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ISSUES IN ACCELERATION RESOLUTION OF ROBOT REDUNDANCY

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Abstract. *Algorithmic methods for solving robot redundancy are considered, providing new strategies for optimizing general kinematic criteria at the acceleration level. Acceleration resolution offers two potential advantages: it allows to take into account dynamic performance and can be directly used with control techniques for accurate tracking. Local first and second-order optimal resolution schemes are described in a convenient unified framework and classified according to their iterative or exact nature. Instances where an equivalence exists between velocity and acceleration methods are pointed out. Existing second-order methods do not apply to the case of mixed criteria, namely objective functions depending both on joint configuration and velocity. A discrete-time formulation is used to design algorithms solving this problem. Simulations with a planar redundant arm show the benefits achieved with the new optimization scheme.*

Keywords. *Robot redundancy; optimization; acceleration control; discrete-time systems.*

Introduction

The potential benefits of using kinematically redundant robots is balanced by the difficulty in designing inverse kinematic schemes with automatic provision of a satisfactory behavior at the joint level. Several methods have been proposed in the last few years, mostly addressing the problem of selecting joint displacement or velocity in a locally optimal way, i.e. with information limited to the current point of the task trajectory. The synthesis of nominal joint accelerations in redundant mechanisms usually requires a more involved analysis but allows to directly face robot dynamics issues, as opposed to velocity schemes which often result in poor dynamic performance. At both orders of resolution, a major point is represented by the choice of the objective function measuring the robot performance, namely the quality of the particular inverse kinematic solution. Independently from the meaning of a specific criterion, its structure and functional dependence will affect the nature of the method to be chosen.

In this paper some algorithmic aspects involved in the redundancy resolution process are examined. A general optimization point of view is assumed in reviewing the basics of currently used methods. By setting up a unified framework, we will be able to give constructive answers to a number of open questions, and in particular: *(i)* what is the relationship between first and second-order methods and when are the obtained joint trajectories identical: *(ii)* when is an algorithm exact, i.e. it provides the local optimal constrained solution in one step; *(iii)* when are properties like *cyclicity* inherited in moving from velocity to acceleration schemes.

Such a general analysis will also point out that no existing second-order redundancy resolution method applies to optimization problems with *mixed criteria*, depending both on joint position and velocity. A *discrete-time* formulation of this problem will be used to develop a new solution algorithm, where joint acceleration is selected considering its effect on the local evolution of both robot configuration and velocity.

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Velocity vs. Acceleration Resolution

Consider a robot manipulator with n joints executing an m-dimensional task, with $d = n - m > 0$ being the degree of redundancy. The associated direct kinematics is

$$\mathbf{p} = \mathbf{f}(\mathbf{q}), \qquad \mathbf{q} \in I\!R^n,\ \mathbf{p} \in I\!R^m, \tag{1}$$

with joint coordinates $\mathbf{q}$ and task variables $\mathbf{p}$. The first and second-order differential kinematics are

$$\dot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \tag{2}$$

$$\ddot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}, \tag{3}$$

where the robot Jacobian $\mathbf{J} = \partial\mathbf{f}/\partial\mathbf{q}$ is an $m \times n$ matrix. In general, a task trajectory $\mathbf{p}(t)$ is given and the inverse kinematic problem consists in finding an associated $\mathbf{q}(t)$ satisfying (1) for each t. In view of the nonlinearity of (1), this problem is solved at a differential level using (2) or (3). Due to the arm redundancy, at each time instant there exists an infinity of solutions $\dot{\mathbf{q}}$ or $\ddot{\mathbf{q}}$ of the form

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}(\mathbf{q})\dot{\mathbf{p}} + (\mathbf{I} - \mathbf{J}^{\dagger}(\mathbf{q})\mathbf{J}(\mathbf{q}))\mathbf{v}, \quad (4)$$

$$\ddot{\mathbf{q}} = \mathbf{J}^{\dagger}(\mathbf{q})\ddot{\mathbf{r}}(\mathbf{q},\dot{\mathbf{q}}) + (\mathbf{I} - \mathbf{J}^{\dagger}(\mathbf{q})\mathbf{J}(\mathbf{q}))\mathbf{a}, \quad (5)$$

where $\mathbf{J}^{\dagger}$ is the unique pseudoinverse of $\mathbf{J}$ (Boullion and Odell, 1971), $\ddot{\mathbf{r}} = \ddot{\mathbf{p}} - \dot{\mathbf{J}}\dot{\mathbf{q}}$, while $\mathbf{v}$ and $\mathbf{a}$ are arbitrary vectors in $I\!R^n$. The $n \times n$ matrix $\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J}$ is the projection operator into the null-space of the Jacobian. In the full row-rank case, the expression of the pseudoinverse is $\mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$.

