Regulation, Inversion Control, and Feedback Equivalence for Flexible Robots

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Summary

- **a world of soft robots**
  - flexible joints, serial elastic actuation (SEA), variable stiffness actuation (VSA), distributed link flexibility, continuum manipulators, ...

- **flexible joint robots**
  - dynamic modeling and structural control properties
  - inverse dynamics and feedback linearization for **trajectory tracking**
  - regulation with partial state feedback and gravity compensation

- **model-based design based on feedback equivalence**
  - exact cancellation of gravity
  - damping injection on the link side
  - environment interaction via **generalized impedance** model

- **an application of flexible joint robots: physical Human-Robot Interaction (pHRI)**
Summary

- flexible link robots
  - dynamic modeling and the role of zero dynamics
  - PD+ for regulation and input-output linearization for joint-level trajectory tracking
  - stable inversion of desired end-effector trajectories
- outlook on control of (planar) soft manipulators
  - using a piecewise continuous curvature (PCC) dynamic model
Classes of soft robots

Robots with **elastic joints**

- design of lightweight robots with **stiff links** for end-effector accuracy
- **compliant elements** absorb impact energy
  - elastic transmissions (HD, cable-driven, ...)
  - soft coverage of links (foam, safe bags)
- **elastic joints** decouple instantaneously the *larger* inertia of the driving motors from *smaller* inertia of the links (involved in contacts/collisions!)
- *relatively* soft joints need more **sensing** (e.g., joint torque) and better **control** to compensate for static deflections and dynamic vibrations

**→ torque-controlled** robots (DLR LWR-III, KUKA LWR-IV & iiwa, Franka, ...)

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Classes of soft robots

Robots with **variable stiffness actuation (VSA)**

- uncertain interaction with dynamic environments (say, *humans*) requires to adjust online the compliant behavior and/or to control contact forces
  - passive joint elasticity & active impedance control used in parallel
- nonlinear flexible joints with **variable (controlled) stiffness** work at best
  - can be made *stiff when moving slow* (**performance**), *soft when fast* (**safety**)
  - enlarge the set of achievable robot compliance in a task-oriented way
- plus: mechanical **robustness**, optimal **energy use**, **explosive motion** tasks, ...
A matter of terminology ...
Different sources of elasticity, though similar robotic systems

- elastic joints vs. SEA (serial elastic actuators)
  - based on the same physical phenomenon: compliance in actuation
  - compliance added on purpose in SEA, mostly a disturbance in elastic joints
  - different range of stiffness: 5-10K Nm/rad down to 0.2-1K Nm/rad in SEA

- joint deformation is often considered in the linear domain
  - modeled as a concentrated torsional spring with constant stiffness at the joint
  - nonlinear flexible joints share similar control properties
  - nonlinear stiffness characteristics & double actuation are needed in VSA
  - a (serial or antagonistic) VSA working at constant stiffness is an elastic joint

- flexible robots are usually classified as underactuated mechanical systems
  - have less commands than generalized coordinates
  - non-collocation of command inputs and controlled outputs
  - however, they are controllable in the first approximation (the easy case!)
Classes of soft robots

Robots with flexible links

- **distributed link deformations**
  - design of very long and slender arms needed in the application
  - use of lightweight materials to save weight/costs
  - due to large payloads (viz. large contact forces) and/or high motion speed

- as for joint elasticity, neglecting link flexibility will limit static (steady-state error) or dynamic (vibrations, poor tracking) performance

- control issue due to non-minimum phase nature of the end-effector output w.r.t. the torque command input ... “it moves in opposite direction at start!”
Classes of soft robots

**Continuum soft manipulators**

- characteristics in construction
  - long, flexible, lightweight, slender arms
  - tendon/cable-driven, multi-segmented, distributed/embedded actuation
  - energy efficient, (intentional) bio-inspired design
- useful in many special robotic applications
  - surgical, underwater, safe human interaction, cluttered environments, ...
- kinematic, quasi-static, and dynamic modeling (with approximations)
- extra control issues due to task hyper-redundancy and under-actuation
Flexible link robots vs. continuum manipulators

What are the actual (control) differences?

- **continuum** manipulators may assume **very complex shapes in 3D**
  - flexible link robots **not!**
- **continuum** manipulators may keep a **body-deformed configuration** under the action of control (apart from gravity)
  - flexible link robots **not!**
- flexible link robots are **always underactuated** mechanical systems
  - continuum manipulators also, but **possibly not!**
- collocated vs. non-collocated control: **both** may or may not have this ...
Dynamic modeling of robots with flexible joints

Lagrangian formulation (so-called reduced model of [Spong, ASME JDSMC 1987])

- open chain robot with $N$ flexible joints and $N$ rigid links, driven by electrical actuators
- use $N$ motor variables $\theta$ (as reflected through the gear ratios) and $N$ link variables $q$
- assumptions
  
  A1) small displacements at joints (elasticity!)
  
