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New Variable Impedance Actuators for the Next Generation of Robots
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Dynamic Gravity Cancellation in Robots with Flexible Transmissions: constant, nonlinear, and variable stiffness

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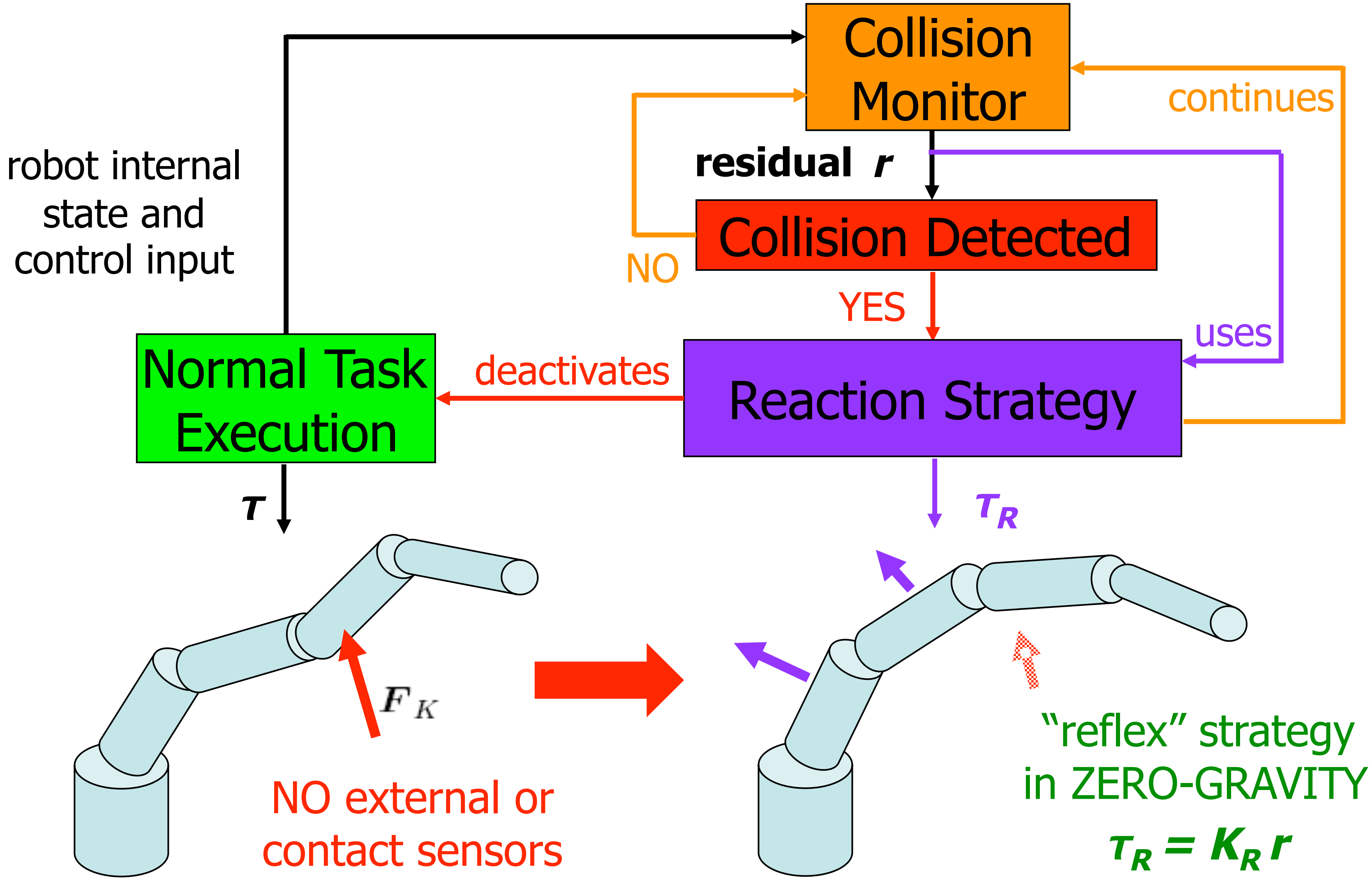
Goals and motivations

- complete cancellation of gravity from the dynamics of a flexible robot by feedback control
  - the robot should behave as in the absence of gravity
- or at least, some relevant output variables should match their behavior under no gravity
  - both in static and dynamic conditions
  - applicability to 1-dof and multi-dof devices

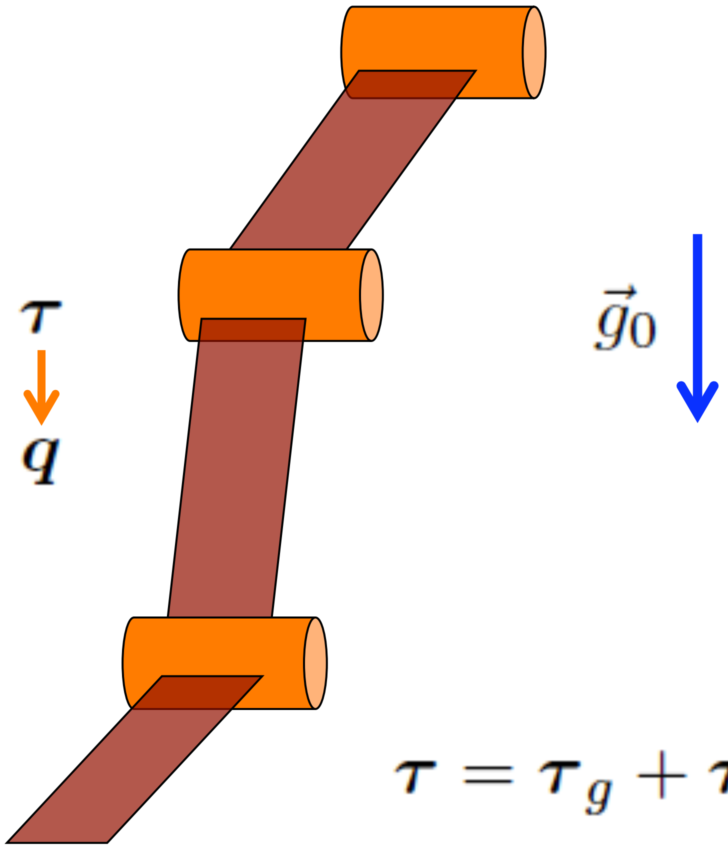
- zero-gravity field for unbiased robot reaction to collisions
  - for safer human-robot interaction tasks
- controllers for regulation tasks without gravity constraints
  - easier tuning of control gains
  - no lower bound restrictions on gains and joint stiffness

Collision detection and reaction

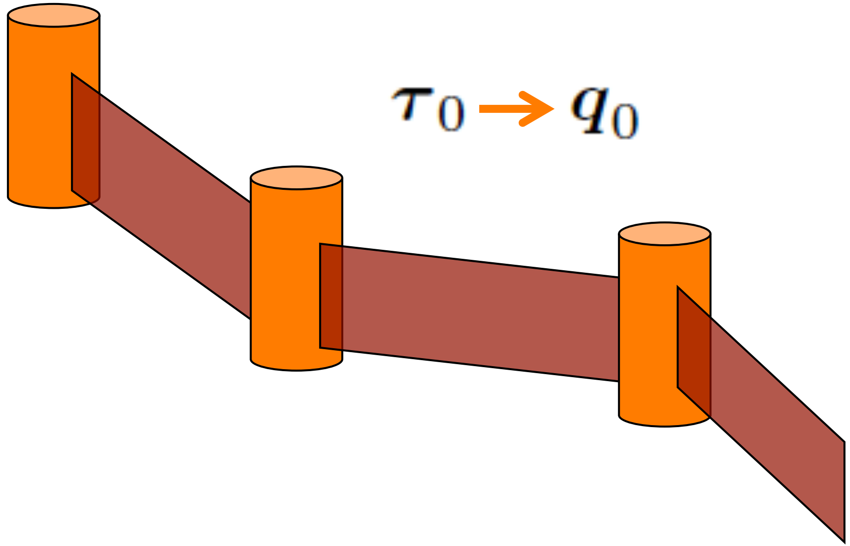
De Luca, Albu-Schäffer, Haddadin: IROS06, IROS08



