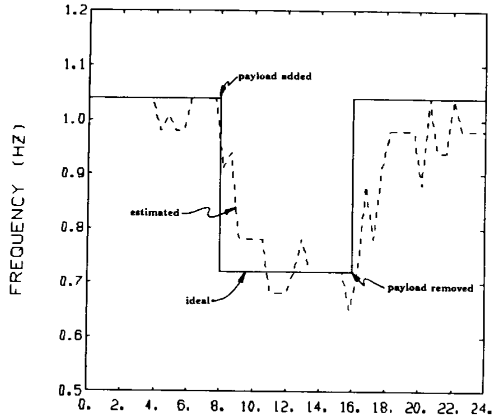

Fig. 5. First modal frequency as detected by TTFE.

and the variation of the frequency where these peaks occur admits a direct functional relationship with the payload carried by the robot. Results for this example system have shown that, practically speaking, any class of desired tracking trajectory (that is, aside from an ideal pure sinusoid which excites only one frequency component) with reasonable amplitude is sufficient to assure an accurate estimate of the first bending frequency; this was illustrated with the use of a very small slew (5°) in the reported experimental results. The gain scheduling controller based on the estimated frequency of the first flexible mode proved to be efficient in terms of computing effort to tune the controller, and compensated satisfactorily for a varying payload. The scheduling scheme also guarantees that the controller used is stable since all possible values for controller parameter combinations are known *a priori* to produce a stable response.

Use of acceleration signals for identification and control has proven to be successful in general flexible structure applications, and specifically for flexible manipulator identification and control in one- and two-link robots at Ohio State. The problem of computation delay experienced in rigid robot applications is much less severe in the case of flexible-link robots due to slower system response. That is, the relatively low modal frequencies (1–2 s periods) dominate the time required in the algebraic loop. Moreover, delay due to mechanical wave propagation is easily accounted for in the identified z-domain transfer function.

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Design of an Exact Nonlinear Controller for Induction Motors

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***Abstract*—A novel approach to the control of induction motors is presented, based on differential-geometric concepts for the control of nonlinear systems. Structural properties of the model are pointed out, and a proper selection of physically meaningful system outputs is indicated which yields, via static state-feedback, exact state linearization and input–output decoupling of the closed-loop system. The proposed approach is used to design a controller for motor torque and flux. Simulation tests are included.**

I. Introduction

The design of a new high-performance controller for drives using an induction motor as an actuator will be considered here. The relevance of this control problem is witnessed by the large number of investigations carried out both from a theoretical and a practical point of view [1]. A major aspect is how to cope with the nonlinear and coupled dynamic behavior of the machine. Typically, the state equations of an induction motor are bilinear [2], while a natural output variable to be set under control, the motor torque, is a nonlinear function of the state.

The control system has to be designed for driving the motor torque along a given profile while keeping limited, even during fast transients, both the machine flux and the current sinked from the inverter supply. In fact, when the modulus of the machine flux exceeds some threshold value, the motor operates in an improper way. On the other hand, the inverter cannot source a current value which is higher than its rated one, even for short time intervals.

In many of the proposed approaches to this problem, the motor flux and the torque are controlled separately. One of the most effective schemes, known as field-oriented (or vector) control [3]–[5], works with a controller which approximately linearizes and decouples the relation between input and output variables. This is obtained by means of a proper selection of state coordinates, under the simplifying hypothesis that the actual machine flux is kept constant and equal to some desired value.

In this paper, the use of control techniques based on the differential-geometric approach is proven to be effective for the design of a full linearizing state-feedback controller for induction machines. The motivation here is twofold. First, this approach leads to a control solution for the induction motor which is exact in the sense that simplifying hypotheses are not taken into account *a priori*. Second, embedding the problem in a general framework allows a systematic investigation of the structural properties of the model and provides a solid theoretical background to other effective methods.

General results on feedback linearization and input–output decoupling are available for the class of smooth nonlinear affine systems [6]–[7]. A comprehensive introduction to these results can be found in [8]. The relative control techniques are quite mature and have already found useful application in different areas [9], [10]. In particular, interesting results have recently appeared in the field of switched reluctance motor control [11].

The nonlinear dynamic model of the motor is introduced in Section II. Section III briefly outlines the design methodology. Section IV contains the main results: the relevant control properties of the dynamic model of the induction machine are pointed out and the synthesis of an exact linearizing and decoupling state-feedback controller is described in detail. In the last two sections, simulation tests are reported and discussed.

