

Autonomous and Mobile Robotics

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Localization

Landmark-Based and SLAM

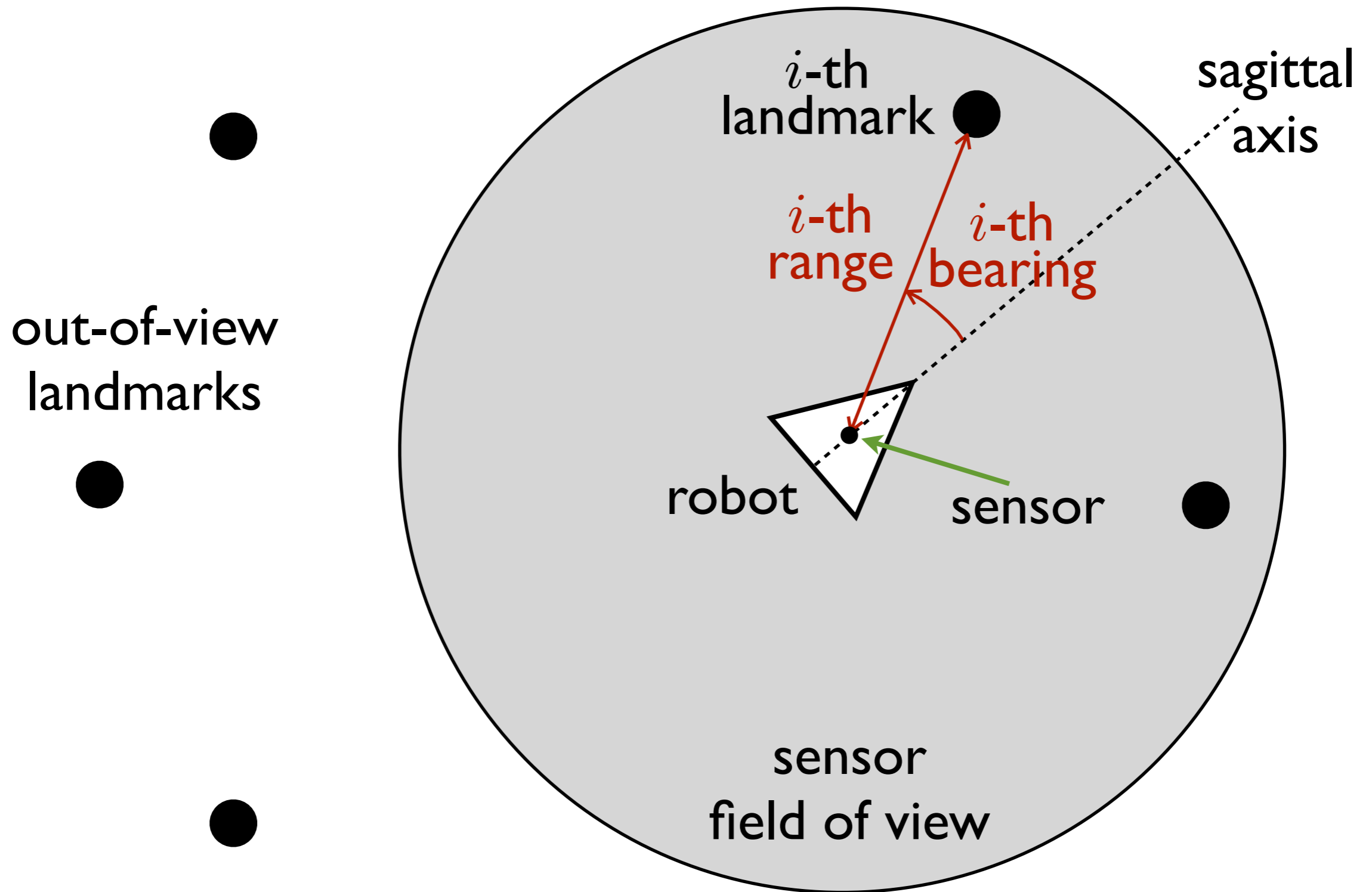
DIPARTIMENTO DI INGEGNERIA INFORMATICA
AUTOMATICA E GESTIONALE ANTONIO RUBERTI



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EKF localization with landmarks

- assume that a unicycle-like robot is equipped with a sensor that measures **range** (relative distance) and **bearing** (relative orientation) to certain **landmarks**
- landmarks may be **artificial** or **natural**
- the position of the landmarks is **fixed** and **known**
- depending on the robot configuration, only a **subset** of the landmarks is actually visible
- suitable sensors are **laser** rangefinders, **depth cameras** or **RFID sensors**



- odometric equations can be used as a **discrete-time model** of the robot; e.g., using Euler method

$$x_{k+1} = x_k + v_k T_s \cos \theta_k + v_{1,k}$$

$$y_{k+1} = y_k + v_k T_s \sin \theta_k + v_{2,k}$$

$$\theta_{k+1} = \theta_k + \omega_k T_s + v_{3,k}$$

where $\mathbf{v}_k = (v_{1,k} \ v_{2,k} \ v_{3,k})^T$ is a **white gaussian** noise with zero mean and covariance matrix \mathbf{V}_k