  A2) axis-balanced motors
  
  A3) each motor is mounted on the robot in a position preceding the driven link
  
  A4) no inertial couplings between motors and links

\[ \begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q, \dot{q}) \dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q - \theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} \]

link equation

motor equation

center of mass of rotors on rotation axes
Single elastic joint
Transfer functions of interest

\[ \tau_{ei} = K_i (\theta_i - q_i) \]

we often look rather at the torque-to-velocity mappings ... (eliminating one integrator)

\[ \begin{align*}
P_{\text{motor}}(s) &= \frac{\theta(s)}{\tau(s)} = \frac{Ms^2 + K}{MBs^2 + (M + B)K} \frac{1}{s^2} \\
\end{align*} \]

- system with stable zeros and relative degree = 2
- passive (zeros precede poles on imaginary axis)
- stabilization can be achieved via output \( \theta \) feedback

\[ \begin{align*}
P_{\text{link}}(s) &= \frac{q(s)}{\tau(s)} = \frac{K}{MBs^2 + (M + B)K} \frac{1}{s^2} \\
\end{align*} \]

- NO zeros!!
- maximum relative degree = 4

[De Luca, Book, Springer Handbook of Robotics, 2016]
Single elastic joint
Transfer functions of interest

- typical anti-resonance/resonance on motor velocity output (minimum phase)
- pure resonance on link velocity output (weak or no zeros)

A (small) motor or link side viscous friction was added in these Bode plots.
Inverse dynamics

**Feedforward action for following a desired trajectory in nominal conditions**

given a desired smooth link trajectory $q_d(t) \in C^4$

- compute symbolically the desired motor acceleration and, therefore, also the desired link jerk (i.e., up to the fourth time derivative of the desired motion)

$$
\begin{pmatrix}
M(q) & 0 \\
0 & B
\end{pmatrix}
\begin{pmatrix}
\ddot{q} \\
\dot{\theta}
\end{pmatrix}
+
\begin{pmatrix}
C(q, \dot{q}) \dot{q} \\
0
\end{pmatrix}
+
\begin{pmatrix}
(\dot{g}(q)) \\
0
\end{pmatrix}
+
\begin{pmatrix}
K(q - \theta) \\
K(\theta - q)
\end{pmatrix}
= \begin{pmatrix}
0 \\
\tau
\end{pmatrix}
$$

$$
\tau_d = B\ddot{\theta}_d + K(\theta_d - q_d)
= BK^{-1} \left[ M(q_d) q^{(4)}_d + 2 \ddot{M}(q_d) q^{(3)}_d + \dddot{M}(q_d) \ddot{q}_d + \frac{d^2}{dt^2} \left( C(q_d, \dot{q}_d) \dot{q}_d + g(q_d) \right) \right]
+ \left[ M(q_d) + B \right] \ddot{q}_d + C(q_d, \dot{q}_d) \dot{q}_d + g(q_d)
$$

- the inverse dynamics can be computed efficiently in $O(N)$ using a modified Newton-Euler algorithm (with link recursions up to the 4th order) [Buondonno, De Luca IROS 2015]

- the feedforward command $\tau_d$ can be used in combination with a PD feedback control on motor position/velocity error to obtain a local but simple trajectory tracking controller
Feedback linearization

Full-state **nonlinear feedback** for accurate trajectory tracking tasks

- the link position \( q \) is a **linearizing (flat) output** (nonlinear equivalent of “no zeros”)

\[
\begin{pmatrix}
M(q) & 0 \\
0 & B
\end{pmatrix}
\begin{pmatrix}
\ddot{\theta} \\
\ddot{q}
\end{pmatrix}
+
\begin{pmatrix}
C(q, \dot{q}) \dot{q} \\
0
\end{pmatrix}
+
\begin{pmatrix}
g(q) \\
0
\end{pmatrix}
+
\begin{pmatrix}
K(q - \theta) \\
K(\theta - q)
\end{pmatrix}
=
\begin{pmatrix}
0 \\
\tau
\end{pmatrix}
\leftrightarrow
q^{(4)} = u
\]

- differentiating twice the link equation and using the motor acceleration yields

\[
\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}
\left(2\dddot{q} + \dddot{q} + \frac{d^2}{dt^2}(C\ddot{q} + g(q))\right)
\]

- an **exactly** linear and I-O decoupled system (“chains of 4 integrators”) is obtained
  - to be stabilized with standard techniques for linear dynamics (pole placement, LQ, ...)
  - requires **higher derivatives** of \( q \)
  - requires **higher derivatives** of the dynamics components

- A \( O(N^3) \) **Newton-Euler** recursive numerical algorithm is available for this problem

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Feedback linearization
Based on the **rigid model** only vs. when including **joint elasticity**

\[
\tau = M(q)(\ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q)) + C(q, \dot{q})\dot{q} + g(q)
\]

\[
\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dddot{M}q^{(3)} + \dddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q))\right)
\]

\[
u = \left(q_d^{[4]} + K_J(\dddot{q}_d - \dddot{q}) + K_A(\dddot{q}_d - \dddot{q}) + K_D(\dddot{q}_d - \dddot{q}) + K_P(q_d - q)\right)
\]

[Spong, ASME JDSMC 1987]
Feedback linearization
Benefits on an industrial KUKA KR-15/2 robot (235 kg) with joint elasticity

conventional industrial robot control

feedback linearization + high-damping

trajectory tracking with model-based control

[Thümmel, PhD 2007]
Visco-elasticity at the joints

Introduces a structural change ...

\[
\begin{bmatrix}
M(q) & 0^* \\
0^* & B
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\dot{\theta}
\end{bmatrix}
+ 
\begin{bmatrix}
C(q, \dot{q}) \dot{q}^* \\
0 \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
g(q) \\
0 \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
K(q - \theta) + D(\dot{q} - \dot{\theta}) \\
K(\theta - q) + D(\dot{\theta} - \dot{q})
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\tau
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>coupling type</th>
<th>control consequence for the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>stiffness</td>
<td>basic elastic coupling, maximum relative degree (= 4) of output ( q )</td>
</tr>
<tr>
<td>damping</td>
<td>reduced relative degree (= 3), only I-O linearization by static feedback</td>
</tr>
<tr>
<td>inertia*</td>
<td>reduced relative degree, exact or I-O linearization needs dynamic feedback</td>
</tr>
</tbody>
</table>