Each choice of $\mathbf{v}(t)$ and $\mathbf{a}(t)$ yields a particular motion of the arm in the joint space, always guaranteeing a correct task execution. These vectors are usually determined through optimization of some performance criterion or by satisfying an augmented task specification (Nakamura, 1991).

The basic issue in selecting (4) versus (5) is the differential order at which redundancy resolution takes place. Indeed, first-order resolution typically provides only *path planning* at the joint level, whereas the second-order scheme gives a *trajectory planning*, i.e. with complete timing information. Correspondingly, equation (4) requires at the current instant only the knowledge of the configuration $\mathbf{q}$, while both $\mathbf{q}$ and $\dot{\mathbf{q}}$ are needed to compute (5). In the latter case, the set $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ can be directly feeded into the robot inverse dynamics computation for control purposes.

In order to obtain joint accelerations from a first-order solution, one may differentiate symbolically (4). This corresponds to a specific choice of the null-space vector $\mathbf{a}$ in the second-order solution (5), as stated in the following

Proposition 1. *Assume that* $\mathbf{p}(t) \in C^2$, *and that the initial joint positions* $\mathbf{q}(0)$ *and velocities* $\dot{\mathbf{q}}(0)$ *are such that* $\mathbf{f}(\mathbf{q}(0)) = \mathbf{p}(0)$, $\mathbf{J}(\mathbf{q}(0))\dot{\mathbf{q}}(0) = \dot{\mathbf{p}}(0)$. *Provided that* $\mathbf{J}(\mathbf{q}(t))$ *is always full rank, the joint trajectory* $\mathbf{q}(t)$ *generated by the first-order solution (4) for any* $\mathbf{v}(t) \in C^1$ *coincides with the one generated by the second-order solution (5) iff*

$$(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\mathbf{a} = (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\left[\dot{\mathbf{J}}^{\dagger}(\dot{\mathbf{p}} - \mathbf{J}\mathbf{v}) + \dot{\mathbf{v}}\right], \quad (6)$$

where $\dot{\mathbf{J}}^{\dagger} = \frac{d}{dt}(\mathbf{J}^{\dagger})$.

Proof. Differentiate (4) to obtain

$$\ddot{\mathbf{q}} = \mathbf{J}^{\dagger}(\ddot{\mathbf{p}} - \dot{\mathbf{J}}\mathbf{v}) + \dot{\mathbf{J}}^{\dagger}(\dot{\mathbf{p}} - \mathbf{J}\mathbf{v}) + (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\dot{\mathbf{v}}. \quad (7)$$

Plugging (4) in (5) to eliminate $\dot{\mathbf{q}}$, and equating with (7) gives

$$(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\mathbf{a} = (\dot{\mathbf{J}}^{\dagger} + \mathbf{J}^{\dagger}\dot{\mathbf{J}}\mathbf{J}^{\dagger})(\dot{\mathbf{p}} - \mathbf{J}\mathbf{v}) + (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\dot{\mathbf{v}}. \quad (8)$$

In the full rank case, differentiating $\mathbf{J}\mathbf{J}^{\dagger} = \mathbf{I}$ with respect to time and premultiplying by $\mathbf{J}^{\dagger}$ leads to $\mathbf{J}^{\dagger}\dot{\mathbf{J}}\mathbf{J}^{\dagger} = -\mathbf{J}^{\dagger}\mathbf{J}\dot{\mathbf{J}}^{\dagger}$. Substituting this in (8), the thesis follows. ■

Equality (6) is limited to null-space projections of joint acceleration vectors. Thus, $\mathbf{a} = \dot{\mathbf{J}}^{\dagger}(\dot{\mathbf{p}} - \mathbf{J}\mathbf{v}) + \dot{\mathbf{v}}$ is a sufficient but not necessary condition for coincidence of first and second-order schemes.