- assume that L landmarks are present, and denote by $(x_{l,i}, y_{l,i})$ the position of the i -th landmark
- let $L_k \leq L$ be the number of landmarks that the robot can actually see at step k

- each of the L_k **measurements** actually contains two components, i.e., a **range** component and a **bearing** component
- assume that for each measurement the **identity** of observed landmark is known (landmarks are **tagged**, e.g., by shape, color or radio frequency)
- we build the **association map** of step k

$$a : \underbrace{\{1, 2, \dots, L_k\}}_{\text{measurements}} \mapsto \underbrace{\{1, 2, \dots, L\}}_{\text{landmarks}}$$

hence, $a(i)$ is the index of the landmark observed by the i -th measurement

- the output equation is

$$\mathbf{y}_k = \begin{pmatrix} \mathbf{h}_1(\mathbf{q}_k, a(1)) \\ \vdots \\ \mathbf{h}_{L_k}(\mathbf{q}_k, a(L_k)) \end{pmatrix} + \begin{pmatrix} w_{1,k} \\ \vdots \\ w_{L_k,k} \end{pmatrix}$$

where

$$\mathbf{h}_i(\mathbf{q}_k, a(i)) = \begin{pmatrix} \sqrt{(x_k - x_{l,a(i)})^2 + (y_k - y_{l,a(i)})^2} \\ \text{atan2}(y_{l,a(i)} - y_k, x_{l,a(i)} - x_k) - \theta_k \end{pmatrix}$$

i-th landmark range
↓
i-th landmark bearing

and $\mathbf{w}_k = (w_{1,k} \dots w_{L_k,k})^T$ is a **white gaussian** noise with zero mean and covariance matrix \mathbf{W}_k

- we want to **maintain** an accurate estimate of the robot configuration in the presence of process and measurement noise: this is the **ideal setting** for KF
- actually, since both process and output equations are **nonlinear**, we must apply the **EKF** and, to this end, the equations must be **linearized**
- process dynamics linearization

$$\mathbf{F}_k = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{q}_k} \right|_{\mathbf{q}_k = \hat{\mathbf{q}}_k} = \begin{pmatrix} 1 & 0 & -v_k T_s \sin \hat{\theta}_k \\ 0 & 1 & v_k T_s \cos \hat{\theta}_k \\ 0 & 0 & 1 \end{pmatrix}$$

- output equation linearization

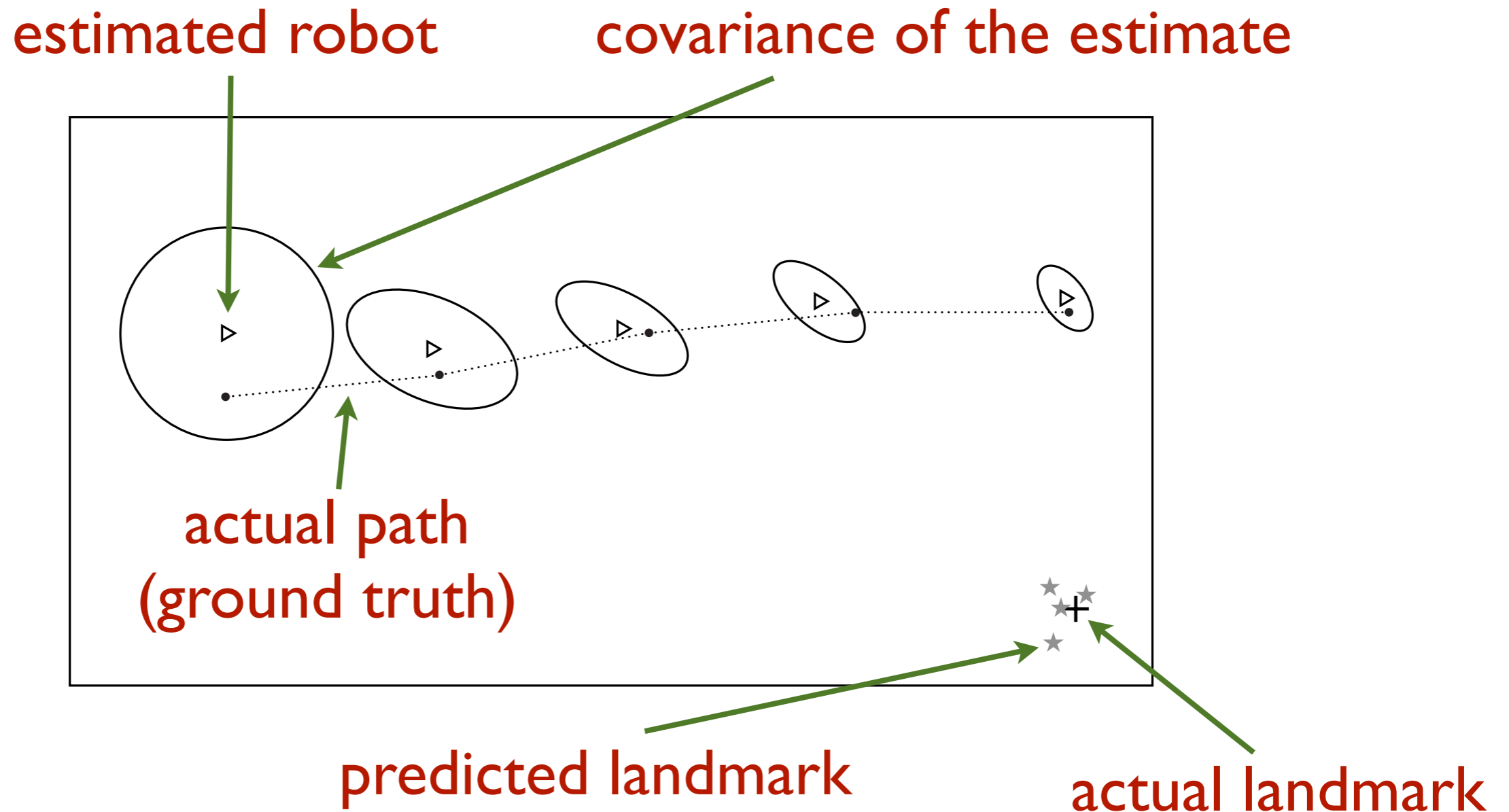
$$\mathbf{H}_{k+1} = \begin{pmatrix} \frac{\partial \mathbf{h}_1}{\partial \mathbf{q}_k} \Big|_{\mathbf{q}_k = \hat{\mathbf{q}}_{k+1|k}} \\ \vdots \\ \frac{\partial \mathbf{h}_{L_k}}{\partial \mathbf{q}_k} \Big|_{\mathbf{q}_k = \hat{\mathbf{q}}_{k+1|k}} \end{pmatrix}$$