* the so-called complete dynamic model includes off-diagonal inertial couplings between motors and links [De Luca, Lucibello, ICRA 1998]
Regulation tasks

Using a minimal PD+ action on the motor side

for a desired constant link position \( q_d \)

- evaluate the associated desired motor position \( \theta_d \) at steady state
- collocated (partial state) feedback preserves passivity, with stiff \( K_P \) gain dominating gravity
- focus on the term for gravity compensation (acting on link side) from motor measurements

\[
\tau = \tau_g + K_P (\theta_d - \theta) - K_D \dot{\theta} \quad K_D > 0
\]

<table>
<thead>
<tr>
<th>( \tau_g )</th>
<th>gain criteria for stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(q_d) )</td>
<td>( \lambda_{\text{min}} \begin{bmatrix} K &amp; -K \ -K &amp; K + K_P \end{bmatrix} &gt; \alpha ) [Tomei, IEEE T-AC 1991]</td>
</tr>
<tr>
<td>( g(\theta - K^{-1}g(q_d)) )</td>
<td>( \lambda_{\text{min}} \begin{bmatrix} K &amp; -K \ -K &amp; K + K_P \end{bmatrix} &gt; \alpha ) [De Luca, Siciliano, Zollo, ASME JDSMC 2004]</td>
</tr>
<tr>
<td>( g(\bar{q}(\theta)), \bar{q}(\theta): g(\bar{q}) = K(\theta - \bar{q}) )</td>
<td>( K_P &gt; 0, \lambda_{\text{min}}(K) &gt; \alpha ) [Ott et al, ICRA 2004]</td>
</tr>
<tr>
<td>( g(q) + BK^{-1}\ddot{q}(q) )</td>
<td>( K_P &gt; 0, K &gt; 0 ) [De Luca, Flacco, CDC 2010]</td>
</tr>
</tbody>
</table>

exact gravity cancellation (with full state feedback)

\[
\alpha = \max_q \left\| \frac{\partial g(q)}{\partial q} \right\|
\]
Exact gravity cancellation
A slightly different view

- for rigid robots this is **trivial**, due to **collocation**

\[ M(q)\ddot{q} + c(q, \dot{q}) + D\dot{q} + g(q) = \tau \]

\[ \tau = \tau_g + \tau_0 \]

\[ \tau_g = g(q) \]

\[ q \equiv q_0 \]

\[ \tau_0 \rightarrow q_0 \]

\[ M(q)\ddot{q} + c(q, \dot{q}) + D\dot{q} = \tau_0 \]
Exact gravity cancellation
... based on the concept of feedback equivalence between nonlinear systems

- for elastic joint robots, non-collocation of input torque and gravity term

\[
\tau = \tau_g + \tau_0
\]
\[
\theta_0 = \theta + K^{-1}g(q)
\]
\[
M(q)\ddot{q} + c(q, \dot{q}) + D_q \dot{q} + g(q) + K(q - \theta) = 0
\]
\[
B\ddot{\theta} + D_\theta \dot{\theta} + K(\theta - q) = \tau
\]

\[
\tau_g = g(q) + D_\theta K^{-1} \dot{g}(q) + BK^{-1} \ddot{g}(q)
\]

state transformation
feedback control
Feedback equivalence

Exploit the system property of being feedback linearizable (without forcing it!)

\[ u = \tilde{a}(x) + \tilde{b}(x)u_0 \]
\[ x_0 = T_0^{-1}(T(x)) = \tilde{T}(x) \]

\[ \dot{x} = f(x) + G(x)u \]

gravity-loaded system

\[ \dot{x}_0 = f_0(x_0) + G_0(x_0)u_0 \]

gravity-free system

feedback transformations

\[ u = \alpha(x) + \beta(x)v \]
\[ z = T(x) \]

static state feedback
+ change of coordinates
both invertible

\[ u_0 = \alpha_0(x_0) + \beta_0(x_0)v \]
\[ z = T_0(x_0) \]

linear, controllable system

\[ \dot{z} = Az + Bv \]

\[ z \approx \text{linearizing outputs} \]
A global PD-type regulator

Exact gravity cancellation + PD law on modified motor variables: A 1-DOF arm

Identical link behavior

Different motor behavior

Gravity-loaded system

Gravity-free system

Gravity-loaded system under PD + gravity cancellation vs.
Gravity-free system under PD (with same gains)

$K_P > 0 \quad K > 0$

Works without strictly positive lower bounds (good also for VSA!)

[De Luca, Flacco, ICRA 2011]
Vibration damping on lightweight robots

DLR-III or KUKA LWR-IV with relatively low joint elasticity (use of Harmonic Drives)

for relatively large joint elasticity (low stiffness), as encountered in VSA systems, vibration damping via joint torque feedback + motor damping is insufficient for high performance!