Rigid robots



trivial, due to collocation

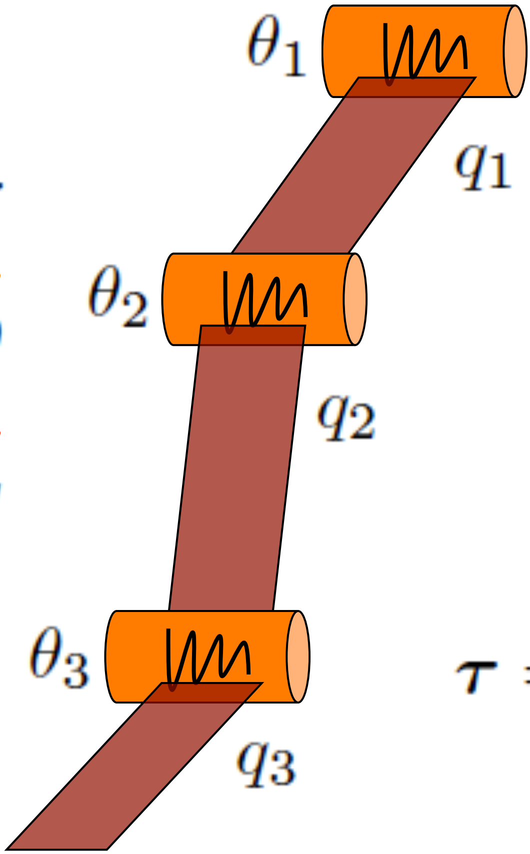


$$\boldsymbol{\tau} = \boldsymbol{\tau}_g + \boldsymbol{\tau}_0 \quad \boldsymbol{\tau}_g = \boldsymbol{g}(\boldsymbol{q}) \quad \boldsymbol{q} \equiv \boldsymbol{q}_0$$

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \quad \Rightarrow \quad \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau}_0$$

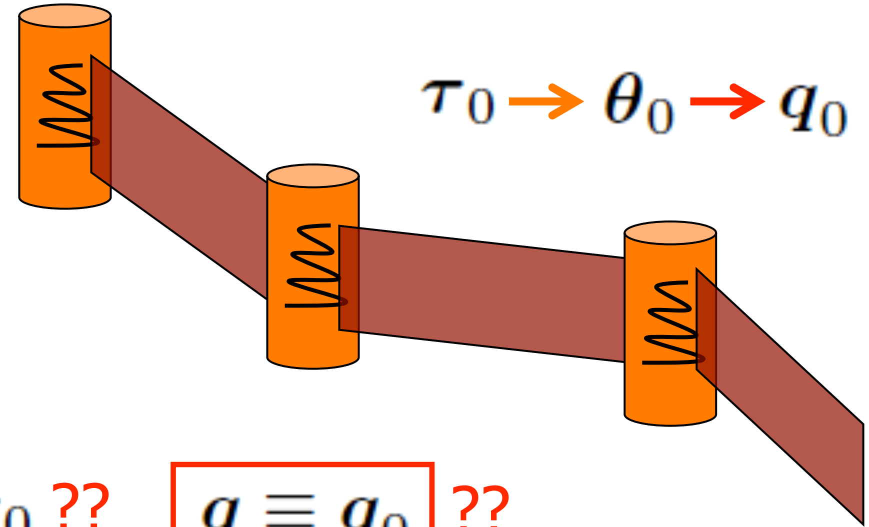
Flexible joint robots

$\boldsymbol{\tau} \downarrow \boldsymbol{\theta} \downarrow \boldsymbol{q}$



$\vec{g}_0$ ↓

non-collocation
of input torque and output



$\boldsymbol{\tau}_0 \rightarrow \boldsymbol{\theta}_0 \rightarrow \boldsymbol{q}_0$

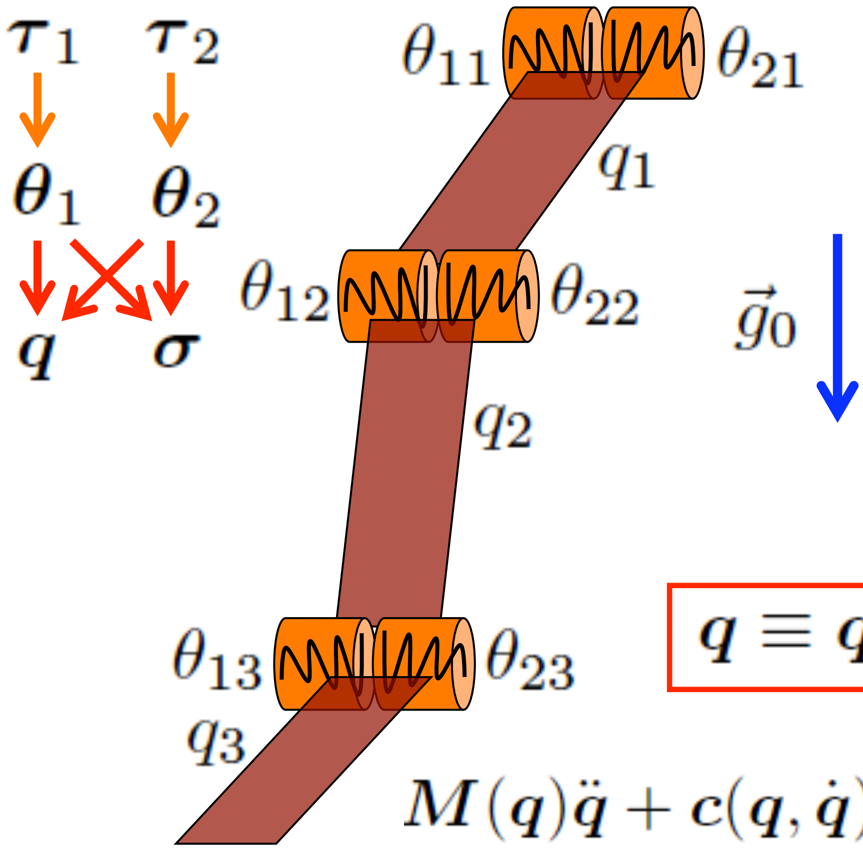
$\boldsymbol{\tau} = \boldsymbol{\tau}_g + \boldsymbol{\tau}_0$?? $\boldsymbol{q} \equiv \boldsymbol{q}_0$??

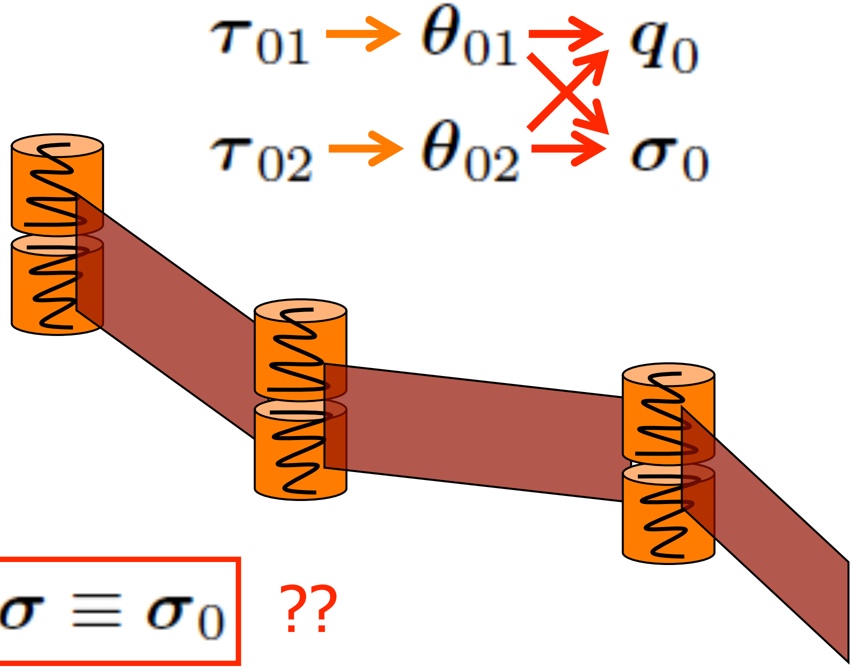
constant or nonlinear
joint stiffness

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}(\boldsymbol{q}-\boldsymbol{\theta}) = \mathbf{0}$$
$$\boldsymbol{B}\ddot{\boldsymbol{\theta}} + \boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q}) = \boldsymbol{\tau}$$

