Autonomous and Mobile Robotics

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Localization

Odometric Localization

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- planning and feedback control require the knowledge of the robot configuration q (e.g., see Motion Control of WMRs: Trajectory Tracking, slide 3)

- in robot manipulators, joint encoders provide a direct measure of q

- WMRs are equipped with incremental encoders that measure only the rotation of the wheels, not the position and orientation of the vehicle

- localization is a procedure for estimating the robot configuration q, typically in real time

- consider a unicycle under constant velocity inputs v_k, ω_k in $[t_k, t_{k+1}]$, as in a digital control implementation; in each sampling interval, the robot moves along an arc of circle of radius v_k/ω_k (a line segment if $\omega_k = 0$)

- assume $\boldsymbol{q}_k$, v_k and ω_k are known; compute $\boldsymbol{q}_{k+1}$ by integration of the kinematic model over $[t_k, t_{k+1}]$

- first possibility: Euler integration

$$
\begin{aligned}
x_{k+1} &= x_k + v_k T_s \cos\theta_k \\
y_{k+1} &= y_k + v_k T_s \sin\theta_k \\
\theta_{k+1} &= \theta_k + \omega_k T_s
\end{aligned}
\qquad\qquad T_s = t_{k+1} - t_k
$$

- x_{k+1} and y_{k+1} are approximate; θ_{k+1} is exact

- second possibility: 2nd order Runge-Kutta integration

$$x_{k+1} = x_k + v_k T_s \cos\left(\theta_k + \frac{\omega_k T_s}{2}\right)$$
$$y_{k+1} = y_k + v_k T_s \sin\left(\theta_k + \frac{\omega_k T_s}{2}\right)$$
$$\theta_{k+1} = \theta_k + \omega_k T_s$$

- the average orientation during $[t_k, t_{k+1}]$ is used

- as a consequence, x_{k+1} and y_{k+1} are still approximate, but more accurate

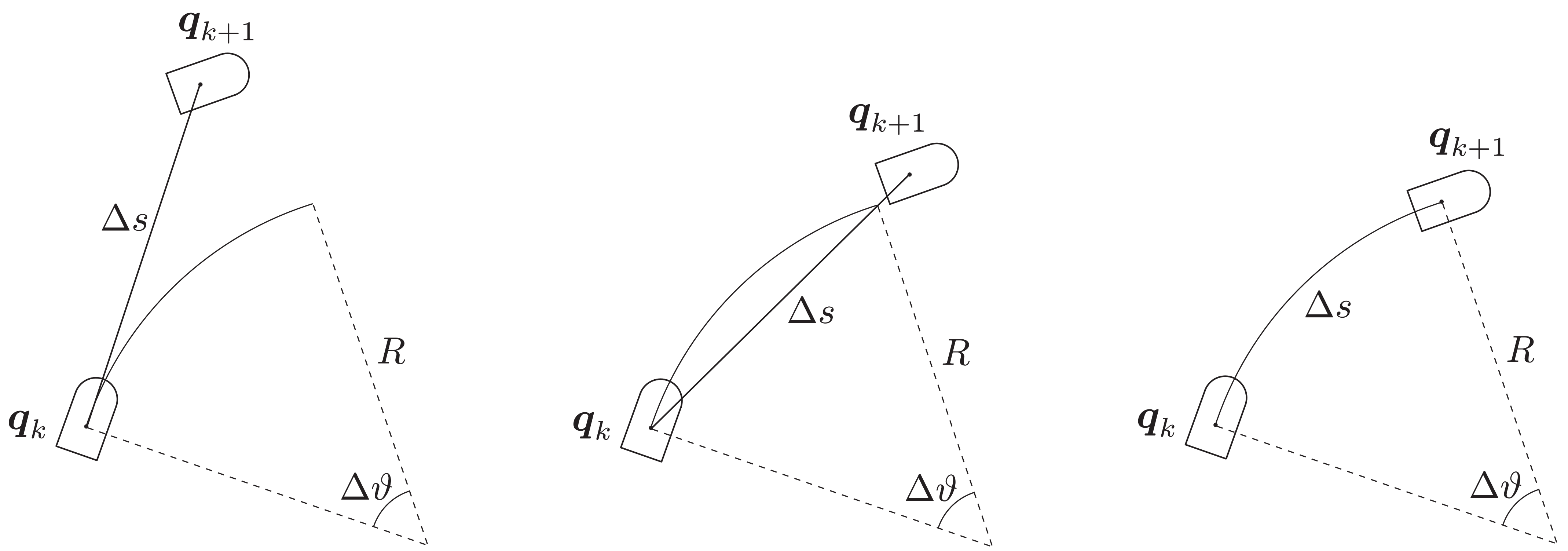
- third possibility: exact integration

$$x_{k+1} = x_k + \frac{v_k}{\omega_k}(\sin\theta_{k+1} - \sin\theta_k)$$
$$y_{k+1} = y_k - \frac{v_k}{\omega_k}(\cos\theta_{k+1} - \cos\theta_k)$$
$$\theta_{k+1} = \theta_k + \omega_k T_s$$

- for $\omega_k = 0$, x_{k+1} and y_{k+1} are still defined and coincide with the solution by Euler and Runge-Kutta