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II. Dynamic Model of the Induction Machine

Based on the two-phase equivalent machine representation and under the standard simplifying modeling hypotheses, the dynamic behavior of a voltage-fed induction motor is described by a set of four nonlinear differential equations [1]. As state variables, it is convenient to use the projections of the stator current and stator flux vectors on a (d, q)-axes reference frame rotating at the same speed of the desired stator flux vector. The i_{ds} and i_{qs} components of the stator current can be obtained on the basis of direct measures, while the flux components φ_{ds} and φ_{qs} can be reconstructed very efficiently be means of a bilinear observer, as shown in [12], [13]. Thus, full state availability can be assumed in the controller design. The projections v_{ds} and v_{qs} of the supply voltage and the slip frequency ω_s, i.e., the difference between the desired angular speed of the flux and the mechanical speed ω, are taken as input variables. The speed ω is a slowly varying parameter, the mechanical dynamics being much slower than the electromagnetic one.

Therefore, setting $x = [i_{ds}, i_{qs}, \varphi_{ds}, \varphi_{qs}]^T$ and $u = [v_{ds}, v_{qs}, \omega_s]^T$, the dynamic equations describing the motor behavior are bilinear and can be written as

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where

$$f(x) = Ax = \begin{bmatrix} -(\alpha+\beta) & 0 & \frac{\beta}{\Lambda_s} & \frac{\omega}{\sigma\Lambda_s} \\ 0 & -(\alpha+\beta) & -\frac{\omega}{\sigma\Lambda_s} & \frac{\beta}{\Lambda_s} \\ -\alpha\sigma\Lambda_s & 0 & 0 & \omega \\ 0 & -\alpha\sigma\Lambda_s & -\omega & 0 \end{bmatrix} x$$

$$g(x) = \begin{bmatrix} 1/\alpha\Lambda_s & 0 & x_2 \\ 0 & 1/\sigma\Lambda_s & -x_1 \\ 1 & 0 & x_4 \\ 0 & 1 & -x_3 \end{bmatrix} = [g_1\, g_2\, g_3(x)]$$

and $\alpha = R_s/(\sigma\Lambda_s)$, $\beta = R_r/(\sigma\Lambda_r)$, $\sigma = 1-(M^2/\Lambda_s\Lambda_r)$. The parameters R_s and R_r are the stator and rotor resistances, Λ_s and Λ_r are the stator and rotor self-inductances, and M is the mutual inductance.

The rotor flux components are linear combinations of the chosen states

$$\varphi_{dr} = (\varphi_{ds} - \sigma\Lambda_s i_{ds})\Lambda_r/M, \qquad \varphi_{qr} = (\varphi_{qs} - \sigma\Lambda_s i_{qs})\Lambda_r/M,$$

while the torque produced by the machine takes the form

$$T_m = \varphi_{ds} i_{qs} - \varphi_{qs} i_{ds}$$

when a motor with one pole pair is considered.

III. Nonlinear Control Design Technique

Consider a square multiinput multioutput nonlinear plant to be controlled:

$$\dot{x} = f(x) + g(x)u = f(x) + \sum_{i=1}^{m} g_i(x)u_i \tag{2}$$

$$y = h(x) = [h_1(x)\cdots h_m(x)]^T \tag{3}$$

where $x \in \mathbb{R}^n$, f and each of the g_i are smooth vector fields, and each output h_j is a smooth function. A common approach is to look for transformations which map this nonlinear control problem into a linear one. In particular, one may try to find a state-space transformation $z = T(x)$ and a state-feedback law of the form

$$u = a(x) + b(x)v \tag{4}$$

with $b(x)$ nonsingular such that the obtained closed-loop system shows a linear behavior in z (feedback linearization), or an input–output linear and decoupled behavior (noninteraction), or both of them (full linearization). Conditions for the solution of the above problems are known, based on the differential-geometric approach [6]–[9]. The sufficient part of these conditions is constructive and leads to a synthesis procedure for the control law.

As in many applications, the outputs (3) for the induction motor system are not completely specified from the beginning. Meaningful outputs can be defined in several ways, related to the control goals. The question that arises is how to select proper output functions which meet the overall performance objectives *and* enable us to solve the above control synthesis problems. The design approach taken here is summarized in four basic steps:

1) check the conditions for feedback linearization of the state equations,
2) derive the set of partial differential equations (PDE's) needed for the construction of the linearizing transformation $T(x)$, and find solutions which can be assumed as system outputs,
3) if needed, complete the set of outputs by choosing other suitable smooth functions of the state, based on the design objectives,
4) compute the control law in the form (4) by applying the decoupling technique [8] to the system with the above outputs.