where

$$\frac{\partial \mathbf{h}_i}{\partial \mathbf{q}_k} \Big|_{\mathbf{q}_k = \hat{\mathbf{q}}_{k+1|k}} = \begin{pmatrix} \frac{\hat{x}_{k+1|k} - x_{l,a(i)}}{\sqrt{(\hat{x}_{k+1|k} - x_{l,a(i)})^2 + (\hat{y}_{k+1|k} - y_{l,a(i)})^2}} & \frac{\hat{y}_{k+1|k} - y_{l,a(i)}}{\sqrt{(\hat{x}_{k+1|k} - x_{l,a(i)})^2 + (\hat{y}_{k+1|k} - y_{l,a(i)})^2}} & 0 \\ \frac{-(\hat{y}_{k+1|k} - y_{l,a(i)})}{(\hat{x}_{k+1|k} - x_{l,a(i)})^2 + (\hat{y}_{k+1|k} - y_{l,a(i)})^2} & \frac{\hat{x}_{k+1|k} - x_{l,a(i)}}{(\hat{x}_{k+1|k} - x_{l,a(i)})^2 + (\hat{y}_{k+1|k} - y_{l,a(i)})^2} & -1 \end{pmatrix}$$

- at this point, just crank the EKF engine

a typical result



data association

- remove the hypothesis that the identity of each observed landmark is known: in practice, landmarks can be **undistinguishable** by the sensor
- the association map must be **estimated** as well
- basic idea: associate each observation to the landmark that minimizes the magnitude of the innovation
- at the $k+1$ -th step, consider the i -th measurement $\mathbf{y}_{i,k+1}$ and compute all the candidate innovations

$$\mathbf{v}_{ij} = \mathbf{y}_{i,k+1} - \mathbf{h}_i(\hat{\mathbf{q}}_{k+1|k}, j)$$

actual
measurement

expected measurement if $\mathbf{y}_{i,k+1}$
referred to the j -th landmark

- the **smaller** the innovation ν_{ij} , the **more likely** that the i -th measurement corresponds to the j -th landmark
- however, the innovation magnitude must be weighted with the **uncertainty** of measurement; in the EKF, this is **encoded** in the matrix

$$\mathbf{S}_{ij} = \mathbf{H}_i(k+1, j) \mathbf{P}_{k+1|k} \mathbf{H}_i(k+1, j)^T + \mathbf{W}_{i, k+1}$$