- for $\omega_k \approx 0$, a conditional instruction may be used in the implementation

geometric comparison



Euler Runge-Kutta exact

- in practice, due to the non-ideality of any actuation system, the commanded inputs v_k and ω_k are not used
- instead, measure the effect of the actual inputs:

$$v_k T_s = \Delta s \qquad \omega_k T_s = \Delta\theta \qquad \frac{v_k}{\omega_k} = \frac{\Delta s}{\Delta\theta}$$

Δs (traveled length) and $\Delta\theta$ (total orientation change) are reconstructed via proprioceptive sensors

- for example, for a differential-drive robot

$$\Delta s = \frac{r}{2}(\Delta\phi_R + \Delta\phi_L) \qquad \Delta\theta = \frac{r}{d}(\Delta\phi_R - \Delta\phi_L)$$

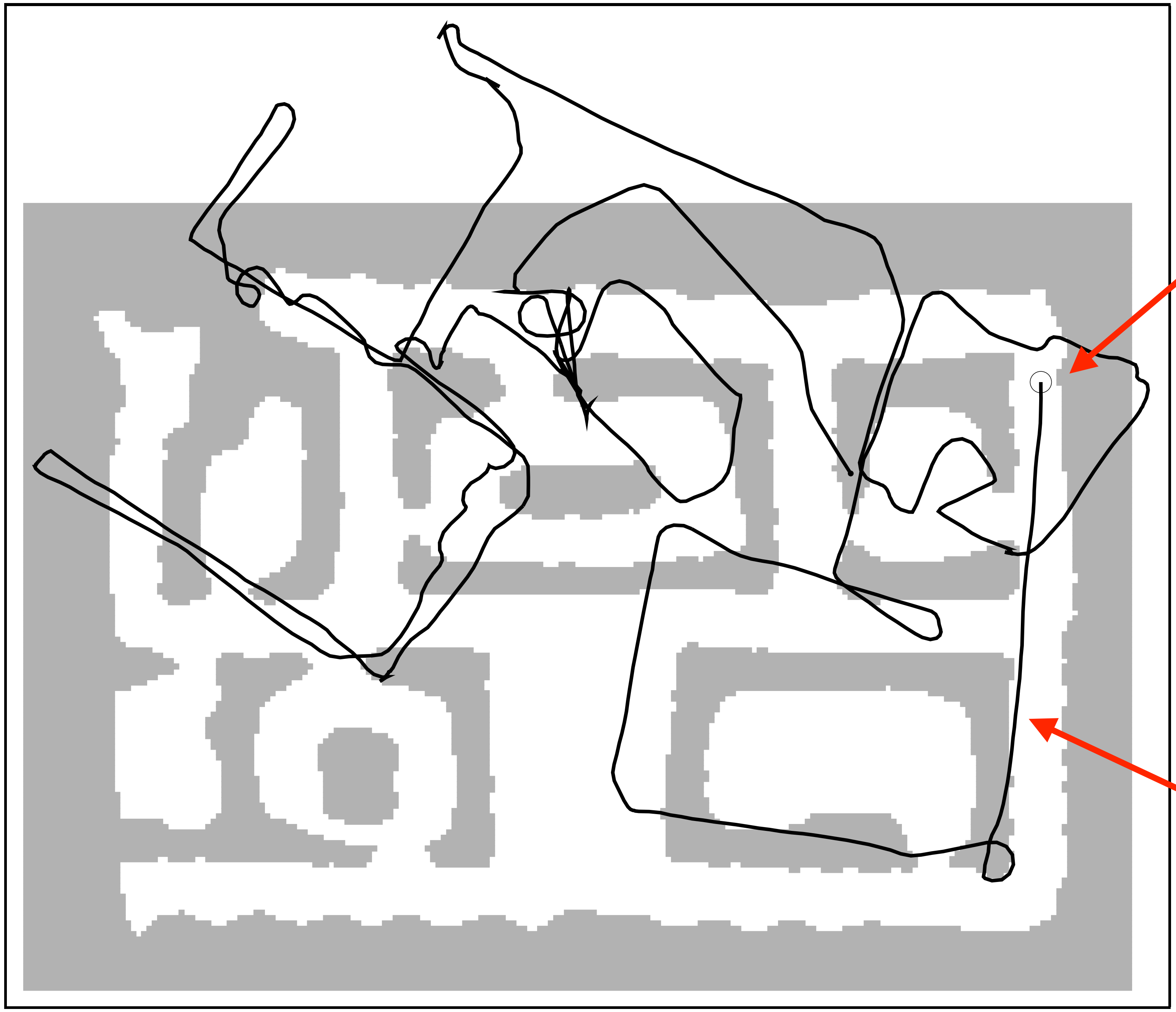
where $\Delta\phi_R$ and $\Delta\phi_L$ are the total rotations measured by the wheel encoders

- maintaining an estimate of the robot configuration by iterative integration of the kinematic model is called odometric localization or dead reckoning

- subject to an error (odometric drift) that grows over time, becoming significant over sufficiently long paths

- causes include wheel slippage (model perturbation), inaccurate calibration of, e.g., wheel radius (model uncertainty) or numerical integration error

- effective localization methods use proprioceptive as well as exteroceptive sensors



robot starts here

path reconstructed
by integration of kinematic model
using encoder measurements

a typical dead reckoning result