The successful completion of these steps leads to the design of a static state-feedback controller which yields full linearization in the closed-loop. A similar philosophy is pursued from a theoretical point of view in [14] for the decoupling problem.

In the following, $L_f h = dh \cdot f$ will denote the Lie derivative of a function h w.r.t. the vector field f, with $L_f^k h = L_f(L_f^{k-1}h)$ and $L_f^0 h = h$. The Lie bracket of two vector fields f and g is defined in local coordinates as $[f, g](x) = (\partial g/\partial x \cdot f - \partial f/\partial x \cdot g)(x)$. A set of vector fields $\{f_i, i = 1, \cdots, k\}$ defines pointwise a distribution Δ as $\Delta(x) = \text{span}\,\{f_i(x), i = 1, \cdots, k\}$; its dimension at x is the dimension of $\Delta(x)$. A distribution is involutive if it is closed under the Lie bracket operation.

IV. Control Design for the Induction Motor

1. Feedback Linearizability

The first step of the design procedure requires us to verify the feedback linearizability of the induction motor state equations (1). Since the state and input dimensions are $n = 4$ and $m = 3$, the general necessary and sufficient conditions for feedback linearization in some (open and dense) region $V \subset \mathbb{R}^4$ [6], [8] particularize in the following conditions on the distributions M_0 and M_1:

(L1) $M_0 = sp\{g_i, i = 1, 2, 3\}$ is involutive and of constant dimension in V,

(L2) $M_1 = sp\{g_i, [f, g_i], i = 1, 2, 3\}$ is of dimension 4 in V.

From $[g_1, g_2](x) = 0$, $[g_1, g_3](x) = -g_2$, $[g_2, g_3](x) = g_1$, involutivity of M_0 follows. Moreover, (L1) holds since

$$\dim M_0(x) = 3, \qquad \forall x \in V \triangleq \{x \in \mathbb{R}^4 | x_1 \neq x_3/\sigma\Lambda_s \text{ or } x_2 \neq x_4/\sigma\Lambda_s\}.$$

Next, from

$$[f, g_1](x) = -A \cdot g_1 = \left[\frac{\alpha + \beta(1-\sigma)}{\sigma\Lambda_s} \;\; \frac{\omega}{\sigma\Lambda_s} \;\; \alpha\omega\right]^T$$

$$[f, g_2](x) = -A \cdot g_2 = \left[-\frac{\omega}{\sigma\Lambda_s} \;\; \frac{\alpha + \beta(1-\sigma)}{\sigma\Lambda_s} \;\; -\omega\alpha\right]^T$$

$$[f, g_3](x) = 0$$

it is possible to show that $\dim M_1(x) = 4$ in V, so that also (L2) holds. The controllability indexes κ_i, $i = 1, 2, 3$ associated with system (1) are computed from the set of integers

$$r_0 \triangleq \dim M_0(x) = 3$$

$$r_1 \triangleq \dim M_1(x) - \dim M_0(x) = 1$$

$$r_k \triangleq \dim M_k(x) - \dim M_{k-1}(x) = 0, \qquad \forall k \geq 2$$

using the definition $\kappa_i \triangleq \#\{r_j | r_j \geq i, j = 0, 1\cdots\}$ [6]. This yields

$$\kappa_1 = 2, \kappa_2 = 1, \kappa_3 = 1.$$

Therefore, in the specified region V of the state space, the induction motor is feedback equivalent to the following linear system in Brunovski canonical form:

$$\dot{z} = \begin{bmatrix} 0 & 1 & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & 0 \end{bmatrix} z + \begin{bmatrix} 0 & \cdot & \cdot \\ 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} v = Az + Bv. \tag{5}$$

2. PDE's Derivation and Solution

The canonical form (5) is obtained in the closed-loop once the proper coordinates $z = T(x)$ are chosen. In order to build the transformation $T(x)$, three smooth functions $h_i(x)$, $i = 1, 2, 3$ have to be found [6] such that in V

$$L_{g_j} L_f^k h_i(x) = 0, \qquad j = 1, 2, 3, \qquad 0 \le k \le \kappa_i - 2, \qquad i = 1, 2, 3.$$