measurement uncertainty
due to prediction uncertainty

measurement uncertainty
due to sensor noise

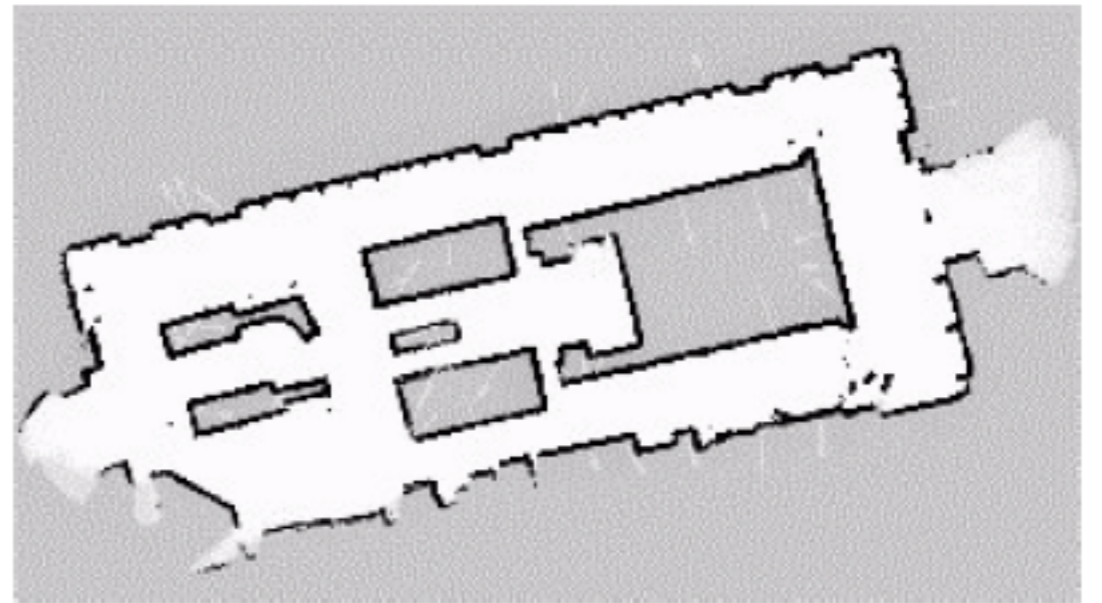
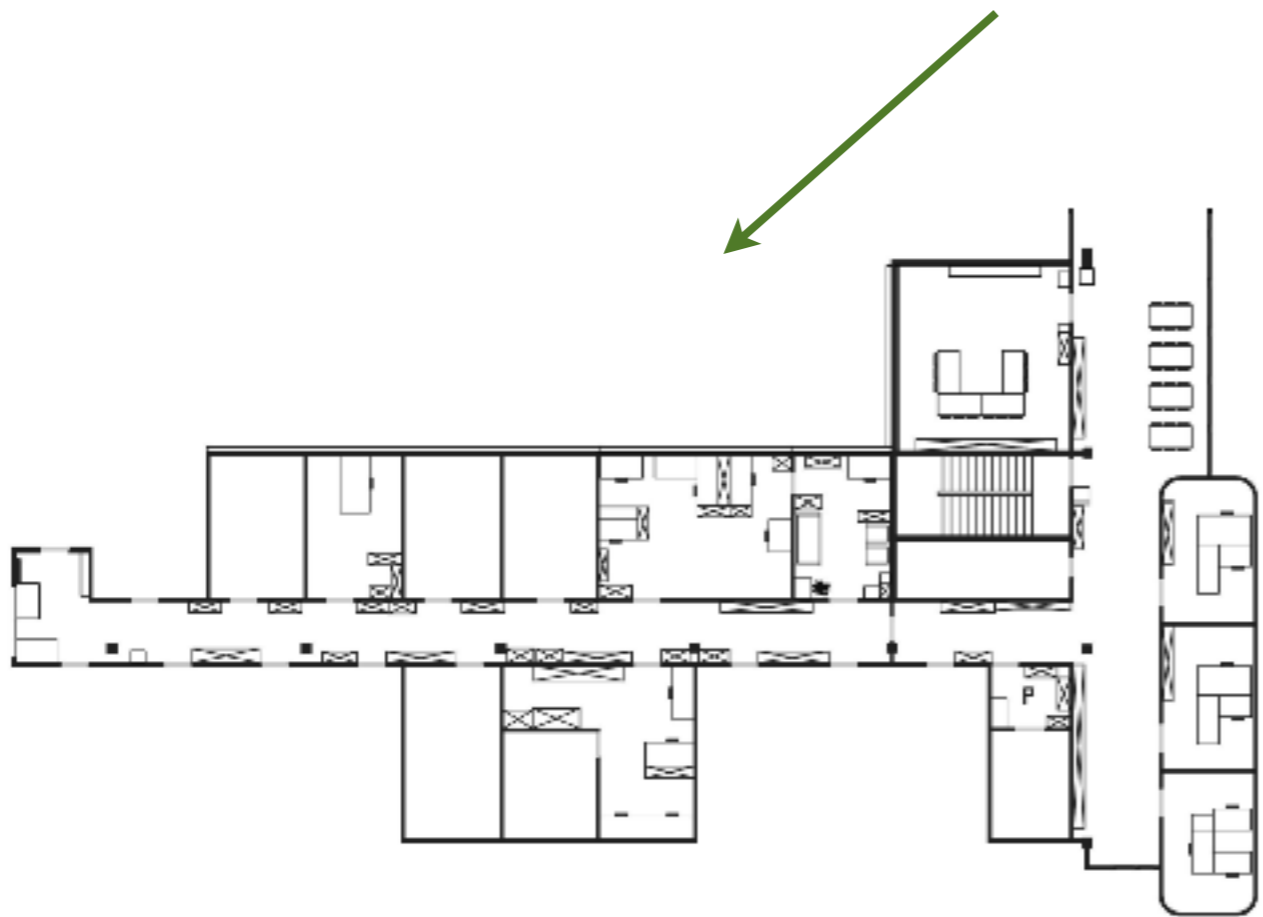
- to determine the association function, let