In the following, we will focus on cases where redundancy is resolved by *local optimization* of a performance criterion. A number of algorithms have been proposed in connection with a variety of criteria measuring arm dexterity (Yoshikawa, 1985), distance from obstacles (Maciejewski and Klein, 1985) or from joint range limits (Liègeois, 1977), joint velocity (Whitney, 1972), joint acceleration (Khatib, 1983) or torque (Hollerbach and Suh, 1987). When the task trajectory is completely known in advance, an alternative approach is to optimize a global (viz. integral) criterion defined over the whole path (Nakamura and Hanafusa, 1987). Besides being unfeasible for on-line application, the resulting algorithms are in general computationally intensive, and will not be considered here.

It can be recognized that the above methods differ in three aspects: the functional dependence of the objective function H, the differential order at which kinematic redundancy is resolved, and the exact[(1)] or iterative nature of the optimization process. Table 1 summarizes the various possibilities. Note that $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$ always denote the *current* joint position and velocity. On the other hand, in first-order schemes (corresponding to the first row) $\dot{\mathbf{q}}$ is the velocity to be chosen at the *current* instant, while $\mathbf{q}$ is the *next* joint position that will be reached as a result of this choice. Similarly, in second-order schemes $\mathbf{q}$ and $\dot{\mathbf{q}}$ are the *next* joint position and velocity that will be reached as a result of the choice of the *current* acceleration $\ddot{\mathbf{q}}$.

First-order methods

Most of the existing techniques fall into class (1.A), i.e. in the *iterative* minimization of a configuration dependent objective $H(\mathbf{q})$ by a velocity scheme. Each step moves towards the constrained optimum of the objective function, possibly reaching it after some iterations. This is usually realized by the *Projected Gradient* (PG) method (Liègeois, 1977)

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}\dot{\mathbf{p}} - \alpha(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\nabla_{\mathbf{q}}H\big|_{\mathbf{q}_0}, \quad (9)$$

where $\alpha > 0$ is a scalar stepsize (Luenberger, 1984). In a discrete-time implementation of the update (9), time derivatives are replaced by finite increments and α is chosen in relation with the sampling time. When the Jacobian has full rank, a convenient alternative to (9) is the *Reduced Gradient* (RG) method proposed by De Luca and Oriolo (1990a). At each instant, the joint vector is partitioned as $(\mathbf{q}_a, \mathbf{q}_b)$, with $\mathbf{q}_a \in I\!R^m$ and $\mathbf{q}_b \in I\!R^{n-m}$, in such a way that

(1) By exact we mean locally exact, not necessarily global.

$\mathbf{J}_a = \partial \mathbf{f}/\partial \mathbf{q}_a$ is nonsingular. Correspondingly, the joint velocity is computed as

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{\mathbf{q}}_a \\ \dot{\mathbf{q}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{J}_a^{-1} \\ \mathbf{0} \end{bmatrix} \dot{\mathbf{p}} - \alpha \begin{bmatrix} \mathbf{J}_R \mathbf{J}_R^T & -\mathbf{J}_R \\ -\mathbf{J}_R^T & \mathbf{I} \end{bmatrix} \nabla_{\mathbf{q}} H \big|_{\mathbf{q}_0}, \tag{10}$$

where $\mathbf{J}_R = \mathbf{J}_a^{-1}\mathbf{J}_b$. The two methods PG and RG provide different updates, and in general the latter is more efficient in approaching the local optimum.

In table 'slot' (1.B), we refer to the possibility of obtaining an *exact* solution to the constrained optimization of a function $H(\mathbf{q})$ at each joint configuration satisfying (1). Assume that the initial configuration $\mathbf{q}(0)$ is optimal, i.e. $(-\mathbf{J}_R^T \;\; \mathbf{I})\nabla_{\mathbf{q}} H = \mathbf{0}$ at $\mathbf{q}(0)$ (Chang, 1987). Imposing the following condition will propagate optimality throughout arm motion:

$$\frac{d}{dt}[(-\mathbf{J}_R^T \;\; \mathbf{I})\nabla_{\mathbf{q}} H] = \mathbf{G}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}. \tag{11}$$

This constraint, used to 'square' system (2), yields the *Extended Jacobian* method (Baillieul, 1985)

$$\dot{\mathbf{q}} = \begin{bmatrix} \mathbf{J}(\mathbf{q}) \\ \mathbf{G}(\mathbf{q}) \end{bmatrix}^{-1} \begin{bmatrix} \dot{\mathbf{p}} \\ \mathbf{0} \end{bmatrix}, \tag{12}$$

provided that no *algorithmic singularities* are encountered. The requirement of starting (12) from a posture where the criterion is maximized ensures that motion will be *cyclic* in the joint space.