In general, this is a set of $m \times (n - m)$ partial differential equations which have to be solved. Solutions are denoted by $h_i(x)$ in view of the fact that these functions will be used as system outputs. Since $\kappa_1 = 2$, $h_1(x)$ has to verify for all $x \in V$

$$L_{g_j} h_1(x) = 0, \qquad j = 1, 2, 3. \tag{6}$$

From $\kappa_2 = \kappa_3 = 1$, no conditions are imposed at this point on $h_2(x)$ and $h_3(x)$. Conditions (6) develop into the set of partial differential equations:

$$\frac{\partial h_1}{\partial x_1} + \sigma \Lambda_s \frac{\partial h_1}{\partial x_3} = 0$$

$$\frac{\partial h_1}{\partial x_2} + \sigma \Lambda_s \frac{\partial h_1}{\partial x_4} = 0$$

$$x_2 \frac{\partial h_1}{\partial x_1} - x_1 \frac{\partial h_1}{\partial x_2} + x_4 \frac{\partial h_1}{\partial x_3} - x_3 \frac{\partial h_1}{\partial x_4} = 0.$$

The following equation is derived from the previous system:

$$(x_4 - \sigma \Lambda_s x_2) \frac{\partial h_1}{\partial x_3} - (x_3 - \sigma \Lambda_s x_1) \frac{\partial h_1}{\partial x_4} = 0$$

where at least one of the two x-dependent coefficients is nonzero in V. In the subregion of V where both coefficients are nonzero, the system of PDE's implies that h_1 has to depend explicitly on all four components of the state. It is easy to see that a feasible solution is

$$h_1(x) = \frac{1}{2}[(x_3 - \sigma \Lambda_s x_1)^2 + (x_4 - \sigma \Lambda_s x_2)^2]. \tag{7}$$

This function is proportional to the squared norm of the rotor flux $\Phi_r^2(x)$. Use of (7) as one of the system output helps ensure correct motor operation. In fact, in order to keep the total motor flux limited, one may control either only the stator or only the rotor flux due to the tight coupling between the two windings.

3. Selection of the Other Outputs

To solve the problem of full linearization, the set of outputs has to be completed by choosing $h_2(x)$ and $h_3(x)$ so that the matrix

$$\begin{bmatrix} L_g L_f^{\kappa_1 - 1} h_1(x) \\ L_g L_f^{\kappa_2 - 1} h_2(x) \\ L_g L_f^{\kappa_3 - 1} h_3(x) \end{bmatrix} = \begin{bmatrix} L_g L_f h_1(x) \\ L_g h_2(x) \\ L_g h_3(x) \end{bmatrix} \tag{8}$$

is nonsingular in a neighborhood of a given x_0 chosen in V. Note that this is always possible, although the choice is not unique [8, p. 242]. Depending on the specific $h_2(x)$ and $h_3(x)$ used, further singularities could be introduced in the control scheme since the above matrix may not have full rank globally on V.

It may seem that the requirements expressed by (6) and (8) impose unnecessary restrictions on the design alternatives. Indeed, these mathematical conditions reveal the presence of physical constraints which are inherent to the motor, thus providing a formal basis to intuition. As an example, a simple and appealing strategy would be to use the torque $T_m(x) = x_2 x_3 - x_1 x_4$, the squared norm of the stator current $I_s^2(x) = x_1^2 + x_2^2$, and of the stator flux $\Phi_s^2(x) = x_3^2 + x_4^2$ as system outputs. However, these output functions would violate both (6) and the nonsingularity of (8). Also, condition (8) would not be fulfilled even if any of these quantities is replaced by $\Phi_r^2(x)$. The reason for this is that the torque is the vector product of current and flux, while rotor flux, stator flux, and stator current are linearly dependent terms. Therefore, it is impossible to assign arbitrary and independent values to these quantities.

The following choice, together with (7), leads instead to a satisfactory design:

$$h_2(x) = T_m(x), \qquad h_3(x) = x_4. \tag{9}$$

These two functions are such to make matrix (8) nonsingular. The motor torque represents a natural control objective. Choosing the q component of the stator flux as output allows us to guarantee the proper orientation of the reference frame with respect to which all motor variables are expressed. Note that if this output is forced to be zero, part of the controller implementation could be simplified by formally replacing $x_4 = 0$ into the equations.

