Description Logics: Modal Logics for Class-Based Modeling

Giuseppe De Giacomo
Dipartimento di Informatica e Sistemistica
Università di Roma "La Sapienza"
Via Salaria 113, 00198 Roma, Italy
degiacomo@dis.uniroma1.it

The research in Artificial Intelligence and Computer Science has always paid special attention to formalisms for the structured representation of information. In Artificial Intelligence, the investigation of such formalisms began with semantic networks and frames, which have been influential for many formalisms proposed in the areas of knowledge representation, databases, and programming languages, and developed towards formal logic-based languages, which will be called here *Description Logics*[1]. Basically, Description Logics represent knowledge in terms of objects (*individuals*) grouped into classes (*concepts*) and pairs of objects grouped into relations (*roles*). Classes are denoted by using appropriate constructs. Interdependencies between classes (such as inclusion, disjointness, etc.) are established by means of assertions (i.e., axioms).

Two main advantages in using structured formalisms for knowledge representation have been advocated, namely, epistemological adequacy, and computational effectiveness. In the last decade, many efforts have been devoted to an analysis of these two aspects. In particular, starting with [9], the research on the computational complexity of the reasoning tasks associated with Description Logics has shown that in order to ensure decidability and/or efficiency of reasoning in all cases, one must renounce to some of the expressive power [43, 45, 47, 46, 25, 26, 27]. These results have led to a debate on the trade-off between expressive power of representation formalisms and worst-case efficiency of the associated reasoning tasks. This issue has been one of the main themes in the area of Description Logics, and has led to at least four different approaches to the design of knowledge representation

[1]Terminological Logics, and Concept Languages are other possible names.

systems.

- In the first approach, the main goal of a Description Logic is to offer powerful mechanisms for structuring knowledge, as well as sound and complete (but possibly non terminating) reasoning procedures. Little attention is paid to both decidability and computational complexity of the reasoning procedures. Systems like OMEGA [1] can be considered as following this approach.

- The second approach advocates a careful design of the Description Logics so as to offer as much expressive power as possible while retaining the possibility of sound, complete, and efficient (often polynomial in the worst case) inference procedures. Much of the research on CLASSIC [51] follows this approach.

- The third approach, similarly to the first one, advocates very expressive languages, but, in order to achieve efficiency, accepts incomplete reasoning procedures. LOOM [44] and KL-ONE [10] are representatives of this approach. No general consensus exists on what kind of incompleteness is acceptable. Perhaps, the most interesting attempts are those which resort to a non-standard semantics for characterizing the form of incompleteness [50, 8, 27].

- Finally, the fourth approach is based on what we can call "the expressiveness and decidability thesis", and aims at defining Description Logics that are both very expressive and decidable, i.e., designed in such a way that sound, complete, and terminating procedures exist for the associated reasoning tasks. Great attention is given in this approach to the complexity analysis for the various sublogics, so as to devise suitable optimization techniques and to single out tractable subcases. This approach is the one followed in the design of KRIS [3], and more recently in the design of FaCT [36, 38], DLP [37], and RACE [60].

Here we focus on the research that adheres to the fourth approach. This aims at both identifying very expressive Description Logics with decidable associated decision problems, and characterizing the computational complexity of reasoning in such Description Logics.

A major advancement in dealing with expressive and decidable Description Logics has been given by Schild who singled out a tight correspondence between Description Logics and Propositional Dynamic Logics[2] [53]. Propositional Dynamic Logics are Modal Logics specifically designed for reasoning about program schemes [41]. The correspondence is based on the similarity between the interpretation structures of the two kinds of logics: at the extensional level, objects in Description Logics correspond to states in Propositional Dynamic Logics, whereas connections between two objects correspond to state transitions. At the intensional level, classes correspond to propositions, and roles corresponds to programs. As a consequence of this correspondence, the large body of research on Propositional Dynamic Logics, and more generally Modal Logics, can be exploited in the context of Description Logics. And, conversely, the work on Description Logics, such as the work on tractability/intractability of Description Logics [26, 12], can be used in the context of Propositional Dynamic Logics and Modal Logics.

