joint stiffness. The measure of $\boldsymbol{\tau}_e$ replaces in all formulas the commanded torque $\boldsymbol{\tau}$.

C. Newton-Euler dynamic model

A different insight on the interaction dynamics can be acquired through the Newton-Euler approach. Given the robot state ($\boldsymbol{q}$, $\dot{\boldsymbol{q}}$) and a nominal $\ddot{\boldsymbol{q}}$, the popular Recursive Newton-Euler Algorithm (RNEA) [16] iterates recursively first from the base to the tip to determine the acceleration of each link, and then from the tip to the base to determine forces and moments exchanged between the links, and henceforth the motor torques required to achieve the nominal $\ddot{\boldsymbol{q}}$ (via projection equations).

In presence of external wrenches, assuming for simplicity that on each link i at most one single external force $\boldsymbol{F}_{ext,i}$ is acting, applied to point $\boldsymbol{p}_i$, possibly together with a pure moment $\boldsymbol{M}_{ext,i}$, the backward recursion takes the form (expressed in world frame)

$$\boldsymbol{F}_i = m_i \boldsymbol{a}_{\boldsymbol{c}_i} \tag{7}$$

$$\boldsymbol{M}_i = \boldsymbol{I}_i \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times (\boldsymbol{I}_i \boldsymbol{\omega}_i) \tag{8}$$

$$\boldsymbol{f}_i = \boldsymbol{f}_{i+1} + \boldsymbol{F}_i - \boldsymbol{F}_{ext,i} \tag{9}$$

$$\begin{aligned}\boldsymbol{n}_i = {} & \boldsymbol{n}_{i+1} + \boldsymbol{r}_{i-1,i} \times \boldsymbol{f}_i + \boldsymbol{r}_{i,\boldsymbol{c}_i} \times \boldsymbol{F}_i \\ & + \boldsymbol{M}_i - \boldsymbol{M}_{ext,i} - \boldsymbol{r}_{i,\boldsymbol{p}_i} \times \boldsymbol{F}_{ext,i},\end{aligned} \tag{10}$$

for $i = n, \ldots, 0$. Note that the iteration $i = 0$ has been added to take into account the typical placement of a F/T sensor at the robot base (see Fig. 1). Each link $i \in \{0, \ldots, n\}$ has been assigned a body-fixed frame with origin $\boldsymbol{O}_i$, according to the standard Denavit-Hartenberg convention[2]. When considering a sensor attached below link 0 (i.e., between the robot base and the world ground), we assume without loss of generality that its reference frame coincides with the global frame, with origin $\boldsymbol{O}_s$. In (7)–(10), the following symbols have been used:

$\boldsymbol{\omega}_i$	angular velocity of frame i
$\boldsymbol{a}_{\boldsymbol{c}_i}$	acceleration of the center of mass (CoM) of link i
$\boldsymbol{r}_{i,\boldsymbol{c}_i}$	position of the CoM of link i w.r.t. $\boldsymbol{O}_i$
$\boldsymbol{r}_{i,\boldsymbol{p}_i}$	position of $\boldsymbol{p}_i$ w.r.t. $\boldsymbol{O}_i$
$\boldsymbol{r}_{i-1,i}$	position of $\boldsymbol{O}_i$ w.r.t. $\boldsymbol{O}_{i-1}$, for $i \in \{1, \ldots, n\}$
$\boldsymbol{r}_{-1,0}$	position of $\boldsymbol{O}_0$ w.r.t. $\boldsymbol{O}_s$, equivalent to $\boldsymbol{r}_{s,0}$
m_i	mass of link i
$\boldsymbol{I}_i$	barycentral inertia tensor of link i
$\boldsymbol{F}_i$	total force acting on the CoM of link i
$\boldsymbol{M}_i$	total moment acting on link i
$\boldsymbol{f}_i$	total force exerted on link i by link $i-1$
$\boldsymbol{n}_i$	total moment exerted on link i by link $i-1$
$\boldsymbol{F}_{ext,i}$	external force acting on link i
$\boldsymbol{M}_{ext,i}$	external moment acting on link i.

The force and moment discharged to the ground F/T sensor are, respectively, $\boldsymbol{f}_g = -\boldsymbol{f}_0$ and $\boldsymbol{n}_g = -\boldsymbol{n}_0$. Remember also that, in the forward recursion, the acceleration of the robot base is initialized with the opposite of the gravitational acceleration, $\boldsymbol{a}_{\boldsymbol{c}_0} = \ddot{\boldsymbol{O}}_0 = -\vec{\boldsymbol{g}}$.

Remark 4. From (9) and (10), we see that the total dynamic effect of an external force $\boldsymbol{F}_{ext,i}$ applied at a point $\boldsymbol{p}_i$ is *the same* as that of the same force applied at $\boldsymbol{O}_i$, plus an additional pure moment $\tilde{\boldsymbol{M}}_{ext,i} = \boldsymbol{r}_{i,\boldsymbol{p}_i} \times \boldsymbol{F}_{ext,i}$.