Objective functions depending on $\dot{\mathbf{q}}$ can be exactly handled with a first-order scheme only when joint velocities appear quadratically (case (1.C)). The current $\mathbf{q}_0$, which may appear in $H(\mathbf{q}_0, \dot{\mathbf{q}})$, shapes the criterion in dependence of the arm configuration. However, the major instance of this class is the minimization of the squared velocity norm $H(\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\dot{\mathbf{q}}$, achieved by the simple Jacobian pseudoinverse solution (Whitney, 1972)

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}\dot{\mathbf{p}}. \tag{13}$$

This is a special case of (4) with no null-space vector, or $\mathbf{v} = \mathbf{0}$. The general relationship between the exact optimization of a *complete* quadratic function (with weighted quadratic term and a linear term in $\dot{\mathbf{q}}$) and (4) will be clarified in the next section. First-order approximate methods for an objective $H(\mathbf{q}, \dot{\mathbf{q}})$ (not reported in Tab. 1) will directly follow from the same analysis.

Second-order methods

The taxonomy of second-order redundancy resolution methods is a transposition of the first-order one. Case (2.A) refers to the iterative optimization of a configuration and velocity-dependent objective function $H(\mathbf{q}, \dot{\mathbf{q}})$, performed at the acceleration level. Surprisingly enough, no method has yet been proposed for such case. A reason for this stands in the mathematical difficulty of optimizing a function of $2n$ differentially related variables using only the n-dimensional vector of joint accelerations. The appropriate framework for this problem is the Calculus of Variations, which applies however only to a globally defined criterion (Martin *et al.*, 1989). Instead, a practical way is to use a simple expansion accounting for the effect of acceleration on the next attained position and velocity. Based on a discrete-time approach, a new optimization algorithm will be developed for this case later on.

No existing methods fall into class (2.B) either, but an extension of the Extended Jacobian approach (12) is straightforward. Assume that the initial velocity $\dot{\mathbf{q}}(0)$ extremizes $H(\mathbf{q}_0, \dot{\mathbf{q}})$, as expressed by the $n - m$ conditions $(-\mathbf{J}_R^T \;\; \mathbf{I})\nabla_{\dot{\mathbf{q}}} H|_{\dot{\mathbf{q}}(0)} = \mathbf{0}$. Optimality will be preserved if the acceleration is such that

$$\frac{d}{dt}[(-\mathbf{J}_R^T \;\; \mathbf{I})\nabla_{\dot{\mathbf{q}}} H] = \mathbf{K}(\mathbf{q}, \dot{\mathbf{q}})\ddot{\mathbf{q}} - \mathbf{s}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}. \tag{14}$$

Using this constraint to 'square' system (3), one can define a *second-order Extended Jacobian* method

$$\ddot{\mathbf{q}} = \begin{bmatrix} \mathbf{J}(\mathbf{q}) \\ \mathbf{K}(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix}^{-1} \begin{bmatrix} \ddot{\mathbf{r}} \\ \mathbf{s} \end{bmatrix}. \tag{15}$$

In contrast with (12), this scheme does not guarantee that cyclic task trajectories map into cyclic joint trajectories. In fact, constraint (14) is not twice integrable in general, i.e. it is *nonholonomic*.