Starting from this correspondence several expressive but still decidable Description Logics have been obtained. These are summarized by Figure 1. The weaker logics are at the bottom of the figure while the stronger ones are at the top. A line between two logics denotes that the logic above is an extension of (in the sense that it has more constructs than) the logic below.

For each of the logics in the picture EXPTIME-decidability of logical implication (and all usual reasoning tasks) has been established. Let us briefly introduce these logics.

$\mathcal{ALC}$ is a very well known Description Logic [55]. It includes boolean constructs (union, intersection, and complement), existential qualification and universal qualification for building complex concept expressions, while roles can only be atomic. $\mathcal{ALC}$ corresponds to the well-known modal logic K_i [53], which is the basic normal multimodal logic [33, 35, 16, 39]. Satisfiability of an $\mathcal{ALC}$ concept (satisfiability of a $\mathcal{K}_i$ formula) is known to be PSPACE-complete while logical implication for $\mathcal{ALC}$ (for K_i) is EXPTIME-complete.

$\mathcal{C}$ is the Description Logic obtained from $\mathcal{ALC}$ by adding the following role constructs: union, chaining, reflexive transitive closure, and identity role over a concept (see [53, 2]). $\mathcal{C}$ corresponds to the Propositional Dynamic Logic PDL, which is the original Propositional Dynamic Logic introduced in [30]. All the usual reasoning tasks in $\mathcal{C}$ (PDL) are known to be EXPTIME-complete.

$\mu\mathcal{ALC}$ is obtained from $\mathcal{ALC}$ by adding two concept constructs denoting the least fixpoint and the greatest fixpoint of concept expressions. Notably, the fixpoint constructs allow for recursive concept definitions. Observe that

[2] We use the term Propositional Dynamic Logics in a slightly more general sense then usual, so as to include the basic multimodal logic K_i, and modal mu-calculus.

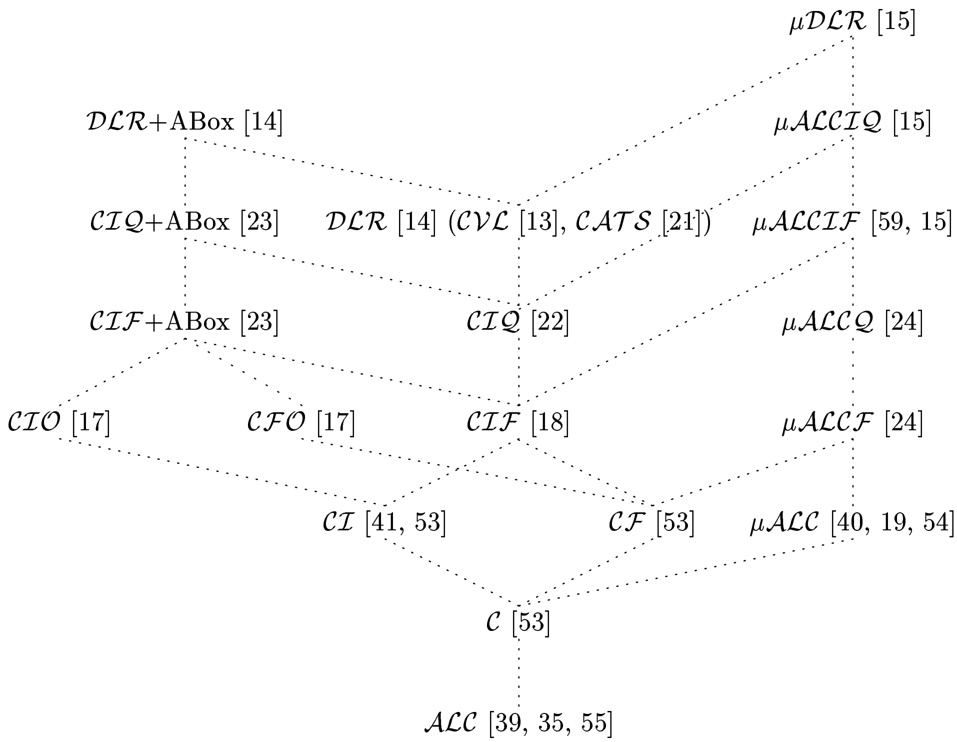

Figure 1: Description Logics derived from the correspondence

even if no role constructs are present, $\mu\mathcal{ALC}$ is actually an extension of $\mathcal{C}$, since all concepts denotable in $\mathcal{C}$ are also denotable in $\mu\mathcal{ALC}$. Indeed using fixpoints we can emulate all role expressions occurring in a $\mathcal{C}$ concept. The corresponding Propositional Dynamic Logic is the *modal mu-calculus* [40], which is known to be decidable and EXPTIME-complete. The correspondence was derived independently by both Schild and the author in [54] and [20] respectively.