III. Combining Real and Virtual Sensors

We will study different possible combinations of real and virtual sensors for the purpose of estimating external force, external moment, and, under certain assumptions, also the contact point. In particular, we will consider that a 6D F/T sensor or a 3D Force sensor, when present, is located at the robot base.

In all cases, we assume that the kinematic and dynamic parameters of the robot are known, that $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ are measured, and that an estimate $\hat{\boldsymbol{\tau}}_{ext}$ has been obtained, as described in Sec. II-B.

A. Estimating the external force with a base force sensor

The force $\boldsymbol{f}_m$ measured by a sensor located at the robot base is given by $\boldsymbol{f}_m = \boldsymbol{f}_g = -\boldsymbol{f}_0$. From (9), we have

$$\boldsymbol{f}_m = \sum_{i=0}^{n} \boldsymbol{F}_{ext,i} - \sum_{i=0}^{n} \boldsymbol{F}_i = \boldsymbol{F}_{ext} + \tilde{\boldsymbol{f}}_g, \tag{11}$$

where $\boldsymbol{F}_{ext}$ is the sum of all external forces acting on the robot, and $\tilde{\boldsymbol{f}}_g$ is the same force that the robot would exert on the ground if it were to follow the same motion but *without* contact forces. The value of $\tilde{\boldsymbol{f}}_g$ can be computed once $\ddot{\boldsymbol{q}}$ is known. For this, it is enough to run RNEA without any contact wrench, and get the ground force resulting from this computation. Therefore, assuming that a single contact occurs on the (possibly unknown) link l, then $\boldsymbol{F}_{ext,l} = \boldsymbol{F}_{ext}$ is extracted from the base force measurement as follows:

1) estimate $\hat{\ddot{\boldsymbol{q}}}$ (either directly or through the residual $\boldsymbol{r}$);
2) compute $\tilde{\boldsymbol{f}}_g$ with RNEA;
3) take $\hat{\boldsymbol{F}}_{ext,l} = \boldsymbol{f}_m - \tilde{\boldsymbol{f}}_g$.

The computational scheme is illustrated in Fig. 1. The estimate of $\boldsymbol{F}_{ext,l}$ obtained in this way provides an excellent mean to detect collisions: if $||\hat{\boldsymbol{F}}_{ext,l}|| > F_{min}$, where F_{min} is a small given threshold, then a collision has been detected. Therefore, an estimate of the contact force, and the associated contact detection, can be performed using only an estimate of $\ddot{\boldsymbol{q}}$, no matter which is the contact link nor what configuration the robot is assuming. However, without further processing, it is not possible to identify the contact point and not even the contact link.

In this framework, computing an estimate of the external joint torque $\boldsymbol{\tau}_{ext}$ using $\hat{\ddot{\boldsymbol{q}}}$ is particularly convenient from an implementation point of view. In fact, the standard RNEA would provide the motor torque at joint i during the backward recursion as

$$\tau_i = \boldsymbol{z}_{i-1}^T \boldsymbol{n}_i, \qquad i = n, \ldots, 1, \tag{12}$$

where $\boldsymbol{z}_{i-1}$ is the unit vector of the i-th joint axis. However, since in the above step 2 we run RNEA without considering contact forces and moments, the output will *not* be $\boldsymbol{\tau}$, the commanded torque that we already know, but rather $\boldsymbol{\tau}_{tot} = \boldsymbol{\tau} + \hat{\boldsymbol{\tau}}_{ext}$. Thus, from the *same* computation performed to obtain $\tilde{\boldsymbol{f}}_g$, we recover also $\hat{\boldsymbol{\tau}}_{ext} = \boldsymbol{\tau}_{tot} - \boldsymbol{\tau}$.

B. Estimating the external moment with a base F/T sensor and known contact point

Dynamic measures are in general not sufficient to estimate an external moment $\boldsymbol{M}_{ext,l}$, because this cannot be easily separated from the moment $\tilde{\boldsymbol{M}}_{ext,l}$ induced by the external force $\boldsymbol{F}_{ext,l}$ (see Remark 4). Therefore, it is necessary to *i)* compute $\boldsymbol{F}_{ext,l}$, as in Sec. III-A, and *ii)* assume that the contact point $\boldsymbol{p}_l$ is known. The latter can be acquired by

[2]If Craig's modified DH notation is adopted [17], it is enough to turn $\boldsymbol{r}_{i-1,i} \times \boldsymbol{f}_i$ into $\boldsymbol{r}_{i,i+1} \times \boldsymbol{f}_{i+1}$ in (10). All subsequent considerations are still valid.

means of contact sensors [18] or vision/depth sensors [8]. At the bare minimum, one should consider only cases in which contact may happen only at some relevant point known in advance, like the tool center point or the end-effector tip.