Variable joint stiffness robots

antagonistic actuation





$q \equiv q_0$ $\sigma \equiv \sigma_0$??

$$\phi_i = q - \theta_i \qquad i = 1, 2$$

$$\begin{aligned} M(q)\ddot{q} + c(q,\dot{q}) + g(q) + \tau_{e1}(\phi_1) + \tau_{e2}(\phi_2) &= 0 \\ B_1\ddot{\theta}_1 - \tau_{e1}(\phi_1) &= \tau_1 \\ B_2\ddot{\theta}_2 - \tau_{e2}(\phi_2) &= \tau_2 \end{aligned}$$

Feedback equivalence

$$u = \tilde{\alpha}(x) + \tilde{\beta}(x)u_0$$
$$x_0 = T_0^{-1}(T(x)) = \tilde{T}(x)$$

$$\dot{x} = f(x) + G(x)u$$

gravity-loaded system

$$\dot{x}_0 = f_0(x_0) + G_0(x_0)u_0$$

gravity-free system

feedback transformations (static state feedback + change of coordinates, both invertible)

$$u = \alpha(x) + \beta(x)v$$
$$z = T(x)$$

$$u_0 = \alpha_0(x_0) + \beta_0(x_0)v$$
$$z = T_0(x_0)$$

$$\dot{z} = Az + Bv$$

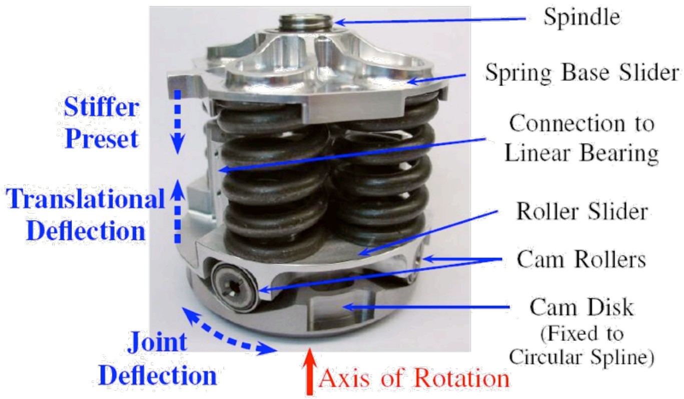
linear, controllable system

z ≈ linearizing outputs

Flexible robots that are feedback linearizable

- robots with elastic joints linearizing output = link position (4)
  - robots with joints having nonlinear flexibility
- robots with VSA-based actuation
  - antagonistic VSA-II
  - DLR-VS joint
  - ...

linearizing output = link position (4) + joint stiffness (2)



Gravity cancellation
in robots with elastic joints

$$
\begin{aligned}
M(q)\ddot{q} + c(q,\dot{q}) + g(q) + D_q\dot{q} + K(q-\theta) &= 0 \\
B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) &= \tau
\end{aligned}
$$

$$q(t) \equiv q_0(t) \quad \forall t \geq 0 \qquad\qquad \tau = \tau_g + \tau_0$$

$$\Rightarrow \quad \boxed{\tau_g = g(q) + D_\theta K^{-1}\dot{g}(q) + BK^{-1}\ddot{g}(q)}$$

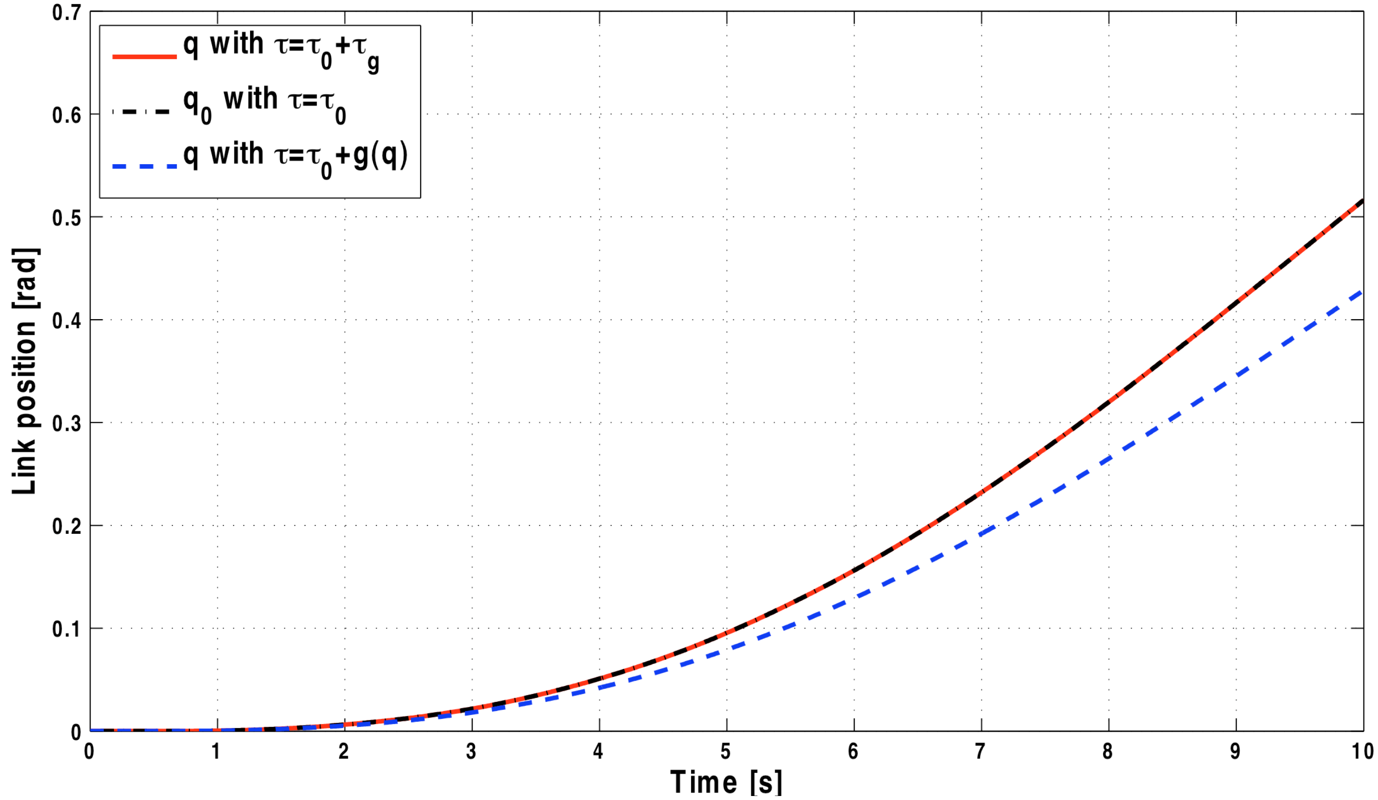
$$
\begin{aligned}
\dot{g}(q) &= \frac{\partial g(q)}{\partial q}\,\dot{q} \\
\ddot{g}(q) &= \frac{\partial g(q)}{\partial q} M^{-1}(q)\big(K(\theta - q) - c(q,\dot{q}) - g(q) - D_q\dot{q}\big) + \sum_{i=1}^{n} \frac{\partial^2 g(q)}{\partial q\, \partial q_i}\,\dot{q}\,\dot{q}_i
\end{aligned}
$$

requires full state feedback

Numerical results
gravity cancellation for 1-dof elastic joint

$$\tau_g = mdg_0 \left\{ \left(1 - \frac{B}{K}\dot{q}^2\right) \sin q - \frac{B}{M}\frac{mdg_0}{K} \sin q \cos q + \frac{MD_\theta - BD_q}{KM} \dot{q} \cos q + \frac{B}{M}(\theta - q) \cos q \right\}$$