The other Description Logics (and corresponding Propositional Dynamic Logics) in Figure 1 are obtained from $\mathcal{C}$ and $\mu\mathcal{ALC}$ (PDL and modal mu-calculus) by adding constructs either on concepts (formulae) or roles (programs). The presence of such constructs is reflected in the name of the logic.

$\mathcal{I}$ in the name of a logic indicates the presence of *inverse roles* (converse programs in Propositional Dynamic Logics). In all Description Logics introduced that include inverse roles, there is a perfect symmetry between atomic roles and their inverses, in the sense that all constructs dealing with atomic roles, deal with inverse of atomic roles as well. Similarly for the corresponding Propositional Dynamic Logics.

$\mathcal{F}$ in the name of a logic indicates the presence of *functional restrictions.* In Description Logics, a functional restriction forces a specified atomic role or its inverse to be functional wrt the individuals that satisfy it. Similarly for the corresponding Propositional Dynamic Logics. Observe the difference between functional restrictions on atomic programs and the assumption that atomic programs are deterministic, characterizing the so called *Deterministic Propositional Dynamic Logics* [5]. The first allows for imposing the functionality of a given program *locally* (i.e., wrt states that are forced to satisfy the restriction), while the latter assumes the functionality of each atomic program once and for all (i.e., for all possible states).

$\mathcal{Q}$ in the name of a logic indicates the presence of *qualified number restrictions.* Qualified number restrictions have a corresponding notion in Modal Logics, the *graded modalities* [58, 57, 28, 29]. Though, to our knowledge, full-fledged Propositional Dynamic Logics that include graded modalities were first studied in the context of Description Logics.

$\mathcal{O}$ in the name of a logic indicates the presence of special atomic concepts (formulae) called *names* denoting exactly a single individual. Note that by means of names, ABoxes (collections of membership assertions), and constructs involving single individuals as ONE-OF or FILLS can be represented. Names corresponds to *nominals* in Modal Logics [7, 31, 6, 52, 11]. Propositional Dynamic Logics with nominals are called *Combinatory Propositional Dynamic Logics* [48, 32, 49]. The results on names obtained in the context of Description Logics closed some open problems related to Combinatory Propositional Dynamic Logics, by characterizing the computational complexity of *Deterministic Combinatory Propositional Dynamic Logic* (which is easily reduced to $\mathcal{CFO}$), and establishing the decidability and characterizing the computational complexity of *Converse Combinatory Propositional Dynamic Logic* (which is easily reduced to $\mathcal{CIO}$). Observe that the computational characterization of the Description Logics with both inverses and functional restrictions remains an open problem. The only known result is the NEXPTIME-hardness of $\mathcal{CIFO}$ [56].

$\mathcal{DLR}$ is a logic that slightly departs from usual Propositional Dynamic Logics, since roles (programs) are replaced by *n-ary relations*. The logic allows for building boolean combinations of relations and for stating assertions on such boolean combinations. Although negation of a n-ary relation is allowed, it is defined essentially as a difference, so as not to introduce, as a side effect, the ability to denote the universal n-ary relation, by means of a boolean expression of relations. This logic has been defined with the intent of providing a formal counterpart of conceptual data models used in databases (such as the Entity-Relationship Model), and is well suited for being integrated with relational databases [14]. By extending $\mathcal{DLR}$ with fixpoints we get $\mu\mathcal{DLR}$ [15]. Interestingly also for $\mathcal{DLR}$ and $\mu\mathcal{DLR}$ (as well as for the less expressive variants $\mathcal{CVL}$ [13] and $\mathcal{CATS}$ [21]) logical implication (and hence all usual reasoning tasks) is EXPTIME-complete.