[Albu-Schäffer et al, IJRR 2007]
Damping injection on the link side

Method for the VSA-driven bimanual humanoid torso David

**PLANT DYNAMICS**

\[
\begin{bmatrix}
M(q) & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{\theta}
\end{bmatrix}
+ 
\begin{bmatrix}
C(q, \dot{q}) \dot{q} \\
0 \\
0 \\
K(q - \theta)
\end{bmatrix}
+ 
\begin{bmatrix}
g(q) \\
0 \\
K(\theta - q)
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\tau
\end{bmatrix}
\]

\[\tau = \tau_0 - D\dot{q} - BK^{-1}D\ddot{q}\]

**CLOSED-LOOP DYNAMICS**

\[
\begin{bmatrix}
M(q) & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\ddot{\theta}_0
\end{bmatrix}
+ 
\begin{bmatrix}
C(q, \dot{q}) \dot{q} \\
0 \\
0 \\
K(q - \theta_0)
\end{bmatrix}
+ 
\begin{bmatrix}
g(q) \\
0 \\
K(\theta_0 - q)
\end{bmatrix}
= 
\begin{bmatrix}
-D\dot{q} \\
\tau_0
\end{bmatrix}
\]

- same principle of feedback equivalence (including state transformation)!
Damping injection on the link side

Method for VSA-driven bimanual humanoid torso David at DLR

[video]

[video]

[Keppler et al, IEEE T-RO 2018]
Environment interaction via impedance control

Matching a generalized (fourth order) impedance model: A simple 1-DOF case

\[ M \ddot{q} + K(q - \theta) = \tau_e \]
\[ B \dot{\theta} + D \dot{\theta} + K(\theta - q) = \tau \]

feedback control

\[ \tau = K(\theta - q) + D \dot{\theta} - BK^{-1} \left\{ (K - K_0)M^{-1}(\tau_e + K(\theta - q)) + K_0B_0^{-1}(\tau_0 - D_0 \dot{\theta}_0 - K(\theta - q)) \right\} \]

state transformation

\[ \dot{\theta}_0 = \dot{q} + K K_0^{-1}(\dot{\theta} - \dot{q}) \]

Now assume that \( M_0 = M \) in order to avoid derivatives of the measured force \( \tau_e \)

\[ M_0 \ddot{q} + K_0(q - \theta_0) = \tau_e \]
\[ B_0 \dot{\theta}_0 + D_0 \dot{\theta}_0 + K_0(\theta_0 - q) = \tau_0 \]

- again, by the principle of feedback equivalence (including the state transformation)
**Torque feedback**

An inner loop that largely reduces motor inertia (and friction)

Consider a pure proportional torque feedback (+ a derivative term for the visco-elastic case)

\[
\tau = BB_0^{-1}u + (I - BB_0^{-1})\tau_j + (I - BB_0^{-1})DK^{-1}\dot{\tau}_j
\]

\[\tau = BB_0^{-1}u + -K_T(I - BB_0^{-1})\tau_j + -K_S(I - BB_0^{-1})DK^{-1}\dot{\tau}_j\]

**physical interpretation:** scaling of the motor inertia and motor friction!

[Ott, Albu-Schäffer, IEEE T-RO 2008]

but also...

**special case** of matching by feedback equivalence!

original motor dynamics

\[B\ddot{\theta} + K(\theta - q) = \tau\]

visco-elastic case

\[B\ddot{\theta} + \tau_j + DK^{-1}\dot{\tau}_j = \tau\]

**after** the torque feedback

\[B_0\ddot{\theta} + K(\theta - q) = u\]

\[B_0\ddot{\theta} + \tau_j + DK^{-1}\dot{\tau}_j = u\]
Full-state feedback

Combining torque feedback with motor PD regulation ("torque controlled robots")

inertia scaling via torque feedback
regulation via motor PD, e.g., with

\[ \tau = (I + K_T)u - K_T \tau_J - K_S \dot{\tau}_J \]

\[ u = g(\bar{q}(\theta)) + K_\theta (\theta_d - \theta) - D_\theta \dot{\theta} \]

⇒ joint level control structure of the DLR (and KUKA) lightweight robots

dynamics feedforward and desired torque command

\[ \tau = \tau_{j,d} - K_T (\tau_J - \tau_{j,d}) - K_S \dot{\tau}_J - K_P (\theta_d - \theta) - K_D \dot{\theta} + \tau_f + \tau_{dob} \text{ (+ integral actions)} \]

motor inertia scaling

setpoint control

vibration damping

friction compensation and/or disturbance observer

**torque control**

\[ K_P = 0 \]
\[ K_D = 0 \]
\[ K_T > 0 \]
\[ K_S > 0 \]
\[ \tau_{j,d} = \tau_d \]

**position control**

\[ K_P > 0 \]
\[ K_D > 0 \]
\[ K_T > 0 \]
\[ K_S > 0 \]
\[ \tau_{j,d} = g(q) \]

**impedance control**

\[ K_P = K_T K_\theta \]
\[ K_D = K_T D_\theta \]
\[ K_T = (BB_d^{-1} - I) \]
\[ K_S = (BB_d^{-1} - I)DK^{-1} \]
\[ \tau_{j,d} = g(\bar{q}(\theta)) \]
Exploiting joint elasticity in pHRI
Detection & selective reaction in torque control mode, with momentum-based residuals

- collision detection & reaction for safety (model-based + joint torque sensing)

Exploiting joint elasticity in pHRI
Human-robot collaboration in torque control mode

- contact force estimation & control (virtual force sensor, anywhere/anytime)

[Magrini et al, ICRA 2015]
Dynamic modeling of a single flexible link

Euler-Bernoulli beam [Bellezza, Lanari, Ulivi, ICRA 1990]