$$\boxed{\tau_0 = \sin 0.1\pi t} \qquad g(q) = mdg_0 \sin q$$



exact reproduction of same link behavior with and without gravity

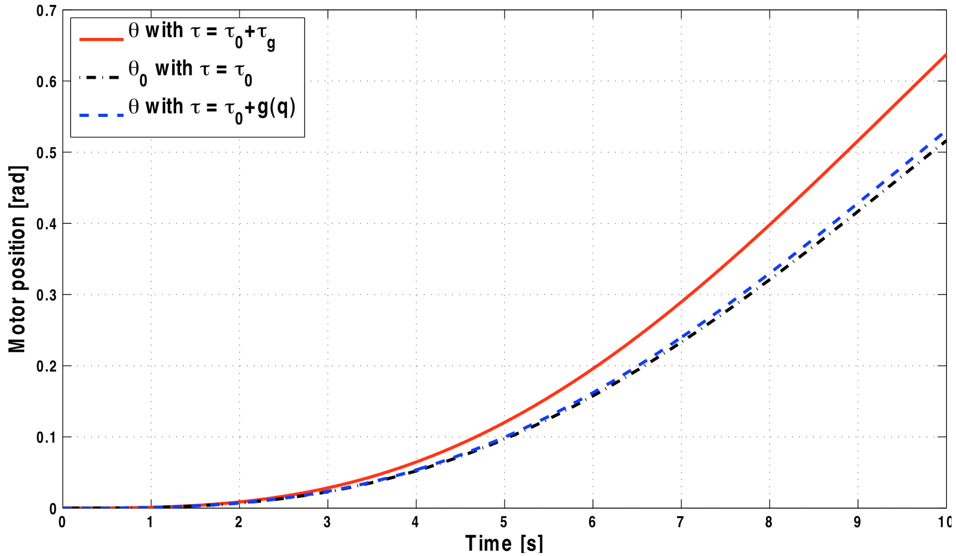
Numerical results

gravity cancellation for 1-dof elastic joint

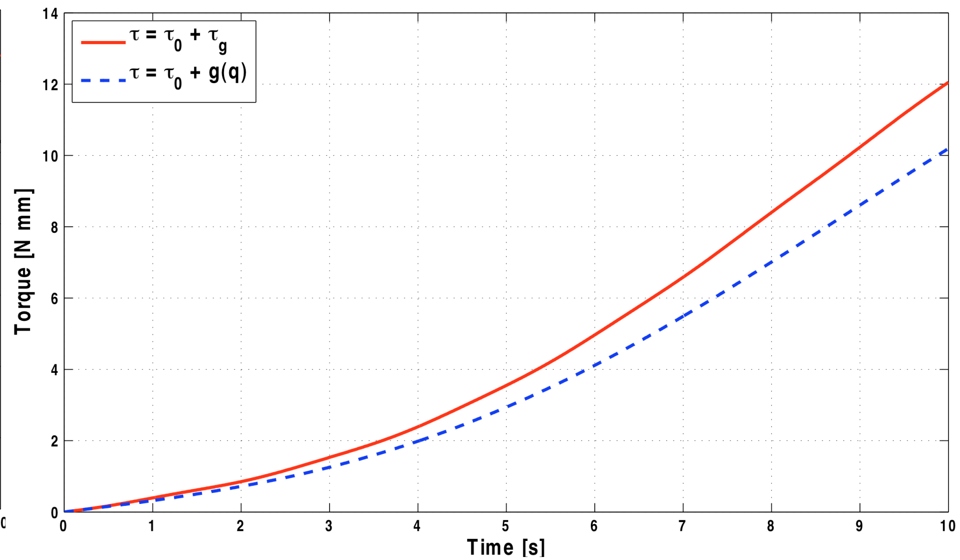
$$\tau_g = mdg_0 \left\{ \left(1 - \frac{B}{K}\dot{q}^2\right)\sin q - \frac{B}{M}\frac{mdg_0}{K}\sin q \cos q + \frac{MD_\theta - BD_q}{KM}\dot{q}\cos q + \frac{B}{M}(\theta - q)\cos q \right\}$$

$$\tau_0 = \sin 0.1\pi t \qquad g(q) = mdg_0 \sin q$$

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q})$$



different motor behavior
with and without gravity



torque comparison w.r.t.
link-based gravity compensation

A new PD-type regulator
for robots with elastic joints

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + D_q\dot{q} + K(q-\theta) = 0$$

$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

$$\tau = \tau_g + \tau_0$$

$$\tau_g = g(q) + D_\theta K^{-1}\dot{g}(q) + BK^{-1}\ddot{g}(q)$$

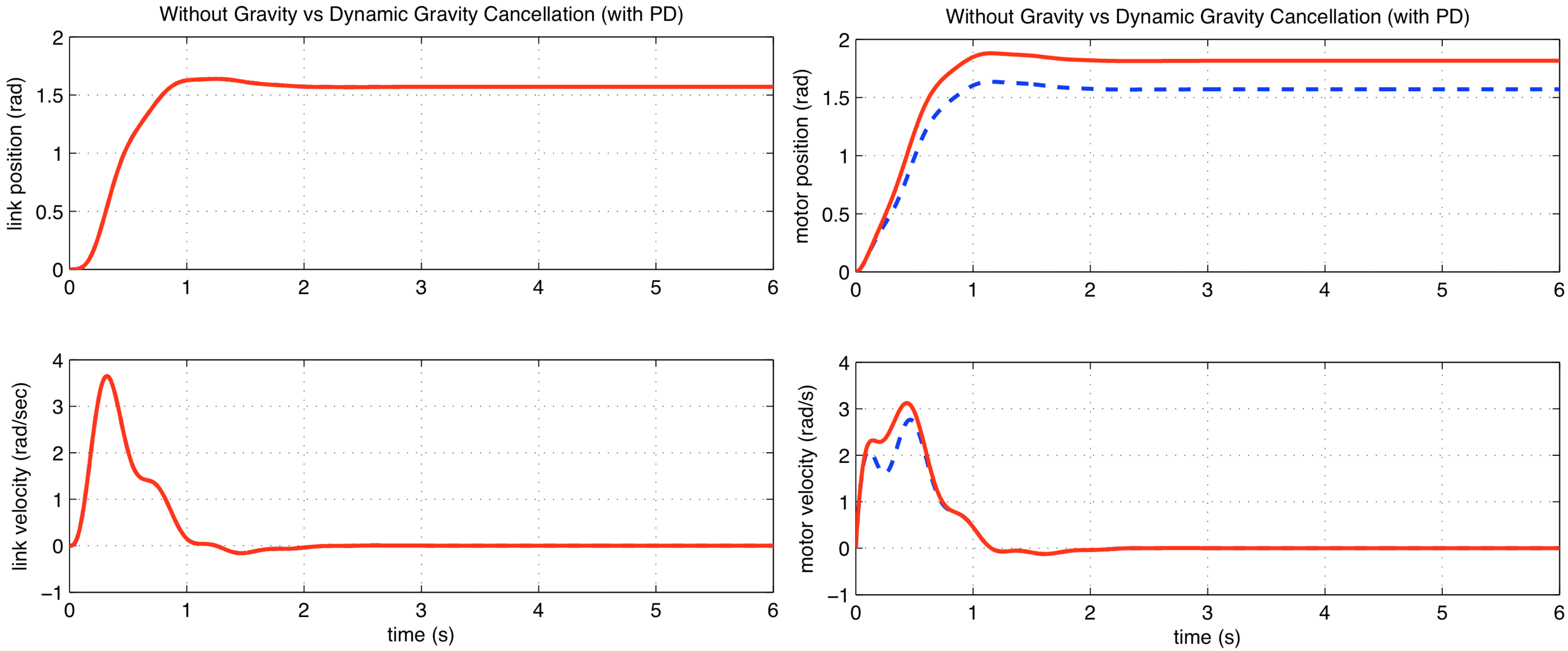
$$\boxed{\tau_0 =} \; K_P(\theta_{d0} - \theta_0) - K_D\dot{\theta}_0$$

$$= \boxed{K_P(q_d - \theta + K^{-1}g(q)) - K_D(\dot{\theta} - K^{-1}\dot{g}(q))}$$

Global asymptotic stability can be shown using a Lyapunov analysis under "minimal" sufficient conditions (also without viscous friction)

$\boxed{K_P > 0}$ $\boxed{K > 0}$ i.e., **no** strictly positive lower bounds and $K_D > 0$