$$\chi_{ij} = \nu_{ij}^T \mathbf{S}_{ij}^{-1} \nu_{ij}$$

and let $a(i) = j$, where j **minimizes** χ_{ij}

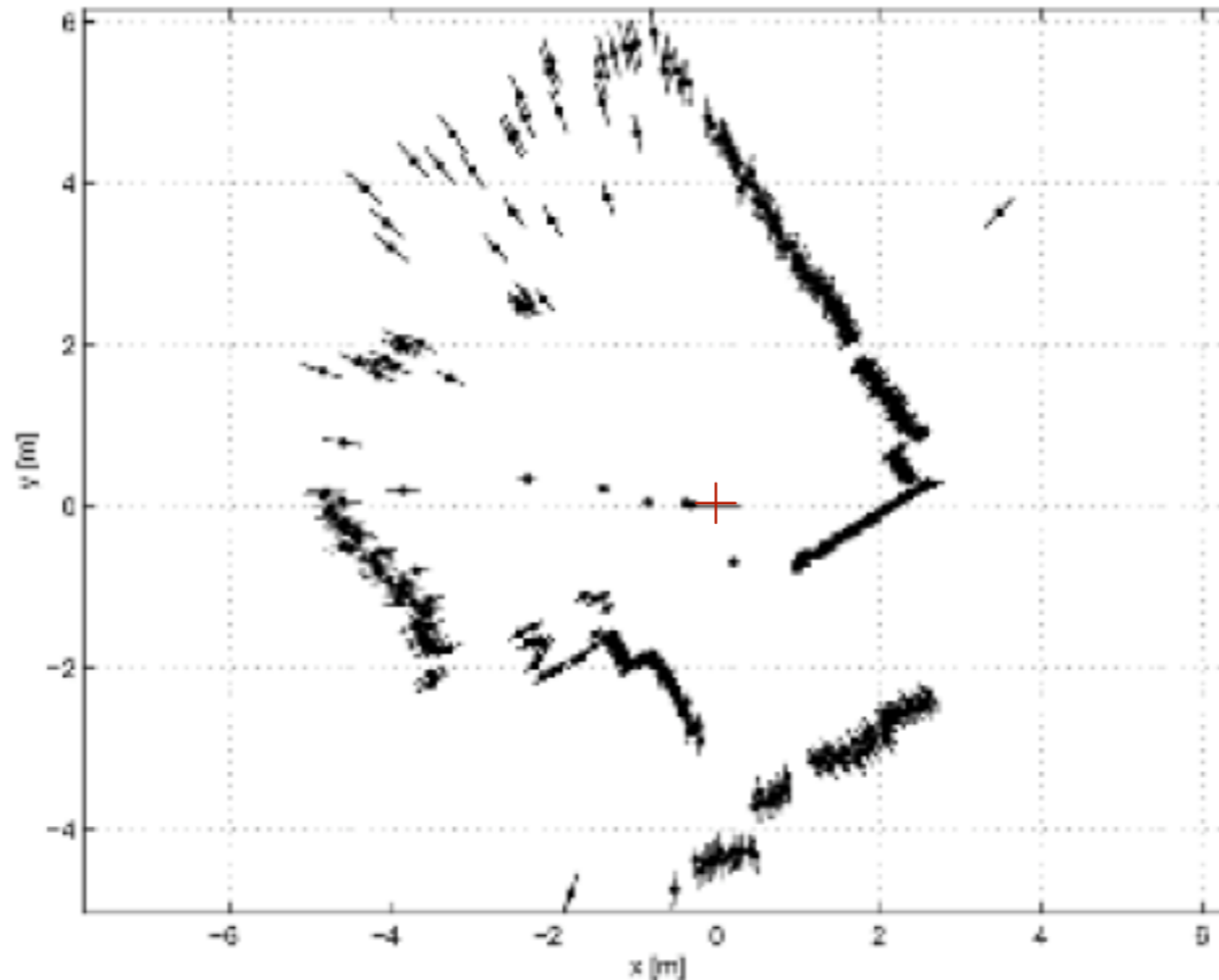
EKF localization on a map

- assume that a **metric map** \mathcal{M} of the environment is known to the robot
- this may be a **line-based map** or an **occupancy grid**



(taken from: Arras, "Feature-based robot navigation in known and unknown environments", 2003)

- assume that the robot is equipped with a **range finder**; e.g., a laser sensor, whose typical scan looks like this (note the uncertainty intervals)



(taken from: Arras, “Feature-based robot navigation in known and unknown environments”, 2003)

- use the **whole scan as output vector**: its components are the range readings in all available directions