Finally, it is worth mentioning that, apart from the results above, the correspondence between Description Logics and Modal Logics is generating quite interesting works, such as [42] on expressiveness characterization of Description Logics that are not propositionally closed, or [34] that looks at the guarded fragments as a form of Description Logics with n-ary relations, or [4] on very general conditions for combining Description Logics constructs without losing decidability, just to mention a few.

References

[1] G. Attardi and M. Simi. Consistency and completeness of OMEGA, a logic for knowledge representation. In *Proc. of the 7th Int. Joint Conf. on Artificial Intelligence (IJCAI'81)*, pages 504–510, Vancouver (Canada), 1981.

[2] F. Baader. Augmenting concept languages by transitive closure of roles: An alternative to terminological cycles. In *Proc. of the 12th Int. Joint Conf. on Artificial Intelligence (IJCAI'91)*, Sydney (Australia), 1991.

[3] F. Baader and B. Hollunder. A terminological knowledge representation system with complete inference algorithm. In *Proc. of the Workshop on Processing Declarative Knowledge (PDK'91)*, volume 567 of *Lecture Notes in Artificial Intelligence*, pages 67–86. Springer-Verlag, 1991.

[4] F. Baader, C. Lutz, H. Sturm, and F. Wolter. Fusions of description logics. In *Proc. of the 2000 Description Logic Workshop (DL 2000)*, pages 21–30, 2000.

[5] M. Ben-Ari, J. Y. Halpern, and A. Pnueli. Deterministic propositional dynamic logic: Finite models, complexity, and completeness. *J. of Computer and System Sciences*, 25:402–417, 1982.

[6] P. Blackburn. Nominal tense logic. *Notre Dame Journal of Formal Logic*, 34(1):56–83, 1993.

[7] P. Blackburn and E. Spaan. A modal perspective on computational complexity of attribute value grammar. *Journal of Logic, Language and Information*, 2:129–169, 1993.

[8] A. Borgida and P. F. Patel-Schneider. A semantics and complete algorithm for subsumption in the CLASSIC description logic. *J. of Artificial Intelligence Research*, 1:277–308, 1994.

[9] R. J. Brachman and H. J. Levesque. The tractability of subsumption in frame-based description languages. In *Proc. of the 4th Nat. Conf. on Artificial Intelligence (AAAI'84)*, pages 34–37, 1984.

[10] R. J. Brachman and J. G. Schmolze. An overview of the KL-ONE knowledge representation system. *Cognitive Science*, 9(2):171–216, 1985.

[11] R. Bull. An approach to tense logic. *Theoria*, 12:171–182, 1970.

[12] D. Calvanese. Reasoning with inclusion axioms in description logics: Algorithms and complexity. In W. Wahlster, editor, *Proc. of the 12th Eur. Conf. on Artificial Intelligence (ECAI'96)*, pages 303–307. John Wiley & Sons, 1996.

[13] D. Calvanese, G. De Giacomo, and M. Lenzerini. Structured objects: Modeling and reasoning. In *Proc. of the 4th Int. Conf. on Deductive and Object-Oriented Databases (DOOD'95)*, volume 1013 of *Lecture Notes in Computer Science*, pages 229–246. Springer-Verlag, 1995.

[14] D. Calvanese, G. De Giacomo, and M. Lenzerini. On the decidability of query containment under constraints. In *Proc. of the 17th ACM SIGACT SIGMOD SIGART Symp. on Principles of Database Systems (PODS'98)*, pages 149–158, 1998.

[15] D. Calvanese, G. De Giacomo, and M. Lenzerini. Reasoning in expressive description logics with fixpoints based on automata on infinite trees. In *Proc. of the 16th Int. Joint Conf. on Artificial Intelligence (IJCAI'99)*, pages 84–89, 1999.

[16] B. F. Chellas. *Modal Logic: An Introduction.* Cambridge University Press, 1980.

[17] G. De Giacomo. *Decidability of Class-Based Knowledge Representation Formalisms.* PhD thesis, Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza", 1995.

[18] G. De Giacomo and M. Lenzerini. Boosting the correspondence between description logics and propositional dynamic logics. In *Proc. of the 12th Nat. Conf. on Artificial Intelligence (AAAI'94)*, pages 205–212. AAAI Press/The MIT Press, 1994.