Numerical results
regulation of a one-link arm with EJ under gravity

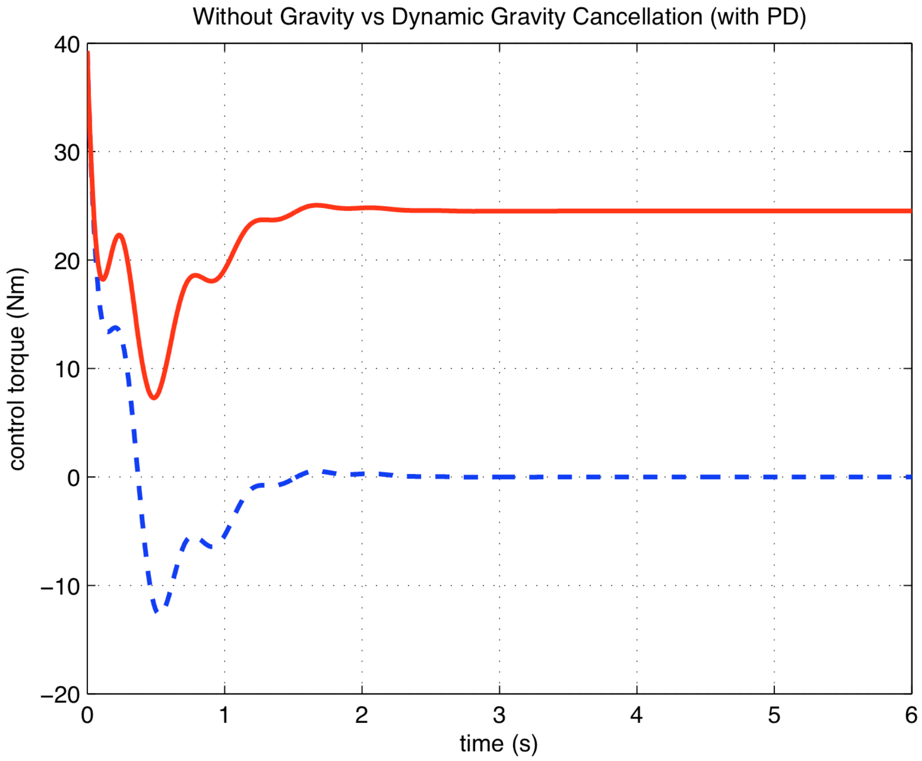


identical dynamic behavior of link
in gravity-loaded system
under PD + gravity cancellation
and in gravity-free system under PD

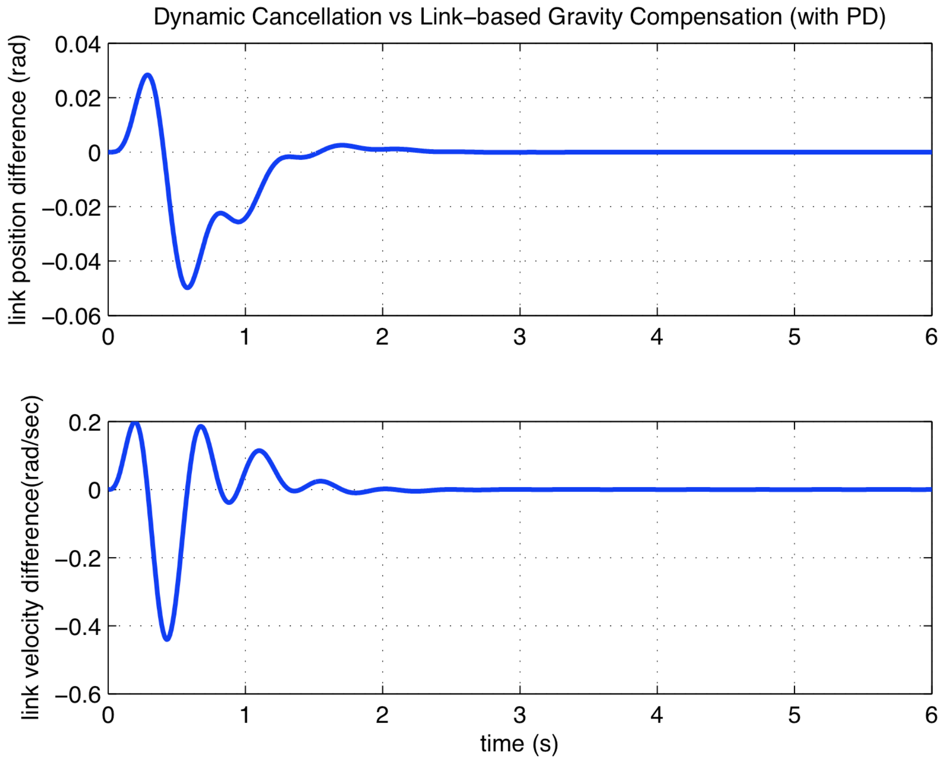
still a different motor behavior

Numerical results

regulation of a one-link arm with EJ under gravity



total control torque profiles
in gravity-loaded system
under PD + gravity cancellation
and in gravity-free system under PD



difference in link behavior
between dynamic gravity cancellation
and link-based compensation g(q)

Gravity cancellation

in robots with nonlinear flexible joints – 1-dof case

$\phi = q - \theta \rightarrow U_e(\phi) \geq 0 \rightarrow \tau_e = \partial U_e / \partial q = \tau_e(\phi) \rightarrow \sigma = \partial \tau_e / \partial q = \sigma(\phi)$

$$\begin{aligned} M\ddot{q} + D_q\dot{q} + g(q) + \tau_e(\phi) &= 0 \\ B\ddot{\theta} + D_\theta\dot{\theta} - \tau_e(\phi) &= \tau \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} M\ddot{q}_0 + D_q\dot{q}_0 + \tau_e(\phi_0) &= 0 \\ B\ddot{\theta}_0 + D_\theta\dot{\theta}_0 - \tau_e(\phi_0) &= \tau_0 \end{aligned}$$

$q(t) = q_0(t)$

for all $t \geq 0$ $\Rightarrow$

$$\begin{aligned} \tau = & \; g(q) + \frac{D_\theta}{\sigma(\phi)}\dot{g}(q) + \frac{B}{\sigma(\phi)}\ddot{g}(q) \\ & + \frac{\sigma(\phi) - \sigma(\phi_0)}{\sigma(\phi)}\left((B + M)\ddot{q} + (D_q + D_\theta)\dot{q}\right) \\ & + \frac{B}{\sigma(\phi)}\left(\frac{\partial\sigma(\phi)}{\partial\phi}\dot{\phi}^2 - \frac{\partial\sigma(\phi_0)}{\partial\phi_0}\dot{\phi}_0^2\right) + \frac{\sigma(\phi_0)}{\sigma(\phi)}\tau_0 \end{aligned}$$

numerically solve $\tau_e(\phi_0) = g(q) + \tau_e(\phi)$ for ϕ_0

Gravity cancellation
in robots with nonlinear flexible joints – 1-dof case

$$\phi = q - \theta \rightarrow U_e(\phi) \geq 0 \rightarrow \tau_e = \partial U_e / \partial q = \tau_e(\phi) \rightarrow \sigma = \partial \tau_e / \partial q = \sigma(\phi)$$

$$\boxed{\tau_e(\phi_0) = g(q) + \tau_e(\phi) = a(q,\theta)}$$

closed-form solution in some particular cases, e.g., quadratic stiffness

$$U_e = \tfrac{1}{2} K \phi^2 + \tfrac{1}{4} K_c \phi^4 \rightarrow \tau_e(\phi) = K\phi + K_c \phi^3 \rightarrow \sigma(\phi) = K + 3K_c\phi^2$$

$$\Rightarrow \quad \phi_0 = \sqrt[3]{\frac{1}{2}\frac{a(q,\theta)}{K_c} + b(q,\theta)} + \sqrt[3]{\frac{1}{2}\frac{a(q,\theta)}{K_c} - b(q,\theta)}$$