Proposition 1 can be used to state the equivalence between first-order methods (1.C) and second-order methods (2.B) in the case of objective functions quadratic in $\dot{\mathbf{q}}$. For example, consider again $H(\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\dot{\mathbf{q}}$, which is optimized at velocity level by (13). On the other hand, (14) can be equivalently expressed now as

$$\frac{d}{dt}[(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\dot{\mathbf{q}}] = \mathbf{0}, \tag{16}$$

	A. Iterative methods	B. Exact methods	C. Exact methods
1. First-order schemes *Find $\dot{\mathbf{q}}$, given $\mathbf{q}_0$*	$H(\mathbf{q})$	$H(\mathbf{q})$	$H(\mathbf{q}_0, \dot{\mathbf{q}})$, *quadratic in* $\dot{\mathbf{q}}$
2. Second-order schemes *Find $\ddot{\mathbf{q}}$, given $\mathbf{q}_0$, $\dot{\mathbf{q}}_0$*	$H(\mathbf{q}, \dot{\mathbf{q}})$	$H(\mathbf{q}_0, \dot{\mathbf{q}})$	$H(\mathbf{q}_0, \dot{\mathbf{q}}_0, \ddot{\mathbf{q}})$, *quadratic in* $\ddot{\mathbf{q}}$

Tab. 1 – Classification of resolution methods resulting in

$$(\mathbf{I}-\mathbf{J}^{\dagger}\mathbf{J})\ddot{\mathbf{q}} = (\mathbf{J}^{\dagger}\dot{\mathbf{J}} + \dot{\mathbf{J}}^{\dagger}\mathbf{J})\dot{\mathbf{q}}. \qquad (17)$$

Using the idempotency of matrix $(\mathbf{I}-\mathbf{J}^{\dagger}\mathbf{J})$, and noting that for any $\ddot{\mathbf{q}}$ there exists a vector $\mathbf{a}$ such that $(\mathbf{I}-\mathbf{J}^{\dagger}\mathbf{J})\ddot{\mathbf{q}} = (\mathbf{I}-\mathbf{J}^{\dagger}\mathbf{J})\mathbf{a}$, the equivalence is shown by applying (6) with $\mathbf{v} = \mathbf{0}$. As a result, this second-order method will be cyclic both in position and velocity only if the pseudoinverse solution (13) is cyclic itself.

Finally, case (B.3) is similar to (A.3), but approaching the problem at the acceleration level gives more flexibility in the definition of objective functions. To achieve good dynamic performance along the task trajectory, the robot dynamic model is introduced

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{u}. \qquad (18)$$

When the squared norm of the joint torque $\frac{1}{2}\mathbf{u}^T\mathbf{u}$ is considered, the objective function remains quadratic and positive definite in terms of the joint acceleration $\ddot{\mathbf{q}}$, being the inertia matrix $\mathbf{B} > 0$. However, local minimization of pure joint torque may lead to unstable 'whipping' phenomena in the joint space (Hollerbach and Suh, 1987). Among the schemes that counterbalance this effect, we mention the square inverse inertia weighted method of Nedungadi and Kazerounian (1989) and the addition of a stabilizing velocity-acceleration term proposed in (De Luca and Oriolo, 1990b). In the latter, the product $\dot{\mathbf{q}}_0\ddot{\mathbf{q}}$ (linear in acceleration) 'completes' the quadratic objective function.

Optimization of a Complete Quadratic Form

In some of the examined redundancy resolution schemes a major role is played by the problem of minimizing a complete quadratic objective function

$$\min_{\mathbf{x}\in\mathbb{R}^n} H(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{W}\mathbf{x} - \mathbf{b}^T\mathbf{x} + c, \qquad (19)$$

with $\mathbf{W}>0$, subject to the linear constraint

$$\mathbf{J}\,\mathbf{x} = \mathbf{y}, \quad \mathbf{y} \in \mathbb{R}^m. \qquad (20)$$

with full rank $\mathbf{J}$ (a constraint qualification condition). Equation (20) typically results from the linearization of a nonlinear constraint (e.g. (1)). The effect of the linear term in (19) is to 'bias' the unconstrained minimum at $\mathbf{x}^* = \mathbf{W}^{-1}\mathbf{b}$.