- beam of length $l$, uniform density $\rho$, Young modulus $E \cdot$ cross-section inertia $EI$ in rotation on a horizontal plane
- actuator inertia $J_0$ at the base and payload mass $m_p$ and inertia $J_p$ at the tip
- various angular variables: $\theta_c(t)$ clamped at base (measured by encoder), $\theta(t)$ pointing at CoM (very convenient!), $\theta_t(t)$ pointing at the tip (measurable and of interest)
- small deformations of pure bending $w(x, t) = \phi(x) \delta(t)$ (with space/time separation)
- Hamilton principle + calculus of variation $\Rightarrow$ PDE equations, with geometric and dynamic boundary conditions

\[
\begin{align*}
J \ddot{\theta}(t) &= \tau(t) \quad J = J_0 + \frac{\rho l^3}{3} + J_p + m_p l^2 \\
E I w'''(x, t) + \rho \left( \ddot{w}(x, t) + x \dddot{\theta}(t) \right) &= 0 \\
w(0, t) &= 0 \\
E I w''(0, t) &= J_0 \left( \ddot{\theta}(t) + \dddot{\theta}(0, t) \right) - \tau(t) \\
E I w''(l, t) &= -J_p \left( \ddot{\theta}(t) + \dddot{\theta}(l, t) \right) \\
E I w'''(l, t) &= m_p \left( l \dddot{\theta}(t) + \dddot{w}(l, t) \right)
\end{align*}
\]
Dynamic modeling of a single flexible link

Characteristic equation and eigenfrequencies

- Infinite countable roots $\beta_i, i = 1, 2, \ldots$ of an eigenvalue problem

\[
(1 - \frac{m_p}{\rho^2} \beta_i^4 (J_0 + J_p))(\cos \beta_i l \sinh \beta_i l - \sin \beta_i l \cosh \beta_i l) - \frac{2m_p}{\rho} \beta_i \sin \beta_i l \sinh \beta_i l - \frac{2J_p}{\rho} \beta_i^3 \cos \beta_i l \cosh \beta_i l \\
- \frac{J_0}{\rho} \beta_i^3 (1 + \cos \beta_i l \cosh \beta_i l) + \frac{J_0 J_p}{\rho^2} \beta_i^6 (\cos \beta_i l \sinh \beta_i l + \sin \beta_i l \cosh \beta_i l) - \frac{J_0 J_p m_p}{\rho^3} \beta_i^7 (1 - \cos \beta_i l \cosh \beta_i l) = 0
\]

- Common assumed modes are special cases
  - Clamped-free: $m_p = 0, J_p = 0, J_0 \to \infty \implies 1 + \cos \beta_i l \cosh \beta_i l = 0$
  - Pinned-free: $m_p = 0, J_p = 0, J_0 = 0 \implies \cos \beta_i l \sinh \beta_i l - \sin \beta_i l \cosh \beta_i l = 0$

- Associated to each root $\beta_i$ there is
  - An eigenfrequency (system vibrations) $\omega_i = \sqrt{EI \beta_i^4 / \rho}$
  - An eigenvector (spatial mode) $\phi_i(x) = A \sin \beta_i x + B \cos \beta_i x + C \sinh \beta_i x + D \cosh \beta_i x$
  - A deformation variable $\delta_i(t)$

- Finite approximation by truncation up to $n_e$ orthonormal modes: $w(x, t) = \sum_{i=1}^{n_e} \phi_i(x) \delta_i(t)$
Dynamic model of a single flexible link

Final equations and system outputs

- **linear dynamic model**
  \[ J\ddot{\theta} = \tau \]
  \[ \ddot{\delta}_i + \omega_i^2 \delta_i = \phi'_i(0)\tau, \quad i = 1, \ldots, n_e \]

- **including modal damping** (\(\zeta_i \in [0,1]\))
  \[ J\ddot{\theta} = \tau \]
  \[ \ddot{\delta}_i + 2\zeta_i\omega_i\dot{\delta}_i + \omega_i^2 \delta_i = \phi'_i(0)\tau, \quad i = 1, \ldots, n_e \]

- **in matrix form**

  \[ q = (\theta, \delta_1, \delta_2, \ldots, \delta_{n_e}) \in \mathbb{R}^{n_e+1} \]

  \[ M = \begin{pmatrix} J & 0 \\ 0 & I_{n_e} \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 2Z\Omega \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 0 \\ 0 & \Omega^2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ \Phi'(0) \end{pmatrix} \]

- **system outputs**

  \[ \theta_c = \theta + \sum_{i=1}^{n_e} \phi'_i(0)\delta_i \]
  \[ \theta_t = \theta + \sum_{i=1}^{n_e} \frac{\phi_i(l)}{l} \delta_i \]