[19] G. De Giacomo and M. Lenzerini. Concept language with number restrictions and fixpoints, and its relationship with μ-calculus. In *Proc. of the 11th Eur. Conf. on Artificial Intelligence (ECAI'94)*, pages 411–415, 1994.

[20] G. De Giacomo and M. Lenzerini. Concept language with number restrictions and fixpoints, and its relationship with mu-calculus. In *Proceedings of the 11th European Conference on Artificial Intelligence (ECAI-94)*, pages 411–415. John Wiley and Sons, 1994.

[21] G. De Giacomo and M. Lenzerini. Enhanced propositional dynamic logic for reasoning about concurrent actions (extended abstract). In *Working Notes of the AAAI 1995 Spring Symposium on Extending Theories of Action: Formal and Practical Applications*, pages 62–67, 1995.

[22] G. De Giacomo and M. Lenzerini. What's in an aggregate: Foundations for description logics with tuples and sets. In *Proc. of the 14th Int. Joint Conf. on Artificial Intelligence (IJCAI'95)*, pages 801–807, 1995.

[23] G. De Giacomo and M. Lenzerini. TBox and ABox reasoning in expressive description logics. In L. C. Aiello, J. Doyle, and S. C. Shapiro, editors, *Proc. of the 5th Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR'96)*, pages 316–327. Morgan Kaufmann, Los Altos, 1996.

[24] G. De Giacomo and M. Lenzerini. A uniform framework for concept definitions in description logics. *J. of Artificial Intelligence Research*, 6:87–110, 1997.

[25] F. M. Donini, M. Lenzerini, D. Nardi, and W. Nutt. The complexity of concept languages. In J. Allen, R. Fikes, and E. Sandewall, editors, *Proc. of the 2nd Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR'91)*, pages 151–162. Morgan Kaufmann, Los Altos, 1991.

[26] F. M. Donini, M. Lenzerini, D. Nardi, and W. Nutt. Tractable concept languages. In *Proc. of the 12th Int. Joint Conf. on Artificial Intelligence (IJCAI'91)*, pages 458–463, Sydney (Australia), 1991.

[27] F. M. Donini, M. Lenzerini, D. Nardi, W. Nutt, and A. Schaerf. Adding epistemic operators to concept languages. In *Proc. of the 3rd Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR'92)*, pages 342–353. Morgan Kaufmann, Los Altos, 1992.

[28] M. Fattorosi-Barnaba and F. De Caro. Graded modalities I. *Studia Logica*, 44:197–221, 1985.

[29] K. Fine. In so many possible worlds. *Notre Dame Journal of Formal Logic*, 13(4):516–520, 1972.

[30] N. J. Fisher and R. E. Ladner. Propositional dynamic logic of regular programs. *Journal of Computer and System Sciences*, 18:194–211, 1979.

[31] G. Gargov and V. Goranko. Modal logic with names. *Journal of Philosophical Logic*, 22:607–636, 1993.

[32] G. Gargov and S. Passy. Determinism and looping in combinatory PDL. *Theoretical Computer Science*, 61:259–277, 1988.

[33] R. Goldblatt. *Logics of time and computation*, volume 7 of *Lecture Notes*. Center for the Study of Language and Information, second edition, 1992.

[34] E. Goncalves and E. Grädel. Decidability issues for action guarded logics. In *Proc. of the 2000 Description Logic Workshop (DL 2000)*, pages 123–132, 2000.

[35] J. Halpern and Y. Moses. A guide to completeness and complexity for modal logics of knowledge and belief. *Artificial Intelligence*, 54:319–379, 1992.

[36] I. Horrocks. Using an expressive description logic: FaCT or fiction? In *Proc. of the 6th Int. Conf. on Principles of Knowledge Representation and Reasoning (KR'98)*, pages 636–647, 1998.

[37] I. Horrocks and P. F. Patel-Schneider. Optimizing description logic subsumption. *J. of Logic and Computation*, 9(3):267–293, 1999.