Defining the Lagrangian as

$$\mathcal{L}(\mathbf{x},\lambda) = H(\mathbf{x}) + \lambda^T(\mathbf{J}\mathbf{x} - \mathbf{y}), \quad \lambda \in \mathbb{R}^m, \qquad (21)$$

the necessary and sufficient conditions for optimality are

$$\begin{aligned}\nabla_{\mathbf{x}}\mathcal{L} &= \mathbf{W}\mathbf{x} - \mathbf{b} + \mathbf{J}^T\lambda = \mathbf{0},\\ \nabla_{\lambda}\mathcal{L} &= \mathbf{J}\mathbf{x} - \mathbf{y} = \mathbf{0},\end{aligned} \qquad (22)$$

from which

$$\begin{aligned}\mathbf{x} &= \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1}(\mathbf{y} - \mathbf{J}\mathbf{W}^{-1}\mathbf{b}) + \mathbf{W}^{-1}\mathbf{b}\\ &= \mathbf{J}^{\dagger}_{\mathbf{W}}\mathbf{y} + (\mathbf{I} - \mathbf{J}^{\dagger}_{\mathbf{W}}\mathbf{J})\mathbf{W}^{-1}\mathbf{b},\end{aligned} \qquad (23)$$

where the definition of weighted pseudoinverse has been used.

Customizing $\mathbf{x}, \mathbf{y}, \mathbf{W}$, and $\mathbf{b}$, one can recover (4) or (5). Indeed, the solution of any quadratic case in Tab. 1 naturally fits in the general form (23). One interesting outcome of the above analysis is that even the approximate update (9), which iteratively minimizes $H(\mathbf{q})$, can be reinterpreted as the *exact* solution minimizing the *complete* quadratic function

$$H(\mathbf{q}_0,\dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\dot{\mathbf{q}} + \alpha\nabla_{\mathbf{q}}H\big|_{\mathbf{q}_0}^T\dot{\mathbf{q}}, \qquad (24)$$

by setting $\mathbf{x}=\dot{\mathbf{q}}$, $\mathbf{y}=\dot{\mathbf{p}}$, $\mathbf{W}=\mathbf{I}$, and $\mathbf{b}=-\alpha\nabla_{\mathbf{q}}H\big|_{\mathbf{q}_0}$ in (23). Therefore, the joint velocity norm is embedded in the PG method as a 'regularizing' term. The relative importance of the two components in (24) is assessed by the choice of α. It follows that only the Extended Jacobian method is available for the exact optimization of a 'pure' configuration dependent objective $H(\mathbf{q})$.

Finally, note that any exact method for quadratic optimization may use the idea of 'reduction' to the space of redundant degrees of freedom, as opposed to 'projection' of vectors in the Jacobian null space, for designing a more efficient algorithm to compute the same solution (De Luca and Oriolo, 1990b).

A Discrete Time Algorithm

In this section, we consider an iterative acceleration scheme for the minimization of a general criterion of the form

$$H(\mathbf{q},\dot{\mathbf{q}}) = \frac{k_1}{2}\dot{\mathbf{q}}^T\dot{\mathbf{q}} + k_2L(\mathbf{q}), \quad k_1, k_2 \geq 0. \qquad (25)$$

When $k_1 = 0$, purely configuration dependent indices $L(\mathbf{q})$ are recovered, which may be also non-quadratic. With the arm in a state $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$, assume that the acceleration $\ddot{\mathbf{q}}$ is kept constant for a sampling time T. Thus, after T seconds the kinematic state of the arm will be

$$\begin{aligned}\mathbf{q} &= \mathbf{q}_0 + \dot{\mathbf{q}}_0T + \frac{1}{2}\ddot{\mathbf{q}}T^2,\\ \dot{\mathbf{q}} &= \dot{\mathbf{q}}_0 + \ddot{\mathbf{q}}T.\end{aligned} \qquad (26)$$

Expanding $L(\mathbf{q})$ around $\mathbf{q}_0$ up to the second order results in

$$\begin{aligned}L(\mathbf{q}) \simeq{}& L(\mathbf{q}_0) + \nabla_{\mathbf{q}}L\big|_{\mathbf{q}_0}^T(\mathbf{q} - \mathbf{q}_0)\\ &+ (\mathbf{q} - \mathbf{q}_0)^T\nabla^2_{\mathbf{q}}L\big|_{\mathbf{q}_0}(\mathbf{q} - \mathbf{q}_0),\end{aligned} \qquad (27)$$

where $\nabla^2_{\mathbf{q}} L$ is the Hessian matrix of $L(\mathbf{q})$. Substituting eqs. (26–27) in (25), the quadratic approximation H' of the criterion H can be expressed as a function of $\ddot{\mathbf{q}}$ only