  **clamped joint level:** always minimum phase  \[ \text{tip level: typically non-minimum phase} \]
Single flexible link

Eigenmodes

- physical data of an Euler-Bernoulli model
  \[ l = 1, \quad \rho = 0.5, \quad EI = 1, \quad J_0 = 0.002 \quad (m_p = J_p = 0) \]
- first four exact mode shapes (normalized) – \( k \)-th mode has \( k \) nodes w.r.t. rigid axis

\[ \phi_1(x) \]
\[ \text{at } f_1 = 3.27 \text{ Hz} \]

\[ \phi_2(x) \]
\[ \text{at } f_2 = 8.89 \text{ Hz} \]

\[ \phi_3(x) \]
\[ \text{at } f_3 = 16.13 \text{ Hz} \]

\[ \phi_4(x) \]
\[ \text{at } f_4 = 28.28 \text{ Hz} \]
Single flexible link
Transfer functions of interest and frequency responses

\[
P_c(s) = \frac{\theta_c(s)}{\tau(s)} = \frac{1}{Js^2} + \sum_{i=1}^{n_e} \frac{\phi_i(0)^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}
\]

\[
P_t(s) = \frac{\theta_t(s)}{\tau(s)} = \frac{1}{Js^2} + \sum_{i=1}^{n_e} \frac{\phi_i'(0)\phi_i(l)/l}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}
\]

\[n_e = 3\] modes

- clamped joint level: always minimum phase
- tip level: typically non-minimum phase

Graphs showing magnitude and phase plots for transfer functions.
in the absence of modal damping

- $n_e = 2$ modes
- $n_e = 3$ modes

**Passivity:**
- Zeros precede poles (in alternate pairs)

**Non-minimum phase zeros:**
- Unstable inverse system!

Clamped joint level
- Tip level

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Single flexible link
Experimental model identification

- in the frequency domain

Sweep joint acceleration excitation signal: plant vs. model

Joint acceleration frequency response: plant vs. model matching (≤ 1%) of resonances at $f_1 = 14.4$, $f_1 = 34.2$, $f_1 = 69.3$ Hz
Dynamic modeling of robots with flexible links

Lagrangian formulation (finite-dimensional)

- open chain robot with $N$ flexible links, each with $n_{e,i}$ deformation variables (a total of $N_e$)
- single-link modeling results embedded with caution for each of the multiple flexible links
- in general, 2D bending + torsion (to limit model complexity, only planar structures here)
- typical use of simpler assumed modes to describe spatial deformation

\[
\begin{align*}
(N + N_e) \times (N + N_e) & \quad \text{full inertia matrix} \\
\begin{pmatrix}
M_{\theta\theta}(\theta, \delta) & M_{\theta\delta}(\theta, \delta) \\
M_{\delta\theta}(\theta, \delta) & M_{\delta\delta}(\theta, \delta)
\end{pmatrix}
\begin{pmatrix}
\ddot{\theta} \\
\ddot{\delta}
\end{pmatrix}
+ 
\begin{pmatrix}
c_{\theta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \\
c_{\delta}(\theta, \delta, \dot{\theta}, \dot{\delta})
\end{pmatrix}
+ 
\begin{pmatrix}
g_{\theta}(\theta, \delta) \\
g_{\delta}(\theta, \delta)
\end{pmatrix}
+ 
\begin{pmatrix}
0 \\
K\delta + D\dot{\delta}
\end{pmatrix}
= 
\begin{pmatrix}
\tau
\end{pmatrix}
\end{align*}
\]

[De Luca, Siciliano, IEEE T-SMC 1991]
Dynamic modeling of robots with flexible links

Simplifications in model (possibly, for control use)

- in matrix form
  \[ q = (\theta, \delta) \in \mathbb{R}^{N+N_e} \]
  \[ M(q)\ddot{q} + c(q, \dot{q}) + g(q) + \begin{pmatrix} 0 \\ D\dot{\delta} + K\delta \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix} \]

- common simplifications in mechanics
  - small deformations (in the linear domain) \( \rightarrow \) \( g_\delta(\theta) \)
  - kinetic energy evaluated in the undeformed \((\delta = 0)\) configuration of the arm \( \rightarrow \) \( M(\theta) \)
  - \( M_{\delta\delta} \) often constant

- flexible link manipulators are underactuated systems
  - less command inputs \( \tau \) than generalized coordinates \( q \)
  - we consider as many controlled outputs \( y \) as commands (‘squaring the I-O problem’)
  - problems, however, with the associated zero dynamics (in a linear or nonlinear setting)
Control problems for flexible link robots
A compact overview (moving in free space) ...

- regulation to a desired equilibrium state \((q, \dot{q}) = (\theta_d, \delta_d, 0,0)\)
  - only the desired joint/rigid variable \(\theta_d\) is assigned: \(\delta_d\) has to be determined
  - \(\theta_d\) may come from a (numerical) kineto-static inversion of a Cartesian pose \(y_d\)
  - forward kinematics of flexible robots is a complete function \(y = \text{kin}(\theta, \delta)\)
  - global stabilization results with joint PD + gravity compensation

- tracking of a joint trajectory \(\theta_d(t)\)
  - the easy case, solved by I-O inversion (stable/minimum phase zero dynamics)
  - solution stiffens the arm at the bases of the flexible links, rejecting vibrations

- tracking of an end-effector trajectory \(y_d(t)\)
  - the difficult case, facing the unstable/non-minimum phase zero dynamics
  - non-causal solution designed in frequency or time domain (feedforward + local stabilizing feedback)
  - causal solution by nonlinear regulation (avoiding critical cancellations)

- rest-to-rest motion between two equilibria in assigned time \(T\)
Control solutions for flexible link robots