With this additional position information, the extraction of $\boldsymbol{M}_{ext,l}$ from a base torque measure $\boldsymbol{n}_m = \boldsymbol{n}_g = -\boldsymbol{n}_0$ proceeds as follows. From (9) and (10), one has

$$\begin{aligned} \boldsymbol{n}_m &= \sum_{i=0}^{n} \boldsymbol{M}_{ext,i} + \sum_{i=0}^{n} \Big(\boldsymbol{r}_{i,\boldsymbol{p}_i} \times \boldsymbol{F}_{ext,i} + \boldsymbol{r}_{i-1,i} \times \sum_{j=i}^{n} \boldsymbol{F}_{ext,j} \Big) \\ &\quad - \sum_{i=0}^{n} \Big(\boldsymbol{M}_i + \boldsymbol{r}_{i,\boldsymbol{c}_i} \times \boldsymbol{F}_i + \boldsymbol{r}_{i-1,i} \times \sum_{j=i}^{n} \boldsymbol{F}_j \Big) \qquad (13) \\ &= \boldsymbol{M}_{ext} + \tilde{\boldsymbol{n}}_{\boldsymbol{F}_{ext}} + \tilde{\boldsymbol{n}}_g , \end{aligned}$$

where $\boldsymbol{M}_{ext}$ and $\tilde{\boldsymbol{n}}_g$ are defined similarly to $\boldsymbol{F}_{ext}$ and $\tilde{\boldsymbol{f}}_g$ in (11). Vector $\tilde{\boldsymbol{n}}_g$ can be computed in the same way as $\tilde{\boldsymbol{f}}_g$, during the same run of RNEA. Vector $\tilde{\boldsymbol{n}}_{\boldsymbol{F}_{ext}}$ in (13) is the additional ground torque resulting from the application of all the external forces $\boldsymbol{F}_{ext,i}$. With a single external force acting on link l, $\tilde{\boldsymbol{n}}_{\boldsymbol{F}_{ext}}$ has the expression

$$\tilde{\boldsymbol{n}}_{\boldsymbol{F}_{ext}} = \left(\boldsymbol{r}_{l,\boldsymbol{p}_l} + \sum_{i=0}^{l} \boldsymbol{r}_{i-1,i} \right) \times \boldsymbol{F}_{ext,l} = \boldsymbol{r}_{s,\boldsymbol{p}_l} \times \boldsymbol{F}_{ext,l} \qquad (14)$$

where $\boldsymbol{r}_{s,\boldsymbol{p}_l}$ is the vector going from $\boldsymbol{O}_s$ to $\boldsymbol{p}_l$. Once $\tilde{\boldsymbol{n}}_g$ and $\tilde{\boldsymbol{n}}_{\boldsymbol{F}_{ext}}$ are known, we have from (13)

$$\hat{\boldsymbol{M}}_{ext,l} = \boldsymbol{n}_m - \tilde{\boldsymbol{n}}_g - \tilde{\boldsymbol{n}}_{\boldsymbol{F}_{ext}} . \qquad (15)$$

Figure 2 summarizes the procedure, highlighting the flow of information from one step to the other.

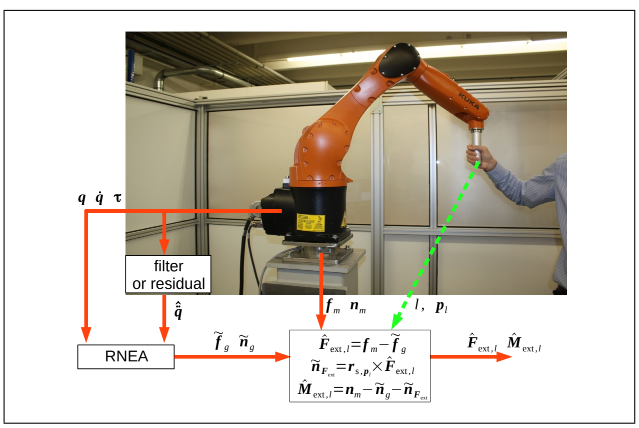

Fig. 2. Obtaining the external force and moment using a base F/T sensor, with known contact point, e.g., at the end-effector tip as shown here.

C. Estimating the contact point

When the contact link l and the associated contact point $\boldsymbol{p}_l$ are both unknown, they can be estimated from dynamic measurements, but *only* under the hypothesis of a pure contact force, i.e., assuming $\boldsymbol{M}_{ext,l} = \boldsymbol{0}$. This assumption is plausible for instance, when an accidental collision occurs with a rigid environment, or when a human is simply pushing on a robot link, without imposing any additional rotation.

Thanks to (6), we can use the residual $\boldsymbol{r}$ to directly isolate the contact link l. With this information now available, and with reference to Fig. 3, we can use the same scheme as in Sec. III-B assuming for the time being that $\boldsymbol{F}_{ext,l}$ is applied at $\boldsymbol{O}_l$. From (13) and (14), we can write in the present case

$$\boldsymbol{n}_m = \boldsymbol{r}_{l,\boldsymbol{p}_l} \times \boldsymbol{F}_{ext,l} + \boldsymbol{r}_{s,l} \times \boldsymbol{F}_{ext,l} + \tilde{\boldsymbol{n}}_g . \qquad (16)$$