$$H'(\ddot{\mathbf{q}}) = \frac{1}{2}\ddot{\mathbf{q}}^T \mathbf{W}\ddot{\mathbf{q}} + \mathbf{b}^T\ddot{\mathbf{q}} + c. \qquad (28)$$

where

$$\mathbf{W} = T^2(k_1\mathbf{I} + \frac{k_2}{4}T^2\nabla^2_{\mathbf{q}}L\big|_{\mathbf{q}_0}),$$
$$\mathbf{b} = T(\frac{k_2}{2}\nabla_{\mathbf{q}}L\big|_{\mathbf{q}_0} + (k_1\mathbf{I} + \frac{k_2}{2}T^2\nabla^2_{\mathbf{q}}L\big|_{\mathbf{q}_0})\dot{\mathbf{q}}_0). \qquad (29)$$

The minimization of $H'(\ddot{\mathbf{q}})$ subject to the kinematic constraint (3), is a special case of problem (19–20). With the above definitions, (23) gives

$$\ddot{\mathbf{q}} = \mathbf{J}^{\dagger}_{\mathbf{W}}\ddot{\mathbf{r}} - (\mathbf{I} - \mathbf{J}^{\dagger}_{\mathbf{W}}\mathbf{J})\mathbf{W}^{-1}\mathbf{b}. \qquad (30)$$

This solution is well-defined only when matrix $\mathbf{W}$ is positive definite; the 'damping' factor k_1 has to be chosen so to satisfy this condition. Since the proposed acceleration scheme includes second-order terms in the Taylor expansion (27), the solution uses the curvature information represented by the Hessian, possibly resulting in a faster convergence when close to optimality.

A simplified version of the algorithm may be devised, neglecting second-order terms in (27). The following solution is obtained (assuming $k_1 \neq 0$)

$$\ddot{\mathbf{q}} = \mathbf{J}^{\dagger}\ddot{\mathbf{r}} - \frac{1}{T}(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})(\dot{\mathbf{q}}_0 + \frac{k_2}{2k_1}T\nabla_{\mathbf{q}}L\big|_{\mathbf{q}_0}). \qquad (31)$$

The null-space acceleration vector is here a linear combination of the two gradients of H w.r.t. $\mathbf{q}$ and $\dot{\mathbf{q}}$. Note that this form requires less computations than differentiating a first-order solution, e.g. (9).

Simulation Results

The discrete-time (DT) algorithm (31) has been compared to the PG method (9) for resolving redundancy in a planar arm with three rotating joints and equal unitary link lengths. The chosen common objective is to maximize manipulability during a motion starting from the initial configuration

$$\mathbf{q}(0) = (-5°, -5°, 170°). \qquad (32)$$

which is close to singularity. The end-effector is required to trace a circular path centered at the origin, at the constant angular speed $\omega = 180°/\text{sec}$. Simulation time is 1 second, with a sampling time $T = 0.01$ seconds. Using absolute joint angles q_i (w.r.t. the x-axis), the Jacobian of this robot is

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} -\sin q_1 & -\sin q_2 & -\sin q_3 \\ \cos q_1 & \cos q_2 & \cos q_3 \end{bmatrix}, \qquad (33)$$

while a convenient manipulability measure is

$$H(\mathbf{q}) = \sin^2(q_2 - q_1) + \sin^2(q_3 - q_2). \qquad (34)$$

Figures 1–3 refer to the first-order PG method with $\alpha = 0.1$, showing the arm motion, the norm of the joint velocity, and the value of the criterion. Note that large joint velocities result, especially to get out of the singularity. A larger α leads in this case to a jerky behavior. For the second-order discrete-time method, the arm has the same initial joint velocity obtained with the PG method

$$\dot{\mathbf{q}}(0) = (179.5°, 180.5°, 180°)/\text{sec}. \qquad (35)$$

Using the coefficients $k_1 = 1$ and $k_2 = 1000$ in (31) provides a motion very similar to Fig. 1. However, joint velocities are significantly reduced (Fig. 4) at practically no expense of performance, as shown by the criterion in Fig. 5.

Conclusions

Optimal first and second-order methods for using the redundant degrees of freedom of a robotic arm have been analyzed, pointing out the relationships among existing techniques and classifying them into exact or iterative.

Three new results were presented.