4. Synthesis of the Controller

Using the above outputs, a fully linearizing state-feedback law is derived by means of the decoupling algorithm [8]. For the application of an input-output decoupling law, the decoupling matrix $A(x)$ is required to be nonsingular. The entries of this $m \times m$ matrix are

$$a_{ij}(x) = L_{g_j} L_f^{r_i - 1} h_i(x)$$

where r_i is the relative degree of the ith output at x_0. By construction, $A(x)$ will have the form (8). In fact, the computed outputs have the set of relative degrees equal to the set of the system controllability indexes [14]. Since

$$\det A(x) = x_3 \beta(\sigma - 1)[(x_4 - \sigma \Lambda_s x_2)^2 + (x_3 - \sigma \Lambda_s x_1)^2]/\sigma \Delta_s$$

and $\beta \neq 0$, $\sigma \neq 1$, $A(x)$ is nonsingular if and only if $x_3 \neq 0$ (nonzero d component of the stator flux) and $h_1(x) \neq 0$ (nonzero rotor flux). These inequalities define the region of validity of the proposed controller. During normal motor operation, these singularities never come into play.

The nonlinear state-feedback control $u = a(x) + b(x)v$ is defined by

$$b(x) = A^{-1}(x) = \begin{bmatrix} \frac{(x_3 - \sigma \Lambda_s x_1)}{2\beta(1-\sigma) h_1(x)} & -\frac{\sigma \Lambda_s (x_4 - \sigma \Lambda_s x_2)}{2 h_1(x)} & 0 \\ \frac{(x_4 - \sigma \Lambda_s x_2)}{2\beta(1-\sigma) h_1(x)} & \frac{\sigma \Lambda_s (x_3 - \sigma \Lambda_s x_1)}{2 h_1(x)} & 0 \\ \frac{(x_4 - \sigma \Lambda_s x_2)}{2 x_3 \beta(1-\sigma) h_1(x)} & \frac{\sigma \Lambda_s (x_3 - \sigma \Lambda_s x_1)}{2 x_3 h_1(x)} & -\frac{1}{x_3} \end{bmatrix}$$

$$a(x) = -A^{-1}(x) [L_f^2 h_1(x) \quad L_f h_2(x) \quad L_f h_3(x)]^T$$

with

$$L_f^2 h_1(x) = \beta\sigma \{\omega \Lambda_s (1 - \sigma) h_2(x) - \Lambda_s [(\alpha + \beta)(1 - \sigma) + 4\beta\sigma]$$
$$\cdot (x_1 x_3 + x_2 x_4) + \Lambda_s^2 [\sigma(\alpha + 2\beta) - \alpha\sigma^2](x_1^2 + x_2^2) + \beta(1 + \sigma)(x_3^2 + x_4^2)\}$$

$$L_f h_2(x) = -(\omega/\sigma \Lambda_s)(x_3^2 + x_4^2) - (\alpha + \beta) h_2(x) + \omega(x_1 x_3 + x_2 x_4) \tag{10}$$

$$L_f h_3(x) = -(\omega x_3 + \alpha\sigma \Lambda_s x_2).$$

Physically interesting terms can be recognized in (10), like the squared norm of the stator current $I_s^2(x)$ and of the flux $\Phi_s^2(x)$ and the scalar product $I_s(x) \cdot \Phi_s(x) = x_1 x_3 + x_2 x_4$. These quantities are independent of the orientation of the reference frame (d, q) and this may help in the implementation.

The resulting closed-loop system is input-output decoupled and linear in the coordinates $z = T(x)$ specified by

$$z_1 = h_1(x), \qquad z_2 = L_f h_1(x), \qquad z_3 = h_2(x), \qquad z_4 = h_3(x).$$

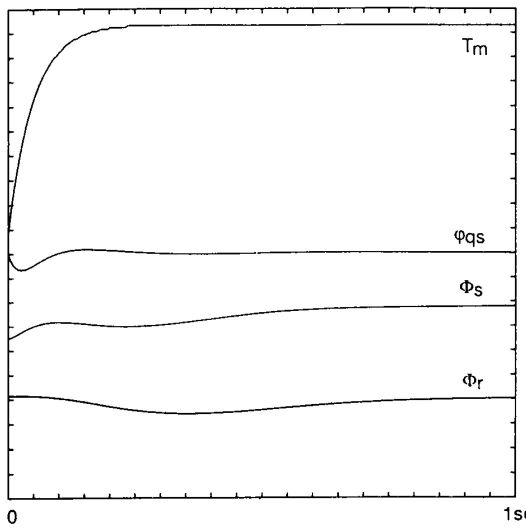

Fig. 1. Time evolution of the controlled outputs and of the stator flux. Φ_s, Φ_r: $6 \cdots 10$ V·s; T_m: $-1300 \cdots 1300$ Nm; φ_{qs}: $-0.05 \cdots 0.05$ V·s.