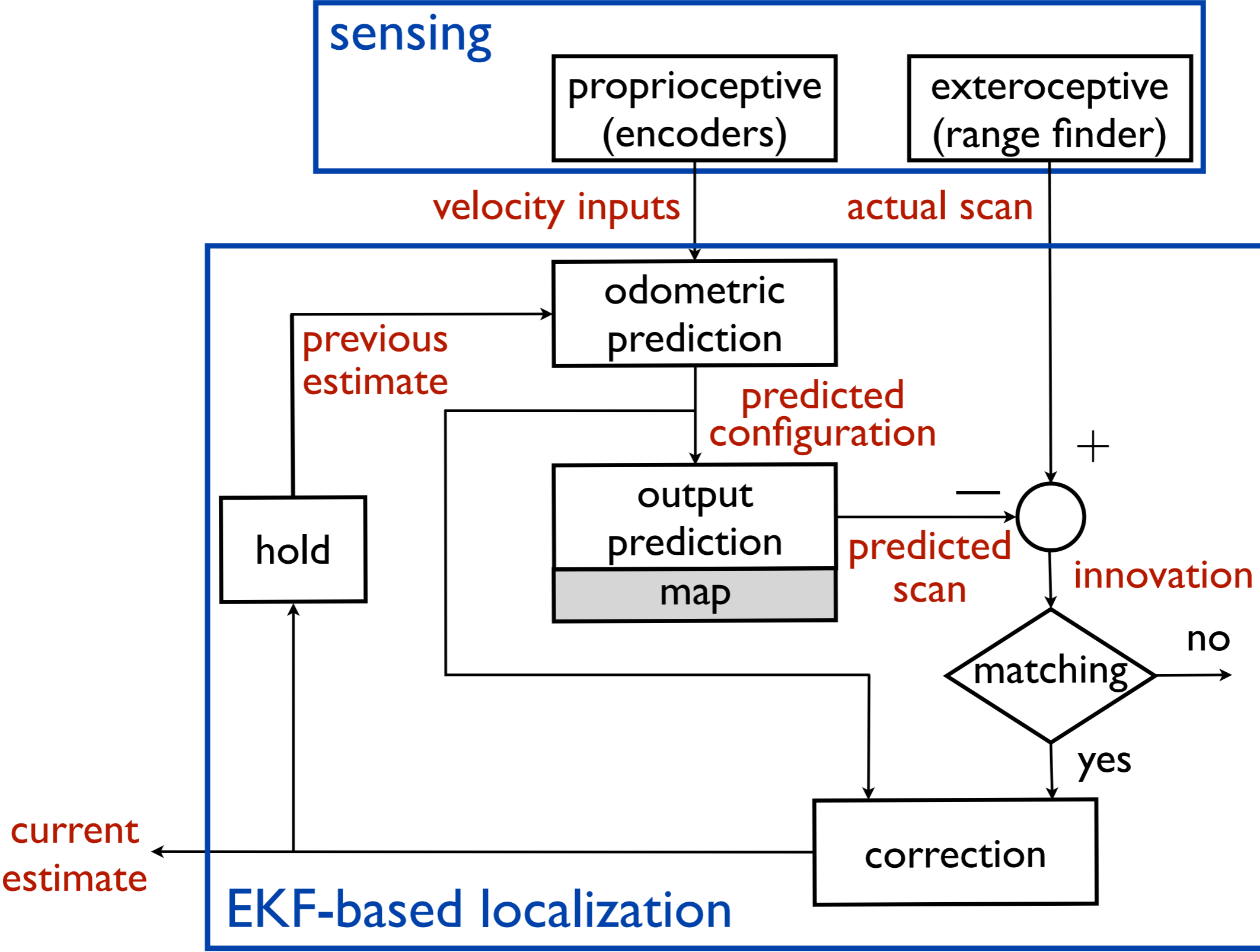
- the **innovation** is then computed as the difference between the **actual scan** and the **predicted scan**

$$\boldsymbol{\nu}_{k+1} = \mathbf{y}_{k+1} - \mathbf{h}(\hat{\mathbf{q}}_{k+1|k}, \mathcal{M})$$