In (16), $\boldsymbol{F}_{ext,l}$ and $\tilde{\boldsymbol{n}}_g$ can be computed as in Sec. III-A and Sec. III-B, respectively, while vector $\boldsymbol{r}_{s,l}$ is known from direct kinematics computations. Therefore, defining the vector

$$\tilde{\boldsymbol{M}}_{ext,l} = \boldsymbol{r}_{s,l} \times \hat{\boldsymbol{F}}_{ext,l} = \boldsymbol{n}_m - \boldsymbol{r}_{s,l} \times \hat{\boldsymbol{F}}_{ext,l} - \tilde{\boldsymbol{n}}_g , \qquad (17)$$

which is known at this stage, we need to solve the 3×3 linear system

$$\tilde{\boldsymbol{M}}_{ext,l} = -\hat{\boldsymbol{F}}_{ext,l} \times \boldsymbol{r}_{l,\boldsymbol{p}_l} = -\boldsymbol{S}(\hat{\boldsymbol{F}}_{ext,l}) \boldsymbol{r}_{l,\boldsymbol{p}_l} \qquad (18)$$

where $\boldsymbol{S}(\boldsymbol{v})$ is the skew-symmetric matrix built from vector $\boldsymbol{v}$, such that $\boldsymbol{S}(\boldsymbol{v})\boldsymbol{x} = \boldsymbol{v} \times \boldsymbol{x}$. This matrix is always singular, and therefore cannot be inverted. However, it can be easily pseudo-inverted as

$$\boldsymbol{S}^{\#}(\boldsymbol{v}) = -\frac{1}{\boldsymbol{v}^T \boldsymbol{v}} \boldsymbol{S}(\boldsymbol{v}) \iff \boldsymbol{v} \neq \boldsymbol{0}, \qquad (19)$$

providing the minimum norm solution to (18)

$$\hat{\boldsymbol{r}}_{l,\boldsymbol{p}_l} = -\boldsymbol{S}^{\#}(\hat{\boldsymbol{F}}_{ext,l}) \tilde{\boldsymbol{M}}_{ext,l} , \qquad (20)$$

which is perpendicular to the *line of action* of the estimated contact force, see Fig. 3. Indeed, an initial estimate of the contact point is given by $\hat{\boldsymbol{p}}_l = \boldsymbol{O}_l + \hat{\boldsymbol{r}}_{l,\boldsymbol{p}_l}$. In general, this $\hat{\boldsymbol{p}}_l$ will not be equal to the actual contact point $\boldsymbol{p}_l$, but allows us to reconstruct it from additional geometric considerations that take into account the form of the colliding link. In fact, the line of action of the contact force can be expressed in parametric form as

$$\boldsymbol{p}(\lambda) = \hat{\boldsymbol{p}}_l + \lambda \frac{\hat{\boldsymbol{F}}_{ext,l}}{\|\hat{\boldsymbol{F}}_{ext,l}\|} , \qquad \lambda \in \mathbb{R}. \qquad (21)$$

The actual contact point $\boldsymbol{p}_l$ will lie on the line of action, i.e., $\boldsymbol{p}_l = \boldsymbol{p}(\lambda^*)$ for some $\lambda^* \in \mathbb{R}$. Intersecting the line of action with a geometric, viz. CAD model of the link will return for a convex link two points lying on the link surface: the point corresponding to the smaller value of λ^* (including sign) denotes a force *pushing* the robot link, while the other one denotes a *pulling* force. When an accidental contact occurs, this is typically of a pushing nature, i.e., the smaller value of λ^* should be chosen.

Remark 5. In a very similar way, we could have estimated $\boldsymbol{r}_{s,\boldsymbol{p}_l}$, the position vector of the contact point with respect to the sensor frame located at the robot base, using directly the right-hand side of (14) and without preliminarily isolating the contact link through the residual. However, the complex geometry of the manipulator would possibly generate more than one intersection between the line of force action and the links, making the actual identification critical.

D. Giving up torque measurements

When the residual indicates a contact at a link $l \geq 3$, it is possible to proceed even without a moment measurement at the base (e.g., using only three load cells for measuring force along three orthogonal directions). Keeping in mind

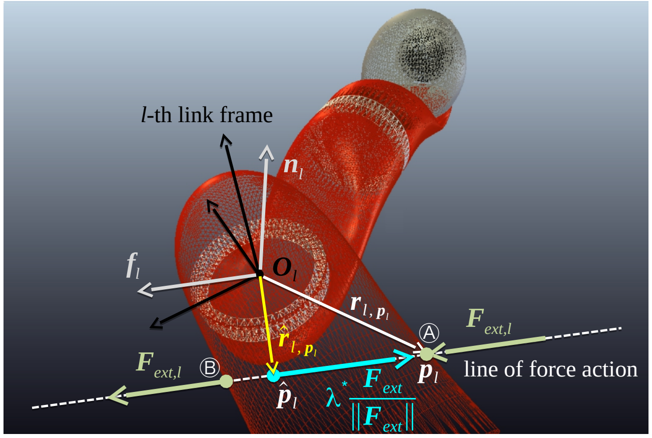

Fig. 3. Estimating the contact point. Force $\boldsymbol{F}_{ext,l}$ hits link l at point $\boldsymbol{p}_l$, labeled as Ⓐ. Vector $\boldsymbol{r}_{l,\boldsymbol{p}_l}$ goes from the link frame origin $\boldsymbol{O}_l$ to $\boldsymbol{p}_l$. From (20), one obtains $\hat{\boldsymbol{r}}_{l,\boldsymbol{p}_l}$, pointing to $\hat{\boldsymbol{p}}_l$ on the line of force action, which is thus reconstructed and represented with a dashed line. Intersecting this line of action with the geometric model of the link surface provides two candidate points, Ⓐ and Ⓑ. Assuming a pushing force ($\lambda^* < 0$), the correct point Ⓐ is recovered.