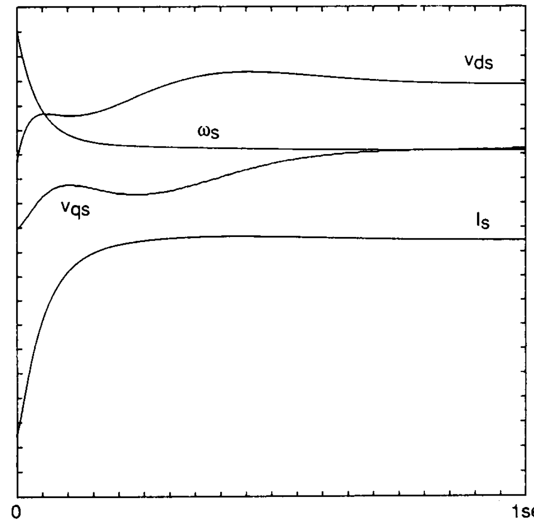

Fig. 2. Time evolution of the control inputs and of the stator current. v_{ds}: $-40 \cdots 40$ V; v_{qs}: $2000 \cdots 2500$ V; ω_s: $-12 \cdots 12$ rad/s; I_s: $0 \cdots 350$ A.

The control synthesis is then completed on the linear side (i.e., in the z coordinates) with the design of a stabilizing external input v.

The overall control computations can be organized so that 29 products and 19 sums are performed for each measurement of the state x and of ω [15]. A very conservative figure of the time needed using a simple digital signal processor is 100 μs, well within the control requirements.

V. Simulation Results

The proposed approach has been tested by simulation on a high-power induction motor with model parameters

$$\alpha = 27.232 \text{ s}, \qquad \beta = 17.697 \text{ s}, \qquad \sigma = 0.064, \qquad \Lambda_s = 0.179 \text{ H}.$$

The rated values of the stator and rotor fluxes are, respectively, equal to 7.3 and 6.88 V·s, while the maximum torque is 1000 Nm. The controller should smoothly regulate the motor torque at $T_{m,\text{des}}$, keep the squared norm of the rotor flux at its rated value $\Phi^2_{r,\text{des}}$, and align the d axis of the reference frame with the desired stator flux vector (i.e., constrain φ_{qs} to 0). Simulations were performed at the mechanical speed of 300 rad/s, close to the motor rated value, starting from a steady state and applying a step change in the desired torque from 100 to 1000 Nm. The results obtained using the nominal parameters show that a sampling time of 1 ms does not induce significant errors.

In an actual drive, the parameters of the machine are not exactly known. In particular, during the normal motor operation, the parameters α and β are affected by drifts as large as 10 and 50 percent of their nominal values, respectively. These maximum variations were used next to verify the robustness of the controller. The design of the external inputs v_i is based on a PID scheme (with $K_p = 235$, $K_i = 450$, $K_d = 22$) for the rotor flux and on a PI control ($K_p = 180$, $K_i = 900$) for the q component of the stator flux. Simple proportional control ($K_p = 50$) is chosen for the torque since the reference signal for this variable is generally imposed by an outer loop designed for the load speed.

Feedback from the full motor state was used in the simulations, without including an observer for the stator flux components. Actually, for the same motor and with the same variations of the parameters, the flux observer described in [12] has a rate of error convergence faster than 20 ms with a residual error $\Delta\Phi_s/\Phi_s \leq 0.001$.

The time evolution of the system variables in the case of a 50 percent higher value for β is shown in Figs. 1 and 2. The maximum errors on the relevant variable ($\Delta\Phi_r = 0.15$, $\Delta\Phi_s = 0.27$, $\Delta\varphi_{qs} = 0.003$, $\Delta T_m = 200$) are well within the acceptable ranges. Moreover, variables which are not directly controlled (i.e., I_s and Φ_s) show neither oscillations nor overshootings. Similar results were also obtained for variations in the parameter α, thus confirming the robustness properties.

VI. Conclusions

It has been shown that the differential-geometric approach can be effectively used for the design of a nonlinear controller for induction motors. The full state linearization and input-output decoupling problems were solved by means of a static nonlinear state feedback. Simulation tests indicate a very satisfactory behavior of the proposed controller and its robustness in the presence of the typical parameter uncertainties of the machine. Moreover, a digital signal processor implementation is feasible due to the sufficiently long sampling times allowed.

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