Main results – 1

- **global asymptotic stabilization** to a desired equilibrium state \((\theta_d, \delta_d, 0,0)\)

\[
\tau = K_P (\theta_d - \theta) - K_D \dot{\theta} + g_\theta (\theta_d, \delta_d)
\]

\[
\delta_d = -K^{-1} g_{\delta}(\theta_d) \quad \lambda_{\text{min}} \left\{ \begin{pmatrix} K_P & 0 \\ 0 & K \end{pmatrix} \right\} > \alpha \quad K_D > 0
\]

possibly by iterative solution of \( \text{kin}(\theta, -K^{-1} g_{\delta}(\theta)) = y_d \)

upper bound on \( \left\| \frac{\partial g(q)}{\partial q} \right\| \) [De Luca, Siciliano, IEEE T-RO 1993a]

- two-link flexible arm with two bending modes for each link under gravity

![Graphs showing joint angles, joint torques, 1st and 2nd link deflections](image)

from \( \theta(0) = (-90^\circ, 0^\circ) \)

to \( \theta_d = (-45^\circ, 0^\circ) \)

\( f_{11} = 1.4, f_{12} = 5.1, \quad f_{21} = 5.2, f_{22} = 32.4 \) [Hz]
Control solutions for flexible link robots

Main results – 2

- tracking of a joint trajectory $\theta_d(t)$ via I-O feedback linearization

$$\tau = (M_{\theta\theta} - M_{\theta\delta} M_{\delta\delta}^{-1} M_{\theta\delta}^T) a + c_\theta + g_\theta - M_{\theta\delta} M_{\delta\delta}^{-1} (c_\delta + g_\delta + K\delta + D\dot{\delta})$$

resulting closed-loop system

$$\ddot{\theta} = a$$

$$\ddot{\delta} = -M_{\delta\delta}^{-1} (M_{\theta\delta}^T a + c_\delta + g_\delta + K\delta + D\dot{\delta})$$

trajectory error (exponential) stabilization

$$a = \ddot{\theta}_d + K_D (\dot{\theta}_d - \dot{\theta}) + K_P (\theta_d - \theta), \quad K_P, K_D > 0$$

- the zero dynamics, when the output $\theta(t) \equiv 0$, is asymptotically stable (via Lyapunov argument)

$$\ddot{\delta} = -M_{\delta\delta}^{-1} (c_\delta + g_\delta + K\delta + D\dot{\delta})$$

- the clamped dynamics, when the output $\theta(t) \equiv \theta_d(t)$, is bounded

$$\ddot{\delta} = -A_2(t) \dot{\delta} + A_1(t) \delta + f_\delta(t)$$

[De Luca, Siciliano, AIAA JGCD 1993b]
Control solutions for flexible link robots

Main results – 3

- tracking of an end-effector trajectory $y_d(t)$
  - non-causal command designed in frequency domain ⇒ desired acceleration as part of a periodic profile, bounded inversion via Fourier transform (or FFT) 
    [Bayo, JRobSyst 1987]
  - ... designed in time domain ⇒ forward/backward time integration of stable/unstable parts of the inverse system 
    [Kwon, Book, ASME JDSMC 1994]
  - both extended from linear to nonlinear case via numerical/iterative methods

- bang-bang acceleration in $T = 2$ s for both system outputs
- control torques, with pre-charge and discharge intervals ($T_r = 2.5$ s)
- stroboscopic motion of the 2R FLEXARM under E-E control
Control solutions for flexible link robots at Sapienza

Main results – 4 (oldies but goldies...)

- stable **nonlinear regulation** of end-effector trajectory for the 2R FLEXARM
- **rest-to-rest** slew motion in **assigned time** for a one-link flexible beam

45° for (rigid) link 1 and 45° for tip of flexible forearm in \( T = 1.5 \) s

[De Luca *et al*, CDC 1990, ICRA 1998]

90° slew in \( T = 2 \) s (flat output design)

[De Luca, Di Giovanni, AIM 2001; De Luca, Caiano, Del Vescovo, ISER 2002]
Control solutions for flexible link robots

More results, including physical interaction

- 3R arm with flexible links TUDOR (TU Dortmund Omni-elastic Robot)
- vibration damping by strain gauge feedback during motion (or after impact)

[Malzahn et al, IEEE ROBIO 2011]

- collision detection and reaction based on generalized momentum observer
  same residual method as in elastic joint robots!!

[Malzahn, Bertram, IFAC World Congr 2014]
Outlook on control of soft manipulators

Continuum planar arms with PCC

- **dynamic modeling assumptions**
  
  A1) [kinematics] approximated as a series of $n$ segments, each with a curvature $q_i$
  
  A2) [inertia] each segment can be described by an equivalent point mass
  
  A3) [impedance] continuous distribution of infinitesimal springs and dampers

- fully actuated on each segment $\Leftrightarrow$ underactuated with $m < n$ input commands

[Della Santina et al, IJRR 2020]
Dynamic model of planar soft manipulator

Full actuation vs. underactuation in PCC model

- **actuated** on each of the $n$ segments

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + Kq + D\dot{q} = \tau \]

with the usual properties ($M > 0$, $\dot{M} - 2C$ skew-symmetric, $g$ bounded in norm, ...)

\[ \Rightarrow \] regulation, curvature trajectory tracking, Cartesian stiffness control, preserving (in nominal conditions) stiffness and damping of the soft system

[Della Santina et al, IJRR 2020]