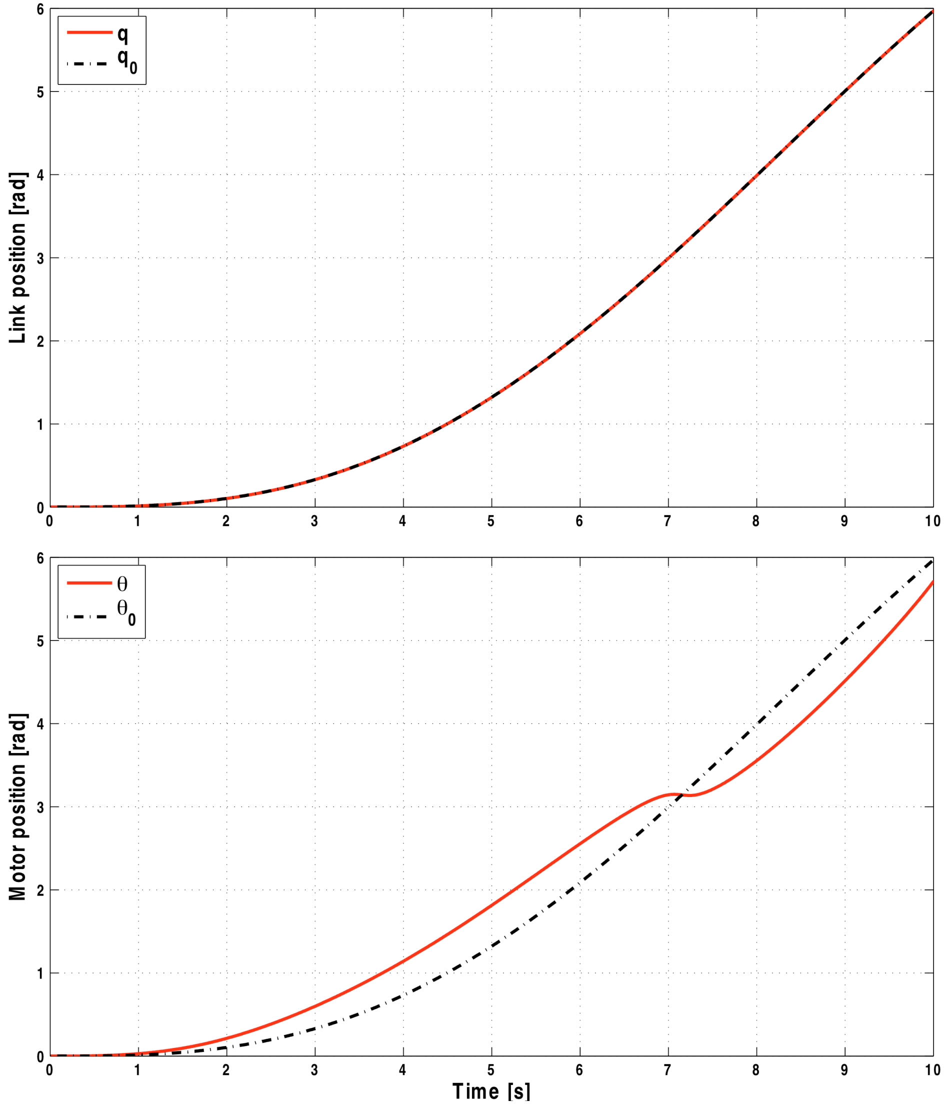
with $b(q,\theta) = \sqrt{\frac{1}{27}\left(\frac{K}{K_c}\right)^3 + \frac{1}{4}\left(\frac{a(q,\theta)}{K_c}\right)^2} > 0$

Numerical results
gravity cancellation in a joint with quadratic stiffness

$$\tau_0 = \sin 0.1\pi t$$

exact reproduction of
same link behavior
with and without gravity

different motor behavior
with and without gravity



Gravity cancellation
in robots with variable stiffness joints – 1-dof case

symmetric, antagonistic arrangement

$$M\ddot{q} + D_q\dot{q} + g(q) + \tau_e(\phi_1) + \tau_e(\phi_2) = 0$$
$$B\ddot{\theta}_1 + D_\theta\dot{\theta}_1 - \tau_e(\phi_1) = \tau_1$$
$$B\ddot{\theta}_2 + D_\theta\dot{\theta}_2 - \tau_e(\phi_2) = \tau_2$$

$$\phi_i = q - \theta_i \quad i = 1, 2$$

total stiffness

$$\sigma_t(\phi_1, \phi_2) = \frac{\partial(\tau_e(\phi_1) + \tau_e(\phi_2))}{\partial q} = \sigma(\phi_1) + \sigma(\phi_2)$$

$$q(t) \equiv q_0(t)$$

AND

$$\sigma_t(t) \equiv \sigma_{t0}(t)$$

$$\forall t \geq 0$$

⇒

$$\mathcal{A}(\phi_1, \phi_2) = \begin{pmatrix} \sigma(\phi_1) & \sigma(\phi_2) \\ \dfrac{\partial\sigma(\phi_1)}{\partial\phi_1} & \dfrac{\partial\sigma(\phi_2)}{\partial\phi_2} \end{pmatrix}$$

⇒ generically non-singular for $\theta_1 \neq \theta_2$

Gravity cancellation

in robots with variable stiffness joints – 1-dof case

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} D_\theta \dot{\theta}_1 - \tau_e(\phi_1) \\ D_\theta \dot{\theta}_2 - \tau_e(\phi_2) \end{pmatrix} + \boxed{\mathcal{A}^{-1}(\phi_1, \phi_2) \cdot}$$

$$\Bigg\{ \boxed{\mathcal{A}(\phi_{10}, \phi_{20})} \left(\begin{pmatrix} \tau_{10} \\ \tau_{20} \end{pmatrix} + \begin{pmatrix} \tau_e(\phi_{10}) - D_\theta \dot{\theta}_{10} \\ \tau_e(\phi_{20}) - D_\theta \dot{\theta}_{20} \end{pmatrix} \right)$$

$$+ B \begin{pmatrix} \ddot{g}(q) + \sum_{i=1}^{2} \left(\frac{\partial \sigma(\phi_i)}{\partial \phi_i} \dot{\phi}_i^2 - \frac{\partial \sigma(\phi_{i0})}{\partial \phi_{i0}} \dot{\phi}_{i0}^2 \right) \\ \sum_{i=1}^{2} \left(\frac{\partial \sigma(\phi_i)}{\partial \phi_i} - \frac{\partial \sigma(\phi_{i0})}{\partial \phi_{i0}} \right) \ddot{q} \\ + \sum_{i=1}^{2} \left(\frac{\partial^2 \sigma(\phi_i)}{\partial \phi_i^2} \dot{\phi}_i^2 - \frac{\partial^2 \sigma(\phi_{i0})}{\partial \phi_{i0}^2} \dot{\phi}_{i0}^2 \right) \end{pmatrix} \Bigg\}$$

ϕ_{10} ϕ_{20}

numerically solve

$$\boxed{\begin{aligned} \tau_e(\phi_{10}) + \tau_e(\phi_{20}) &= -M\ddot{q} - D_q \dot{q} = a_1(q, \theta_1, \theta_2) \\ \sigma(\phi_{10}) + \sigma(\phi_{20}) &= \sigma_t(q, \theta_1, \theta_2) \end{aligned}}$$

Gravity cancellation
in variable quadratic stiffness joint

numerical solution in the particular case of (double) quadratic stiffness

$$\begin{aligned} K(\phi_{10}+\phi_{20})+K_c(\phi_{10}^3+\phi_{20}^3) &= a_1(q,\theta_1,\theta_2) \\ 2K+3K_c(\phi_{10}^2+\phi_{20}^2) &= \sigma_t(q,\theta_1,\theta_2) \end{aligned}$$

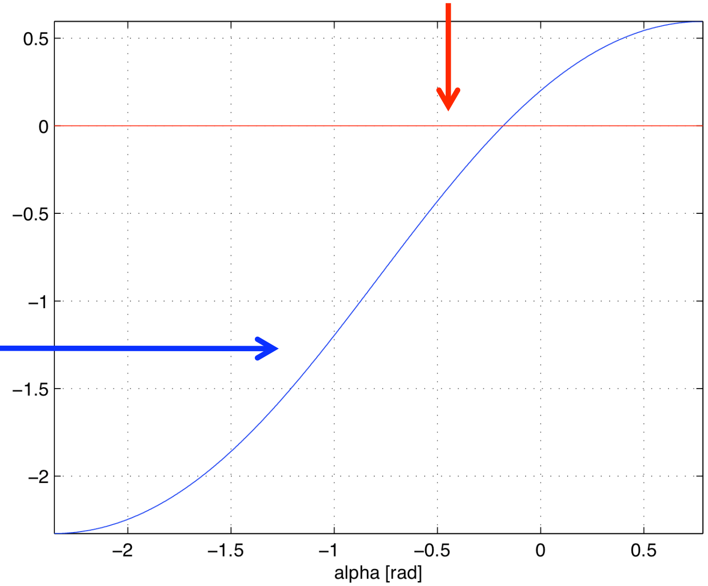
$$\frac{\sigma_t-2K}{3K_c} := R^2 \geq 0 \longrightarrow \begin{cases} \phi_{10} = R\cos\alpha \\ \phi_{20} = R\sin\alpha \end{cases} \quad \alpha \in [0, 2\pi)$$