Remarks 1 and 2, define the two partial Jacobians $\boldsymbol{J}_{\boldsymbol{v},l} \in \mathbb{R}^{3\times l}$ and $\boldsymbol{J}_{\boldsymbol{\omega},l} \in \mathbb{R}^{3\times l}$, such that

$$\dot{\boldsymbol{O}}_l = \boldsymbol{J}_{\boldsymbol{v},l}(\boldsymbol{q})\dot{\boldsymbol{q}}_{1:l}, \qquad \boldsymbol{\omega}_l = \boldsymbol{J}_{\boldsymbol{\omega},l}(\boldsymbol{q})\dot{\boldsymbol{q}}_{1:l}, \tag{22}$$

where $\boldsymbol{v}_{1:l}$ denotes the first l elements of a vector $\boldsymbol{v}$. Then, by defining

$$\bar{\boldsymbol{M}}_{ext,l} = \boldsymbol{M}_{ext,l} + \tilde{\boldsymbol{M}}_{ext,l} = \boldsymbol{M}_{ext,l} + \boldsymbol{S}(\boldsymbol{r}_{l,\boldsymbol{p}_l})\boldsymbol{F}_{ext,l}, \tag{23}$$

we obtain from Remark 4

$$\boldsymbol{\tau}_{ext,1:l} = \boldsymbol{J}^T_{\boldsymbol{v},l}(\boldsymbol{q})\boldsymbol{F}_{ext,l} + \boldsymbol{J}^T_{\boldsymbol{\omega},l}(\boldsymbol{q})\bar{\boldsymbol{M}}_{ext,l}. \tag{24}$$

Equation (24) follows also by duality, being

$$\begin{aligned} \dot{\boldsymbol{p}}_l &= \frac{\mathrm{d}}{\mathrm{d}t}\Big(\boldsymbol{O}_l + \boldsymbol{r}_{l,\boldsymbol{p}_l}\Big) = \dot{\boldsymbol{O}}_l + \boldsymbol{\omega}_l \times \boldsymbol{r}_{l,\boldsymbol{p}_l} \\ &= \Big(\boldsymbol{J}_{\boldsymbol{v},l}(\boldsymbol{q}) - \boldsymbol{S}(\boldsymbol{r}_{l,\boldsymbol{p}_l})\boldsymbol{J}_{\boldsymbol{\omega},l}(\boldsymbol{q})\Big)\dot{\boldsymbol{q}}_{1:l} = \boldsymbol{J}_{\boldsymbol{v}_{\boldsymbol{p}}}(\boldsymbol{q})\dot{\boldsymbol{q}}. \end{aligned} \tag{25}$$

Thus, we reconstruct $\bar{\boldsymbol{M}}_{ext,l}$ as

$$\bar{\boldsymbol{M}}_{ext,l} = (\boldsymbol{J}^T_{\boldsymbol{\omega},l}(\boldsymbol{q}))^{\#}(\hat{\boldsymbol{\tau}}_{ext,\,1:l} - \boldsymbol{J}^T_{\boldsymbol{v},l}(\boldsymbol{q})\hat{\boldsymbol{F}}_{ext,l}), \tag{26}$$

where $\hat{\boldsymbol{F}}_{ext,l}$ is computed as in Sec. III-A. At this stage, one can consider two situations. If the contact point $\boldsymbol{p}_l$ is known (and thus, $\boldsymbol{r}_{l,\boldsymbol{p}_l}$), we can reconstruct also an estimate $\hat{\boldsymbol{M}}_{ext,\,l}$ from (23). Otherwise, if we assume $\boldsymbol{M}_{ext,l} = \boldsymbol{0}$, we recover the scenario in Sec. III-C to estimate $\boldsymbol{p}_l$.

Remark 6. When the above procedure is carelessly applied for contacts at a link $l < 3$, a very poor estimation results, as some information is lost in the null space of $\boldsymbol{J}^T_{\boldsymbol{\omega},l}$. Similar problems arise when the robot encounters a singularity of the angular Jacobian $\boldsymbol{J}^T_{\boldsymbol{\omega},l}$. To gain robustness and avoid unbounded values of the estimates when close to singularities, a damped least squares method [19] is used.

