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ZMP Constraint Restriction for Robust Gait Generation in Humanoids

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Abstract— **We present an extension of our previously proposed IS-MPC method for humanoid gait generation aimed at obtaining robust performance in the presence of disturbances. The considered disturbance signals vary in a range of known amplitude around a mid-range value that can change at each sampling time, but whose current value is assumed to be available. The method consists in modifying the stability constraint that is at the core of IS-MPC by incorporating the current mid-range disturbance, and performing an appropriate restriction of the ZMP constraint in the control horizon on the basis of the range amplitude of the disturbance. We derive explicit conditions for recursive feasibility and internal stability of the IS-MPC method with constraint modification. Finally, we illustrate its superior performance with respect to the nominal version by performing dynamic simulations on the NAO robot.**

I. Introduction

Enabling humanoid robots to move in an unstructured and uncertain environment is a rather complex control problem. While interesting results have been achieved in humanoid locomotion, a great effort is still required to improve the robot's behavior with respect to uncertainties and disturbances.

Most techniques for humanoid gait generation are based on the Zero Moment Point (ZMP, the point on the ground for which the horizontal components of the contact moments become zero), which ensures dynamic balance if it is kept inside the robot's support polygon. The ZMP can be indirectly controlled by generating an appropriate Center of Mass (CoM) trajectory, which is then kinematically tracked. However, the dynamics relating CoM and ZMP are complex and simplified models are in general used instead. The most widespread is the Linear Inverted Pendulum (LIP) [1].

When constraints are taken into account, for example on the ZMP, the linear Model Predictive Control (MPC) formulation has proven to be very effective [2], [3], [4], [5], [6]. Nonlinear extensions have also been used successfully [7], [8]. In particular, interesting variations allowing CoM height variations have been presented [9], [10], [11], [12].

To avoid the divergence of the CoM with respect to the ZMP, we have introduced an explicit *stability constraint* in the MPC formulation [13], leading to the Intrinsically Stable MPC (IS-MPC). This constraint, based on the idea of boundedness [14], [15], [16], extends the classic terminal constraint used in MPC formulations for set-point control. It has been shown that, using some preview information, the IS-MPC ensures recursive feasibility (i.e., the ability to recursively guarantee a solution satisfying the constraints) and internal stability.

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In humanoid gait generation, robustness remains a critical issue due to model uncertainty and external disturbances [17], [18]. In a general MPC scheme, disturbances can cause constraint violations, and possibly lead to instability. If a bound on the disturbance is available, a possible solution is to restrict the constraints based on a robust positive invariant set [19], [20], [21], [22]. In order for this set to exist, the system needs to be stabilized. This idea has been applied to humanoids in [23], [24] where the LIP dynamics is stabilized around a reference trajectory provided by the MPC. The robust positive invariant set can then be computed and a constraint restriction is found accordingly.

In the presence of external disturbances a possible alternative, complementary to the previous approach, consists of using an estimate of the disturbance, provided by a disturbance observer, to counteract the disturbance itself. Examples in humanoid balance or gait generation are given by [25], [26], [27]. We explored a closely related idea in [28], where a modified version of the stability constraint led to a form of *indirect compensation* of the disturbance.

In this paper we present an extension of our IS-MPC method for humanoid gait generation aimed at obtaining robust gait generation in the presence of disturbances. This is achieved by explicitly including the mid-range disturbance in the stability constraint, and by performing an appropriate ZMP restriction during the control horizon, based on the disturbance range amplitude. We also present conditions which guarantee recursive feasibility and stability for the IS-MPC with constraint modification.

The paper is organized as follows. In Sect. II we briefly recall the IS-MPC scheme and the corresponding stability constraint. The class of considered disturbances is defined in Section III where both the robust stability constraint and the restricted ZMP constraint are also introduced. Recursive feasibility and stability are also analyzed in this new context. Dynamic simulations are presented in Sect. IV. Section V addresses conclusions and future work.

II. Background

Internal stability is a fundamental issue in humanoid gait generation. In fact, even if dynamic balance is guaranteed through the ZMP criterion, the CoM can still diverge with respect to the ZMP, ultimately leading to a failure due the robot's inability to realize the gait.

A. Nominal model

Denote the position of the humanoid CoM and ZMP as (x_c, y_c, z_c) and $(x_z, y_z, 0)$, respectively. The balance of moments around the ZMP provides a relationship between the evolution of the latter and that of the CoM [29]. Assuming that motion takes place on flat horizontal ground with the CoM traveling at constant height $\bar{z}_c$, and neglecting angular momentum contributions around the CoM, we obtain the simplified model known as Linear Inverted Pendulum (LIP), which has identical and decoupled x-axis (sagittal) and y-axis (coronal) dynamics. For illustration, consider the sagittal dynamics

$$\ddot{x}_c = \eta^2 (x_c - x_z), \tag{1}$$

with $\eta = \sqrt{g/\bar{z}_c}$, where g is the gravity acceleration. Note that the ZMP position x_z acts as an input in this model.