[38] I. Horrocks, U. Sattler, and S. Tobies. Practical reasoning for expressive description logics. In H. Ganzinger, D. McAllester, and A. Voronkov, editors, *Proc. of the 6th Int. Conf. on Logic for Programming and Automated Reasoning (LPAR'99)*, number 1705 in Lecture Notes in Artificial Intelligence, pages 161–180. Springer-Verlag, 1999.

[39] G. E. Hughes and M. J. Cresswell. *An Introduction to Modal Logic*. Methuen, London, 1968.

[40] D. Kozen. Results on the propositional μ-calculus. *Theoretical Computer Science*, 27:333–354, 1983.

[41] D. Kozen and J. Tiuryn. Logics of programs. In J. Van Leeuwen, editor, *Handbook of Theoretical Computer Science*, pages 790–840. Elsevier Science Publishers, 1990.

[42] N. Kurtonina and M. de Rijke. Expressiveness of concept expressions in first-order description. *Artificial Intelligence*, 107(2):303–333, 1999.

[43] H. J. Levesque and R. J. Brachman. Expressiveness and tractability in knowledge representation and reasoning. *Computational Intelligence*, 3:78–93, 1987.

[44] R. MacGregor. The evolving technology of classification-based knowledge representation systems. In J. F. Sowa, editor, *Principles of Semantic Networks*, pages 385–400. Morgan Kaufmann, Los Altos, 1991.

[45] B. Nebel. Computational complexity of terminological reasoning in BACK. *Artificial Intelligence*, 34(3):371–383, 1988.

[46] B. Nebel. *Reasoning and Revision in Hybrid Representation Systems*, volume 422 of *Lecture Notes in Artificial Intelligence*. Springer-Verlag, 1990.

[47] B. Nebel. Terminological reasoning is inherently intractable. *Artificial Intelligence*, 43:235–249, 1990.

[48] S. Passy and T. Tinchev. PDL with data constraints. *Information Processing Letters*, 20:35–41, 1985.

[49] S. Passy and T. Tinchev. An essay in combinatory dynamic logic. *Information and Computation*, 93:263–332, 1991.

[50] P. F. Patel-Schneider. A hybrid, decidable, logic-based knowledge representation system. *Computational Intelligence*, 3(2):64–77, 1987.

[51] P. F. Patel-Schneider, D. L. McGuiness, R. J. Brachman, L. A. Resnick, and A. Borgida. The CLASSIC knowledge representation system: Guiding principles and implementation rational. *SIGART Bull.*, 2(3):108–113, 1991.

[52] A. Prior. *Past, Present and Future*. Oxford University Press, 1967.

[53] K. Schild. A correspondence theory for terminological logics: Preliminary report. In *Proc. of the 12th Int. Joint Conf. on Artificial Intelligence (IJCAI'91)*, pages 466–471, Sydney (Australia), 1991.

[54] K. Schild. Terminological cycles and the propositional μ-calculus. In J. Doyle, E. Sandewall, and P. Torasso, editors, *Proc. of the 4th Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR'94)*, pages 509–520, Bonn (Germany), 1994. Morgan Kaufmann, Los Altos.

[55] M. Schmidt-Schauß and G. Smolka. Attributive concept descriptions with unions and complements. Technical Report SR-88-21, Fachbereich Informatik, Universität Kaiserslautern, Kaiserslautern (Germany), 1988.

[56] S. Tobies. The complexity of reasoning with cardinality restrictions and nominals in expressive description logics. *J. of Artificial Intelligence Research*, 12:199–217, 2000.

[57] W. Van der Hoek. On the semantics of graded modalities. *Journal of Applied Non-Classical Logics*, 2(1):81–123, 1992.

[58] W. Van der Hoek and M. de Rijke. Counting objects. *Journal of Logic and Computation*, 5(3):325–345, 1995.

[59] M. Y. Vardi. Reasoning about the past with two-way automata. In *Proc. of the 25th Int. Coll. on Automata, Languages and Programming (ICALP'98)*, volume 1443 of *Lecture Notes in Computer Science*, pages 628–641. Springer-Verlag, 1998.

[60] R. M. Volker Haarslev. Expressive abox reasoning with number restrictions, role hierarchies, and transitively closed roles. In *Proc. of the 7th Int. Conf. on Principles of Knowledge Representation and Reasoning (KR'2000)*, pages 273–284, 2000.