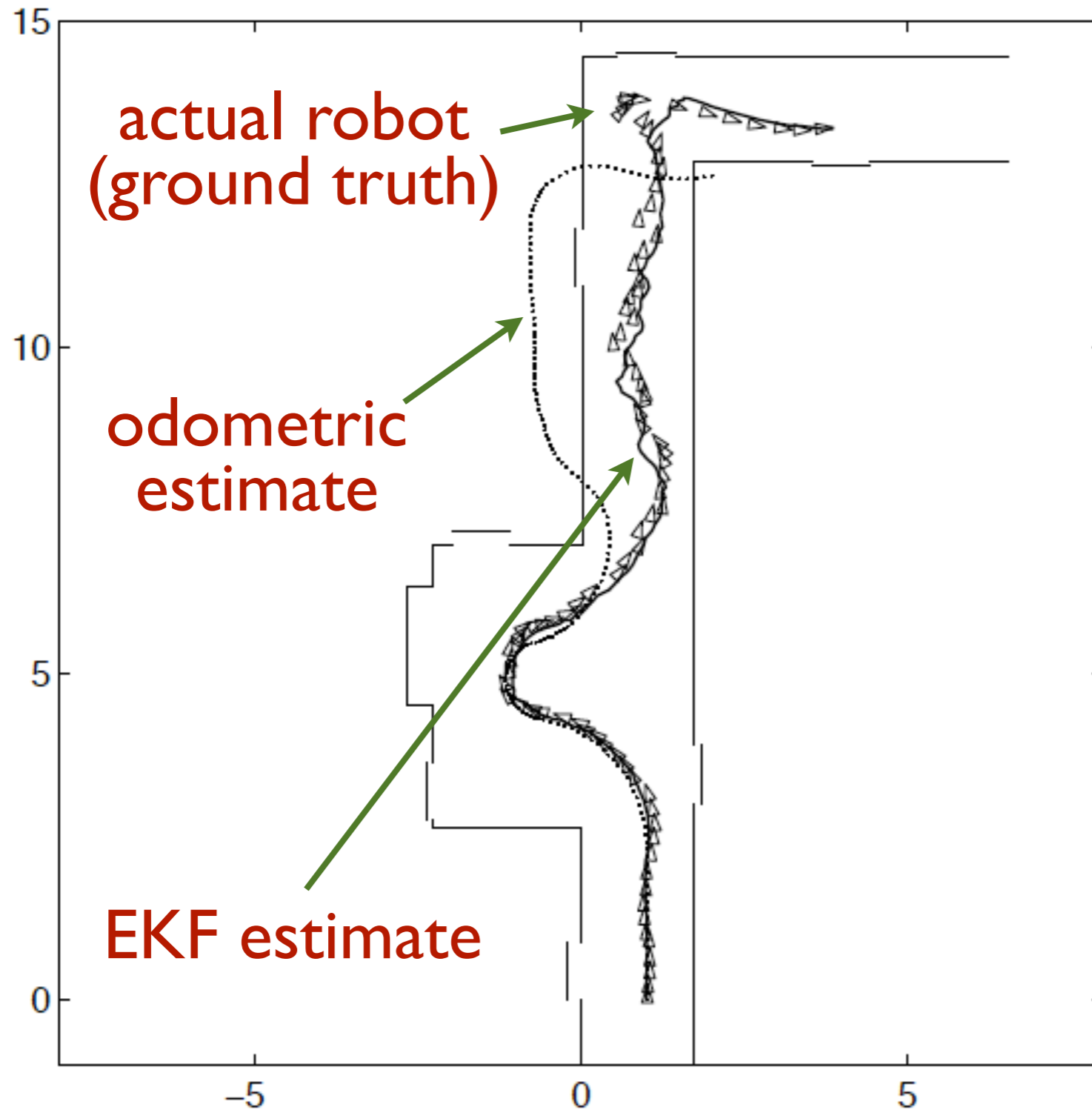
where $\mathbf{h}(\)$ computes the predicted scan by placing the robot at a configuration in the map

- note that **no data association** is needed; on the other hand, **aliasing** may severely displace the estimate
- both the process dynamics (i.e., the robot kinematic model) and the output function \mathbf{h} are **nonlinear**, and therefore the **EKF** must be used

architecture



a typical result



- robotized wheelchair with high slippage
- 5 ultrasonic sensors with 2 Hz rate
- shadow zone behind the robot

EKF SLAM

- remove the hypothesis that the environment is known a priori: as it moves, the robot must use its sensors to **build a map** and at the same time **localize** itself
- **SLAM**: Simultaneous **L**ocalization **A**nd **M**ap-building
- in **probabilistic** SLAM, the idea is to **estimate** the **map features** in addition to the robot configuration
- here we discuss a **simple landmark-based** version of the problem which can be solved using KF or EKF

- assumptions:
 - the robot is an **omnidirectional point-robot**, whose configuration is then a cartesian position
 - L landmarks are distributed in the environment (their position is unknown)
 - the robot is equipped with a sensor that can **see**, **identify** and **measure** the relative position of **all** landmarks wrt itself (infinite FOV + no occlusions)
- define an extended state vector to be estimated

$$\mathbf{x} = \left(\begin{array}{cccc} x & y & x_{l1} & y_{l1} & \dots & x_{lL} & y_{lL} \end{array} \right)^T$$

robot
landmark l
...
landmark L
position
position
...
position

- since the landmarks are fixed, the **discrete-time model** of the robot+landmarks system is ($T_s=1$ w.l.o.g.)

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x,k} \\ u_{y,k} \end{pmatrix} + \begin{pmatrix} v_{x,k} \\ v_{y,k} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

where $u_{x,k}$ and $u_{y,k}$ are the robot velocity inputs while $\mathbf{v}_{xy,k} = (v_{x,k} \ v_{y,k})^T$ is a white gaussian noise with zero mean and covariance matrix $\mathbf{V}_{xy,k}$

- this is clearly a **linear** model of the form

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{v}_k$$

where $\mathbf{u}_k = (u_{x,k} \ u_{y,k})^T$ is the input vector while $\mathbf{v}_k = (v_{1,k} \ v_{2,k} \ \dots \ v_{2+2L,k})^T$ is a white gaussian noise with zero mean and covariance matrix

$$\mathbf{V}_k = \begin{pmatrix} \mathbf{V}_{xy,k} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

- the i -th measurement contains the relative position of the i -th landmark wrt the sensor

$$\mathbf{y}_{i,k} = \begin{pmatrix} x_{li,k} - x_k \\ y_{li,k} - y_k \end{pmatrix} + \mathbf{w}_{i,k}$$

where $\mathbf{w}_{i,k}$ is a white gaussian noise with zero mean and covariance matrix $\mathbf{W}_{i,k}$

- it is a linear equation

$$\mathbf{y}_{i,k} = \mathbf{C}_i \mathbf{x}_k + \mathbf{w}_{i,k}$$

with

$$\mathbf{C}_i = \begin{pmatrix} -1 & 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 \end{pmatrix}$$