E. Giving up force and torque measurements

When the residual indicates a contact at a link $l \geq 6$, it is possible to completely discard the need of a base F/T sensor. Defining from (22) the Jacobian $\boldsymbol{J}_l = (\boldsymbol{J}^T_{\boldsymbol{v},l}\ \boldsymbol{J}^T_{\boldsymbol{\omega},l})^T \in \mathbb{R}^{6\times l}$

TABLE I
COMPARISON OF DIFFERENT SCENARIOS

	l	Known $\boldsymbol{p}_l$	Unknown $\boldsymbol{p}_l$ with pure contact force
$\boldsymbol{F}/\boldsymbol{T}$ sensor at the base	any	Estimate $\boldsymbol{W}_{ext,l}$	Estimate $\boldsymbol{F}_{ext,l}$ and $\boldsymbol{p}_l$
$\boldsymbol{F}$ sensor at the base	any	Estimate $\boldsymbol{F}_{ext,l}$	Estimate $\boldsymbol{F}_{ext,l}$
	≥ 3	Estimate $\boldsymbol{W}_{ext,l}$	Estimate $\boldsymbol{F}_{ext,l}$ and $\boldsymbol{p}_l$
No sensor at the base	≥ 6	Estimate $\boldsymbol{W}_{ext,l}$	Estimate $\boldsymbol{F}_{ext,l}$ and $\boldsymbol{p}_l$

and from (23) the wrench $\bar{\boldsymbol{W}}_{ext,l} = (\boldsymbol{F}^T_{ext,l}\ \bar{\boldsymbol{M}}^T_{ext,l})^T$, we solve

$$\bar{\boldsymbol{W}}_{ext,l} = (\boldsymbol{J}^T_l(\boldsymbol{q}))^{\#}\,\boldsymbol{\tau}_{ext,1:l}. \tag{27}$$

Similar considerations apply as in Sec. III-D. Figure 4 shows an application of this scheme to determine the a priori unknown contact point.

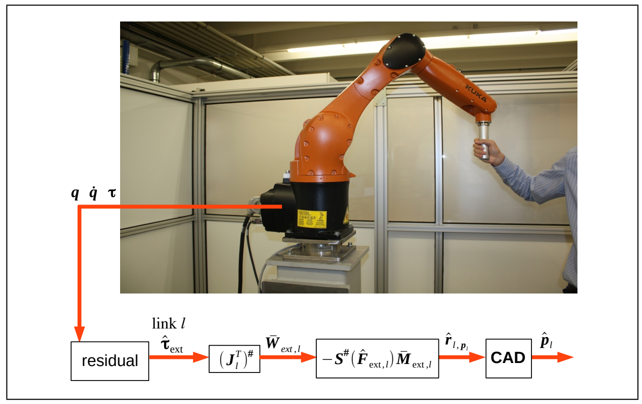

Fig. 4. Obtaining the external force and its point of application without using the base F/T sensor, for a contact occurring on link 6 (or beyond).

F. Summary of cases

Combining the results of the above sections, we obtain 6 different application cases, depending on which real or virtual sensors are being used and which assumption is made on the external wrench. With an F/T sensor at the robot base, and knowing the contact point, it is possible to estimate the complete external wrench. When the contact point is unknown, it is possible to localize this point, *if* we assume a pure contact force. When no torque measurement is available at the base, the same results can be achieved using an estimate of $\boldsymbol{\tau}_{ext}$, but only when the contact is on link $l \geq 3$. When no base measurements are available at all, the same is true for contacts on link 6 or beyond. These results are summarized in Tab. I.

G. Joint acceleration vs. residual

As explained in Sec. II-B, the estimates of $\ddot{\boldsymbol{q}}$ and $\boldsymbol{\tau}_{ext}$ are functionally equivalent. Therefore, all scenarios in Tab. I can be implemented using any of the two estimates $\hat{\ddot{\boldsymbol{q}}}$ or $\hat{\boldsymbol{\tau}}_{ext}$. The latter is usually obtained from (6) using the residual $\boldsymbol{r}$ in (4). However, the following cases hold.

- With a F/T sensor at the base and knowledge of the contact point, $\hat{\boldsymbol{\tau}}_{ext}$ is not needed and $\hat{\ddot{\boldsymbol{q}}}$ will be sufficient.

With unknown contact point, $\hat{\boldsymbol{\tau}}_{ext}$ is only required to detect the contact link.

- With only a force sensor at the base, it is possible to measure the external force, and thus detect the contact, by using only $\hat{\ddot{\boldsymbol{q}}}$. However, to reconstruct the external moment or the unknown contact point, $\hat{\boldsymbol{\tau}}_{ext}$ is necessary.
- With no base sensor, $\hat{\boldsymbol{\tau}}_{ext}$ is strictly necessary while $\hat{\ddot{\boldsymbol{q}}}$ is of no direct use.

IV. Simulation Results

The validity of the above estimation methods was tested through extensive numerical simulations, using a dynamic model of a 7R robot with the kinematics similar to a KUKA LWR. The robot kinematic and dynamic parameters[3] are listed in Tab. II. To include model uncertainty, our estimation schemes are based on dynamic parameters corrupted by additive random errors within $\pm 1\%$ of the nominal values.