1) Under the full rank assumption for the robot Jacobian, a necessary and sufficient condition was given for the equivalence between a velocity and an acceleration solution method.

2) A second-order version of the Extended Jacobian technique was introduced for the exact optimization of objective functions depending on joint velocity. Optimality of the initial state of the robot arm is required in this case.

3) An iterative algorithm has been proposed for optimizing at the acceleration level a criterion depending on joint position and velocity, based on a discrete-time approach.

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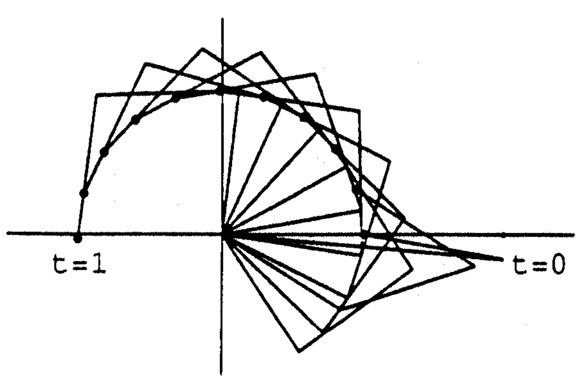

Fig 1 – Arm motion

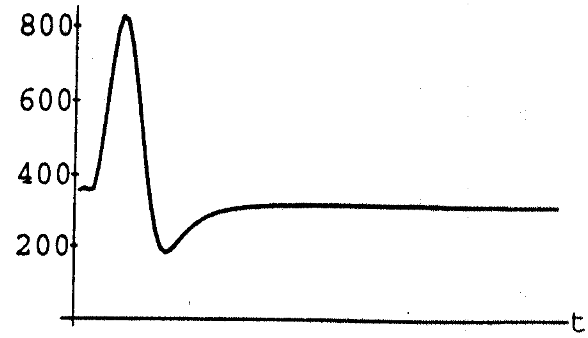

Fig 2 – Joint velocity norm with the PG method

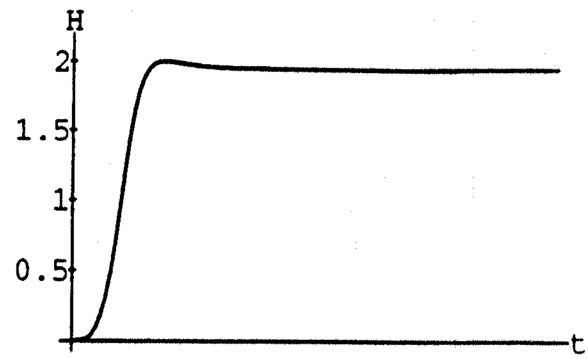

Fig 3 – Manipulability with the PG method

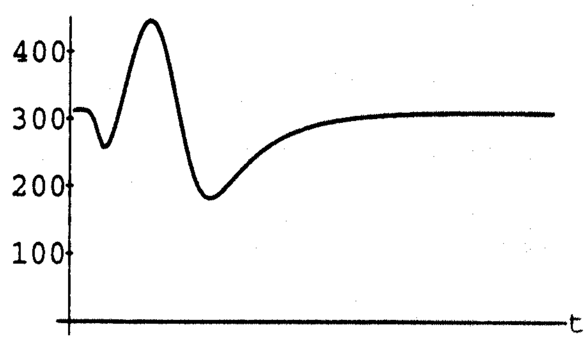

Fig 4 – Joint velocity norm with the DT method

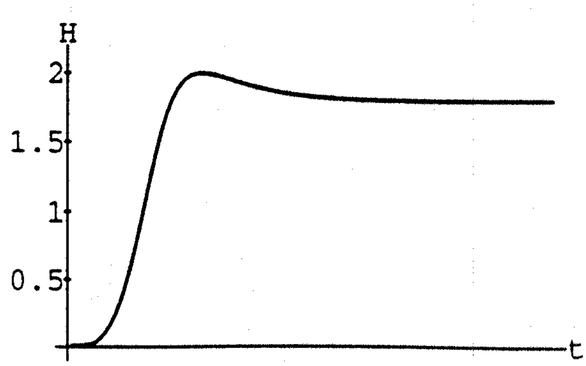

Fig 5 – Manipulability with the DT method