↑
($2i+1$)-th column

- stack all measurements to create the output vector

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{w}_k$$

where

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_L \end{pmatrix} \quad \mathbf{w}_k = \begin{pmatrix} \mathbf{w}_{1,k} \\ \vdots \\ \mathbf{w}_{L,k} \end{pmatrix}$$

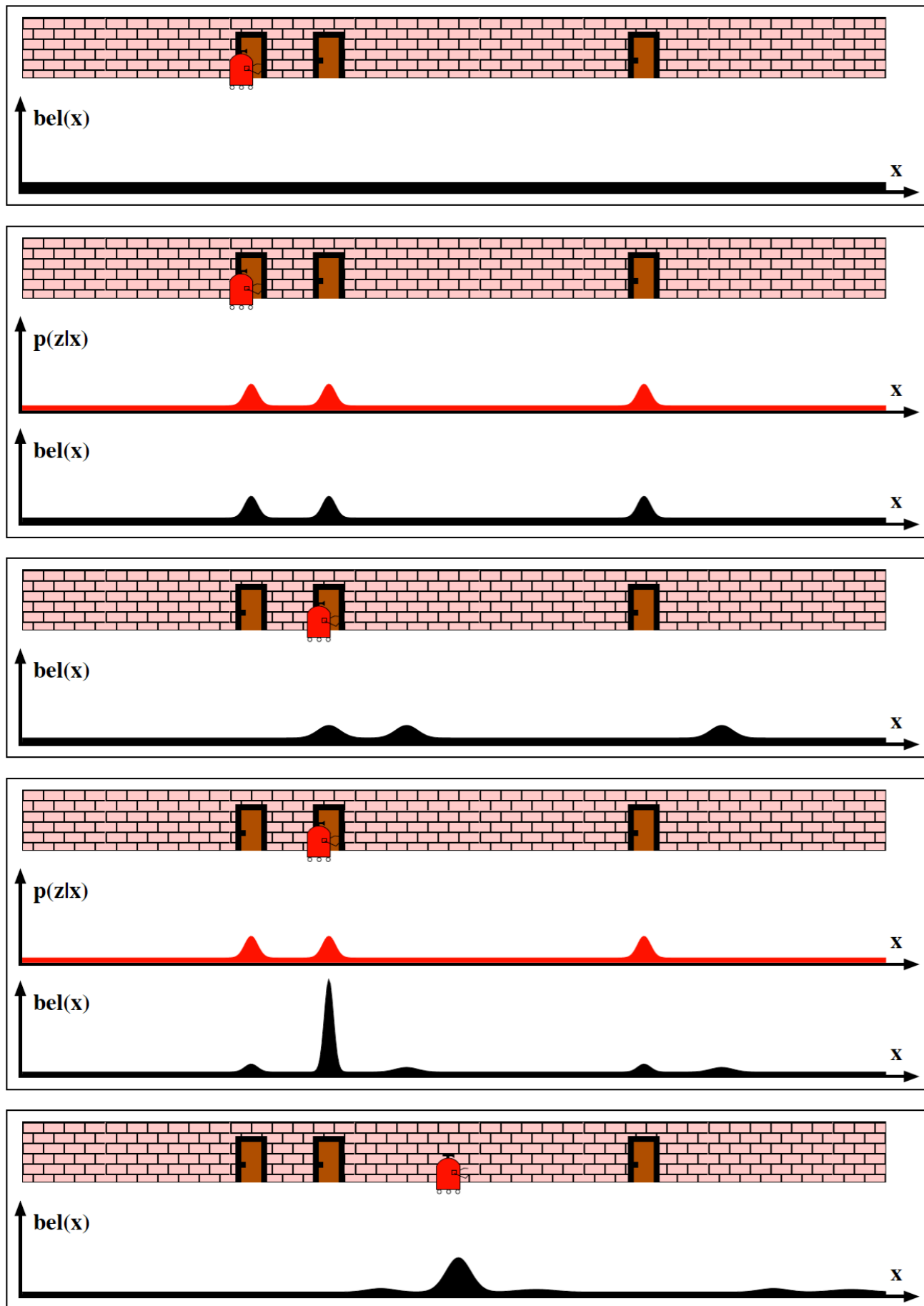
and the covariance of the measurement noise is

$$\mathbf{W}_k = \begin{pmatrix} \mathbf{W}_{1,k} & 0 & \dots & 0 \\ 0 & \mathbf{W}_{2,k} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{W}_{L,k} \end{pmatrix}$$

- at this point, just crank the KF engine

how realistic is KF/EKF localization?

- KF/EKF assume that the probability distribution for the state is **unimodal**, and in particular a gaussian
- this requires an **accurate** estimate of the robot **initial configuration** and also relatively **small uncertainties** (**position tracking** problem)
- however, if the robot is released at an **unknown** (or poorly known) position, the probability distribution for the state becomes **multimodal** in the presence of aliasing (**kidnapped robot** problem)



- need to track **multiple hypotheses**
- more general Bayesian estimators (e.g., **particle filters**) must be used