TABLE II
Kinematic and Dynamic Parameters

	1	2	3	4	5	6	7
α_i rad	$\pi/2$	$-\pi/2$	$-\pi/2$	$\pi/2$	$\pi/2$	$-\pi/2$	0
a_i cm	0	0	0	0	0	0	0
d_i cm	0	0	40	0	39	0	0
θ_i rad	q_1	q_2	q_3	q_4	q_5	q_6	q_7
m_i kg	2.7	2.7	2.7	2.7	1.7	1.6	0.3
${}^i\boldsymbol{r}^x_{i,c_i}$ cm	0.1340	0.1340	0.1340	0.1340	0.0993	0.0259	0
${}^i\boldsymbol{r}^y_{i,c_i}$ cm	8.7777	2.6220	8.7777	2.6220	11.1650	0.5956	0
${}^i\boldsymbol{r}^z_{i,c_i}$ cm	2.6220	8.7777	2.6220	8.7777	2.6958	0.5328	6.3
${}^i\boldsymbol{I}^{xx}_i$ g m^2	16.341	16.341	16.341	16.341	9.818	3.012	0.102
${}^i\boldsymbol{I}^{xy}_i$ g m^2	0.003	0.001	0.003	0.001	0.001	0	0
${}^i\boldsymbol{I}^{xz}_i$ g m^2	0.001	0.003	0.001	0.003	0.001	0	0
${}^i\boldsymbol{I}^{yy}_i$ g m^2	5.026	16.173	5.026	16.173	3.708	3.023	0.102
${}^i\boldsymbol{I}^{yz}_i$ g m^2	3.533	3.533	3.533	3.533	3.094	0.019	0
${}^i\boldsymbol{I}^{zz}_i$ g m^2	16.173	5.026	16.173	5.026	9.092	3.414	0.158

In the reported results, motion along a desired sinusoidal trajectory in the joint space is controlled by a feedback linearization scheme using the perturbed dynamic parameters and with a PID stabilizing part. The robot is subject to a persistent external force that acts on link 6, resulting in an initial deviation from the desired trajectory, as shown in Fig. 5. The error is quite large for joint 6, but is eventually recovered thanks to the integral term in the controller.

A pure external force is applied to a fixed point and is constant in the reference frame of link 6, with values ${}^6\boldsymbol{F}_{ext,6} = (20\ 0\ 0)^T$ [N] and ${}^6\boldsymbol{r}_{6,\boldsymbol{p}_6} = (0\ 0\ 0.1)^T$ [m]. These values vary over time if expressed in the world frame, and the resulting external joint torques are time-varying as well, due to the change of robot configuration during motion.

We show here two representative cases, namely estimating the contact force and the location of the contact point, with or without using an F/T base sensor. In both schemes, the estimator uses the residual vector to estimate $\boldsymbol{\tau}_{ext}$ and, from this, to compute the joint acceleration.

Figures 6 and 7 show the outputs of the estimation performed using the base F/T sensor. Both estimations are correct and rather insensitive to motion and perturbed dynamics. We remark that, since the contact Jacobian is never used in this scheme, the estimator is not affected by the crossing of kinematic singularities.

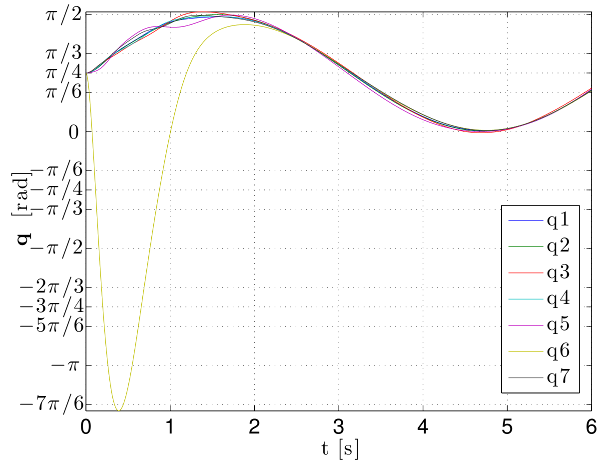

Fig. 5. Joint trajectory followed by the robot.

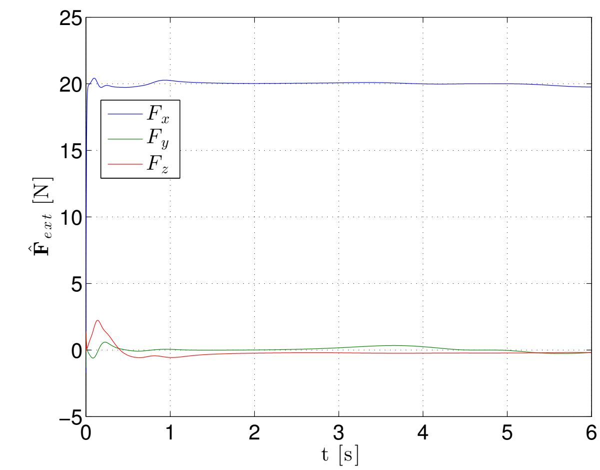

Fig. 6. External force estimated with the base F/T sensor.