$$(\cos\alpha+\sin\alpha)+\frac{\sigma_t-2K}{3K}(\cos^3\alpha+\sin^3\alpha) = \frac{a_1}{KR}$$

one of the two smooth branches, obtained with

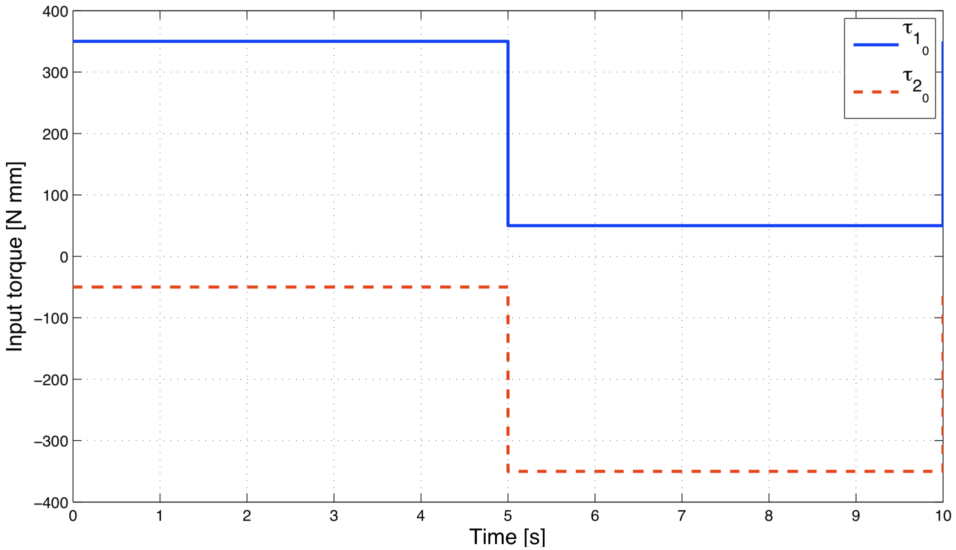
$$K = 100 \quad K_c = 500 \quad \sigma_t = 220$$

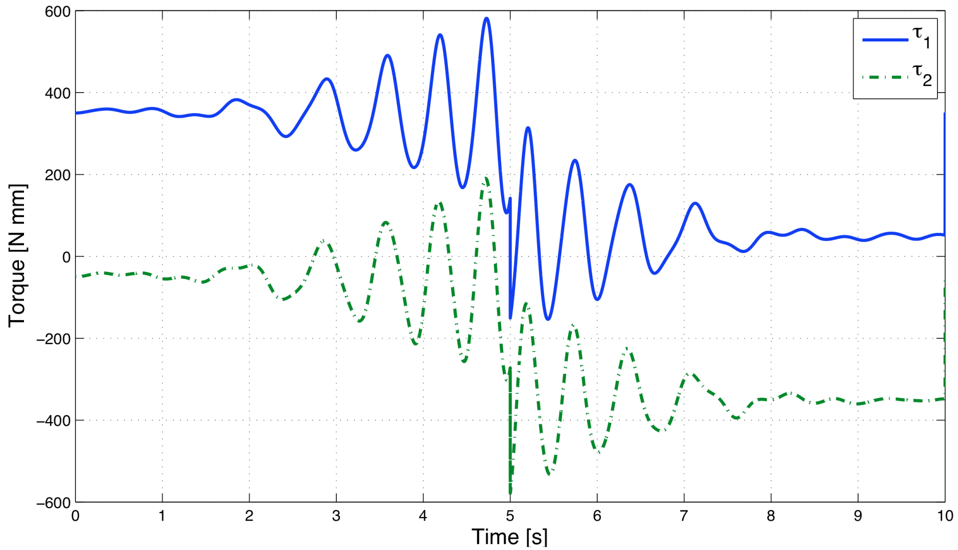
$a_1 = 10$



Numerical results

gravity cancellation in joint with variable quadratic stiffness





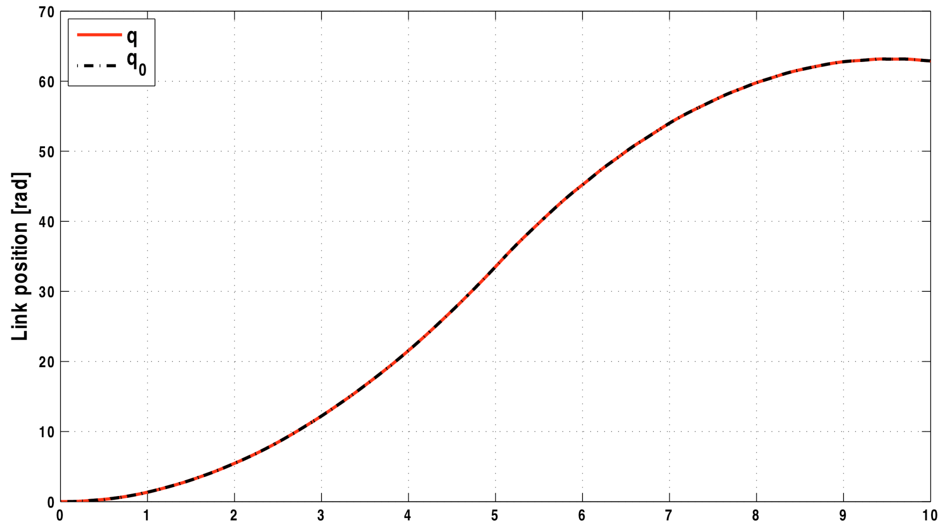
bang-bang (open loop)
torques sent to the VSA joint
in the absence of gravity

torques applied to the VSA joint
in the presence of gravity
with dynamic gravity cancellation

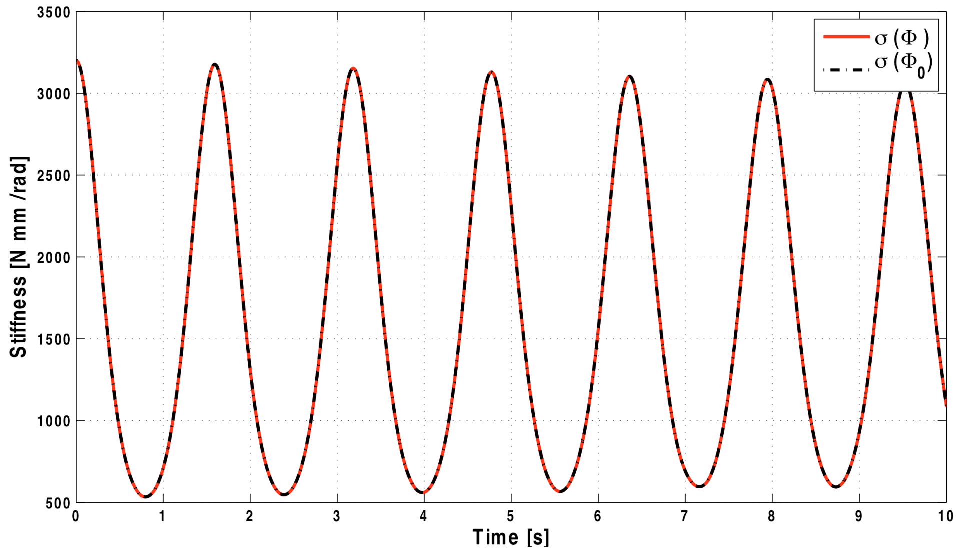
Numerical results

gravity cancellation in joint with variable quadratic stiffness

exact reproduction of
same link behavior
with and without gravity



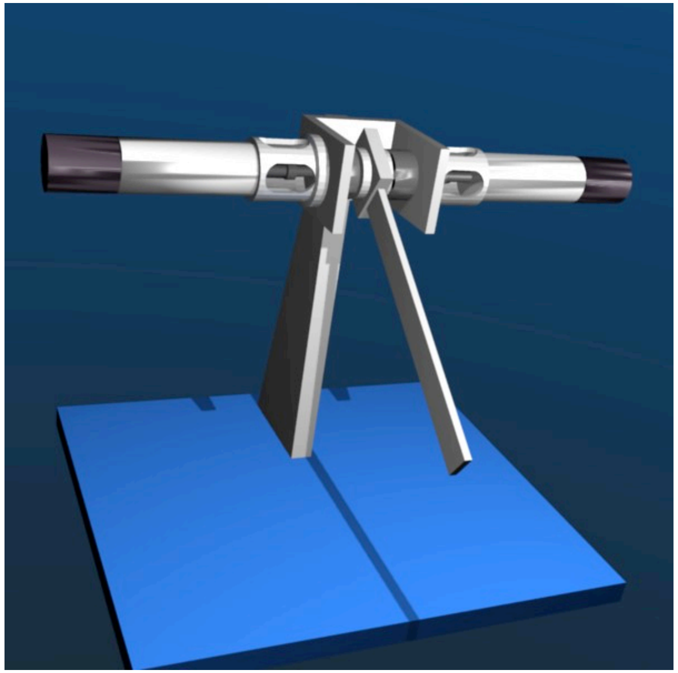
exact reproduction of
same stiffness behavior
with and without gravity