- **underactuated** with only $m < n$ input commands
  - let $q = (q_a, q_u)$, possibly after relabeling of segments, being $q_a \in \mathbb{R}^m$ the curvature of active segments and $q_u \in \mathbb{R}^{n-m}$ that of the unactuated segments
  - dropping dependencies, with active commands $\tau \in \mathbb{R}^m$ and suitable partitions

\[
\begin{pmatrix}
M_{aa} & M_{au} \\
M_{au}^T & M_{pu}
\end{pmatrix}
\begin{pmatrix}
\ddot{q}_a \\
\ddot{q}_u
\end{pmatrix}
+
\begin{pmatrix}
C_{aa} & C_{au} \\
C_{ua} & C_{uu}
\end{pmatrix}
\begin{pmatrix}
\dot{q}_a \\
\dot{q}_u
\end{pmatrix}
+
\begin{pmatrix}
g_a \\
g_u
\end{pmatrix}
+
\begin{pmatrix}
K_a & 0 \\
0 & K_u
\end{pmatrix}
\begin{pmatrix}
q_a \\
q_u
\end{pmatrix}
+
\begin{pmatrix}
D_a & 0 \\
0 & D_u
\end{pmatrix}
\begin{pmatrix}
\dot{q}_a \\
\dot{q}_u
\end{pmatrix}
=
\begin{pmatrix}
\tau \\
0
\end{pmatrix}
\]

\[ \Rightarrow \] a few preliminary results ... [joint work with Pietro Pustina, 2021]
Regulation and trajectory tracking

Full actuation: moving from joint configuration space to local curvature space

- **regulation** to a (quasi-static) \( q_d \)

\[
\tau = K q_d + D \dot{q}_d + g(q) + K_P (q_d - q) + K_D (\dot{q}_d - \dot{q})
\]

regulatin to a (quasi-static) \( q_d \)

- **tracking** of \( q_d(t) \), with \( \dot{q}_d \neq 0, \ddot{q}_d \neq 0 \)

\[
\tau = K q_d + D \dot{q}_d + g(q) + C(q, \dot{q}) \dot{q}_d + M(q) \ddot{q}_d + K_P (q_d - q) + K_D (\dot{q}_d - \dot{q})
\]

tracking of \( q_d(t) \), with \( \dot{q}_d \neq 0, \ddot{q}_d \neq 0 \)

feedforward (soft robot stiffness & damping)

gravity feedback cancellation

robustifying PD action

[Della Santina et al, IJRR 2020]

passivity-based tracking controller

video
Zero dynamics and regulation

Underactuated planar PCC model, without and with gravity

- **Zero dynamics** when the output is $y = q_a \in \mathbb{R}^m$
  - in the **absence** of gravity ($g(q) \equiv 0$), the unique state $(q_u, \dot{q}_u) = (0,0)$ is globally asymptotically stable for the zero dynamics of the soft robot
  - in the **presence** of gravity (e.g., in a vertical plane), the trajectories of the zero dynamics remain bounded and converge to $(q_u, \dot{q}_u) = (q_{u,eq}, 0)$, being $q_{u,eq}$ a solution of
    \[
    K_u q_u + g_u(0, q_u) = 0
    \]
  - proofs by Lyapunov/La Salle analysis

- **Regulation** to $q_d = (q_{a,d}, 0) \in \mathbb{R}^n, q_{a,d} \in \mathbb{R}^m$, in the **absence** of gravity
  \[
  \tau = K_P (q_{a,d} - q_a) - K_D \dot{q}_a + K_a q_{a,d} \quad K_P, K_D > 0
  \]

- **Regulation** to $q_d = (q_{a,d}, q_{p,d}) \in \mathbb{R}^n, q_{a,d} \in \mathbb{R}^m$, in the **presence** of gravity
  \[
  \begin{align*}
  \tau &= K_P (q_{a,d} - q_a) - K_D \dot{q}_a + g_a(q_d) + K_a q_{a,d} \quad K_P > 0, \text{ sufficiently large} \\
  \tau^g &= K_P (q_{a,d} - q_a) - K_D \dot{q}_a + g_a(q_{a,d}, q_u) + K_a q_{a,d} \\
  K_u q_u + g_u(q_{a,d}, q_u) &= 0
  \end{align*}
  \]
Simulation results

Underactuation with $n = 3$ segments, $m = 2$ actuated: $q_a = (q_1, q_3)$, $q_u = q_2$

- regulation to $q_{a,d} = (0,0)$ from $q(0) = (-\pi, -\pi, \pi)$ using $\tau^g$, in the presence of gravity

- tracking of $q_{a,d}(t) = (\sin t, \cos t)$ starting from $q(0) = (-\pi, -\pi, 0)$, using a partial feedback linearization control $\tau^{PFL}$, in the presence of gravity

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Take home messages

Control of soft robots in 2021+

- a “soft explosion” is revamping the mature field of flexible robot control
  - consideration of dynamics in the control design/performance of soft robots
  - combine (learned) feedforward and feedback to achieve robustness
  - iterative learning (on repetitive tasks) is available for flexible manipulators
  - optimal control (min time, min energy, max force, ...) still open for fun

- revisiting model-based control design
  - do not fight against the natural dynamics of the system
    - it is unwise to stiffen what was designed/intended to be soft on purpose
    - still, don’t give up too much of desirable performance!

- ideas assessed for flexible joints and links may migrate to other classes of soft-bodied robots (and applications)
  - keep in mind intrinsic task constraints and control limitations (e.g., instabilities in system inversion of tip trajectories for flexible link robots)
  - locomotion, shared manipulation, physical interaction in complex tasks, ...
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pdf and videos: see also
[www.diag.uniroma1.it/deluca/Publications.php](http://www.diag.uniroma1.it/deluca/Publications.php)
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