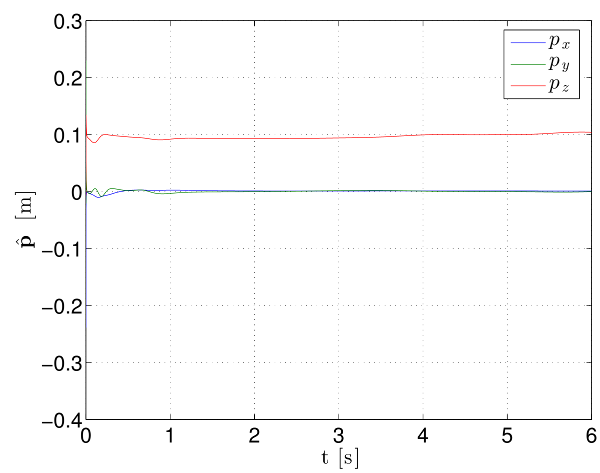

Fig. 7. Contact point estimated with the base F/T sensor.

Figures 8 and 9 show the estimation results using only virtual sensing (no base F/T sensor). Both the external force and the contact point are estimated reasonably well except around time $t = 2$ s, when the robot is passing close to a kinematic singularity. This is confirmed by a peak

[3] For compactness, we used the inertia units [g m^2] = 10^{-3}·[kg m^2].

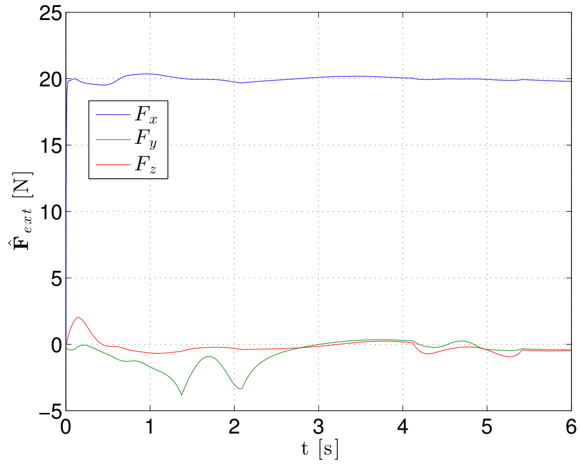

Fig. 8. External force estimated without F/T sensor.

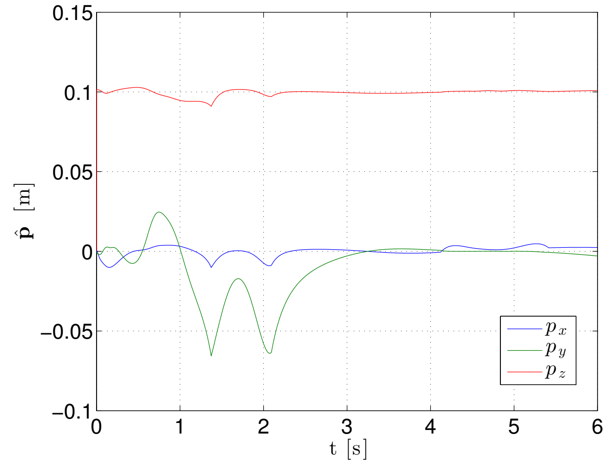

Fig. 9. Contact point estimated without F/T sensor.

in the evolution of the condition number of the Jacobian $\boldsymbol{J}_l(\boldsymbol{q})$, as shown in Fig. 10. Instead, a second near-singularity situation occurring around $t = 5$ s barely affects estimation performance.

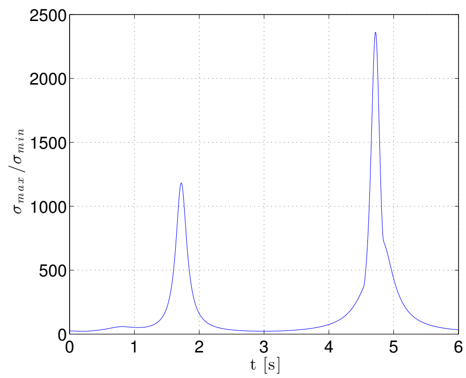

Fig. 10. Condition number of the Jacobian $\boldsymbol{J}_l(\boldsymbol{q})$.

V. Conclusions

We have considered several different methods for the estimation of external forces and moments applied to a robot, together with the unknown contact position.

Using a F/T sensor at the base allows us to accurately estimate a pure contact force acting on any (known) link of the manipulator, no matter how close to the base, together with the unknown position of its application point. Information on the contact link is naturally provided by the residuals. On the other hand, estimation of the full contact wrench requires necessarily the contact point to be known *a priori*; use of the residuals does not not help in avoiding this need. However, the residuals can compensate the lack of torque measurements at the robot base, and even of all force/torque measurements, provided that the contact occurs sufficiently far from the base in the kinematic chain.

Based on the promising estimation results obtained so far in realistic simulations, which were performed using slightly perturbed dynamic parameters, we plan to implement the proposed schemes in the next future on a KUKA Agilus industrial robot.

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