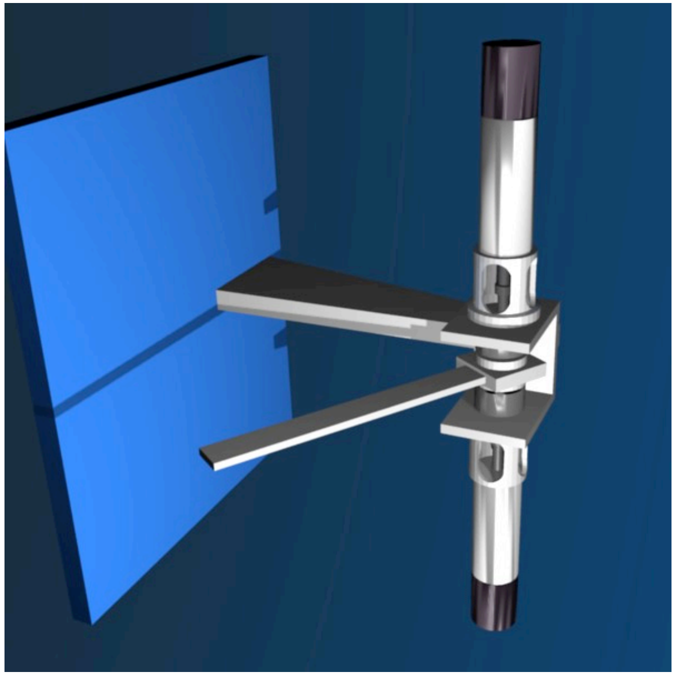
Gravity cancellation
for VSA-II driving a single link

- bi-directional antagonistic arrangement of two motors with a nonlinear flexible transmission by UniPisa
  - Grashof neutral four-bar linkage + linear spring (two for each side)

$$\beta(\phi_i) = \arcsin\left(C \sin\left(\frac{\phi_i}{2}\right)\right) - \frac{\phi_i}{2} \qquad \tau_e(\phi_i) = 2K\,\beta(\phi_i)\,\frac{\partial \beta(\phi_i)}{\partial \phi_i}$$

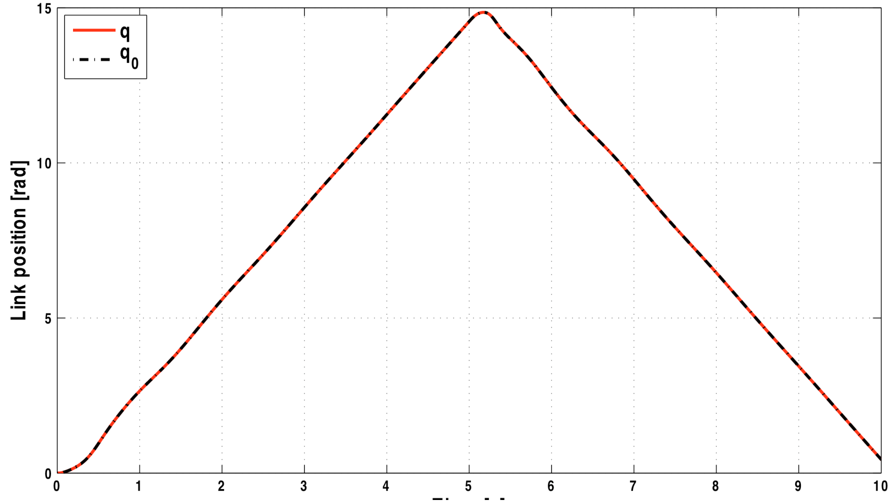


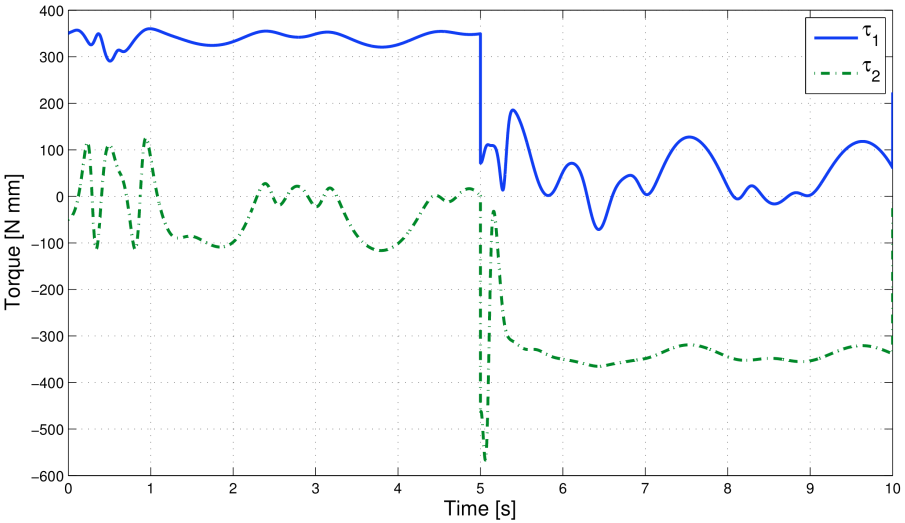
via feedback



Numerical results
gravity cancellation on the VSA-II joint

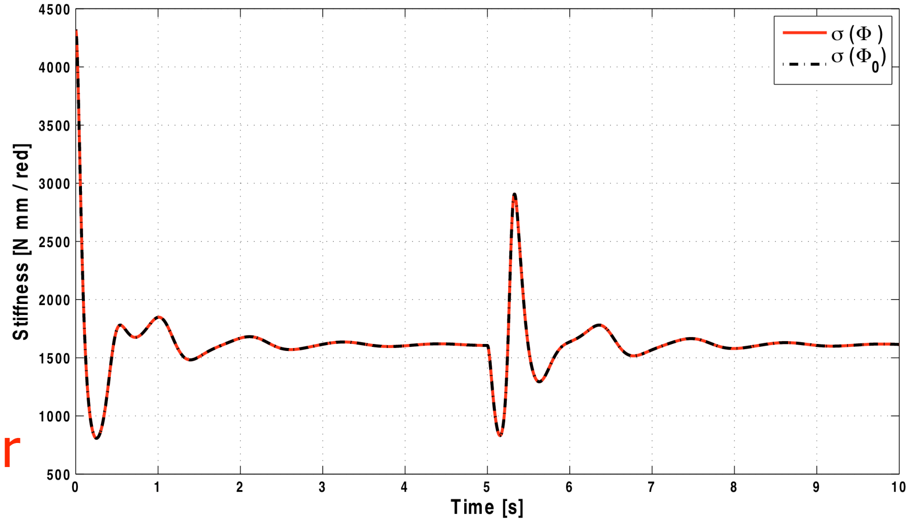
exact reproduction
of link behavior





applied torques for gravity cancellation

exact reproduction
of stiffness behavior



Conclusions

- achieved cancellation of gravity from link dynamics (and stiffness, if variable) of robots with flexible transmissions
  - works even in highly dynamic conditions
- it is a by-product of FL (feedback linearization)
  - but much simpler (especially in the multi-dof case)
  - compromise between FL and energy-based Lyapunov methods
- allows the definition of natural torque-based reaction schemes to collisions
  - also for VSA-based robots (as opposed to IROS'09)
- leads to novel regulation control designs
  - without (larger than zero) lower bounds on gains and stiffness
- unifies the handling of robot stiffness in response to contacts, independently from gravity