The LIP dynamics (1) can be decomposed into a stable and an unstable subsystem using the following change of coordinates:

$$x_u = x_c + \dot{x}_c/\eta \tag{2}$$

$$x_s = x_c - \dot{x}_c/\eta. \tag{3}$$

The unstable component x_u, also known as *divergent component of motion* [30] or *capture point* [31], evolves as

$$\dot{x}_u = \eta\,(x_u - x_z).$$

B. IS-MPC for the nominal case

Intrinsically Stable MPC(IS-MPC) is a scheme for humanoid gait generation that uses a stability constraint to guarantee that the CoM remains bounded with respect to the ZMP, i.e., that the gait is internally stable. Below we summarize IS-MPC for the simplified case in which the footsteps are assigned and have constant orientation. Since their positions are not decision variables, no kinematic constraints must be enforced in this case. See [32] for further details.

The prediction model is a dynamically extended LIP

$$\begin{pmatrix} \dot{x}_c \\ \ddot{x}_c \\ \dot{x}_z \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \eta^2 & 0 & -\eta^2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_c \\ \dot{x}_c \\ x_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \dot{x}_z, \tag{4}$$

with the ZMP velocity $\dot{x}_z$ now acting as input. We assume piecewise-constant inputs, i.e., $\dot{x}_z(t) = \dot{x}_z^i$ for $t \in [t_i, t_{i+1}]$, with $\delta = t_{i+1} - t_i$ the duration of a sampling interval. The control horizon of the MPC is $T_c = C \cdot \delta$.

Dynamic balance is guaranteed by the following constraint on the ZMP

$$x_z^m(t) \le x_z(t) \le x_z^M(t), \qquad t \in [t_k, t_{k+C}], \tag{5}$$

with the upper and lower bounds extracted from the footstep plan at time t. Note that the size of admissible ZMP region is constant

$$x_z^M(t) - x_z^m(t) = d,$$

provided that a moving constraint is used during double support [5].

In spite of the instability of the LIP dynamics, x_u (and hence x_c) would remain bounded with respect to x_z if the following condition were satisfied [14]:

$$x_u^k = \eta \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau. \tag{6}$$

This is however a non-causal relationship, because it links x_u^k, the value of x_u at the current instant t_k, to the profile of x_z after t_k.

To derive a causal version of (6), observe that the profile of x_z inside T_c is obtained by integration of the velocities $\dot{x}_z^k, \ldots, \dot{x}_z^{k+C-1}$, i.e., the MPC decision variables; whereas the profile of x_z outside T_c depends on the unknown velocities $\dot{\tilde{x}}_z^{k+C}, \dot{\tilde{x}}_z^{k+C+1}, \ldots$, collectively referred to as the *tail*, which must be instead conjectured. In particular, an *anticipative tail* can be built by using the information encoded in the footstep plan up to a preview horizon $T_p = P \cdot \delta$ and zeroing all successive velocities. The resulting causal *stability constraint* takes the form

$$\sum_{i=0}^{C-1} e^{-i\eta\delta} \dot{x}_z^{k+i} = -\sum_{i=C}^{P-1} e^{-i\eta\delta} \dot{\tilde{x}}_z^{k+i} + \frac{\eta}{1 - e^{-\eta\delta}} (x_u^k - x_z^k), \tag{7}$$

where $\dot{\tilde{x}}_z^{k+i}$, $i = C, \ldots, P-1$ are the conjectured velocities in the preview horizon.

Collecting the MPC decision variables as

$$\dot{X}_z^k = (\dot{x}_z^k \;\; \ldots \;\; \dot{x}_z^{k+C-1})^T$$

the following Quadratic Programming (QP) problem is solved at each iteration of IS-MPC:

$$\left\{ \begin{array}{l} \min\limits_{\dot{X}_z^k} \|\dot{X}_z^k\|^2 \qquad \text{subject to:} \\ \bullet\ \text{ZMP constraint (5);} \\ \bullet\ \text{stability constraint (7).} \end{array} \right.$$

Once the solution is found, the first sample $\dot{x}_z^k$ of the optimal input sequence is used to integrate the prediction model (4). This results in a reference trajectory for the humanoid CoM that is tracked using a standard kinematic controller.

It can be shown that IS-MPC at t_k is feasible if and only if

$$x_u^{k,m} \le x_u^k \le x_u^{k,M}, \tag{8}$$

where

$$x_u^{k,m} = \eta \int_{t_k}^{t_{k+C}} e^{-\eta(\tau - t_k)} x_z^m d\tau + \eta \int_{t_{k+C}}^{\infty} e^{-\eta(\tau - t_k)} \tilde{x}_z d\tau$$

$$x_u^{k,M} = \eta \int_{t_k}^{t_{k+C}} e^{-\eta(\tau - t_k)} x_z^M d\tau + \eta \int_{t_{k+C}}^{\infty} e^{-\eta(\tau - t_k)} \tilde{x}_z d\tau$$

are the limits of the *feasibility region*.

Based on the feasibility analysis, in [32] it is proven that IS-MPC with the anticipative tail is recursively feasible if the preview horizon T_p is sufficiently long. Moreover, it is shown that recursive feasibility also implies internal stability of the CoM with respect to the ZMP.

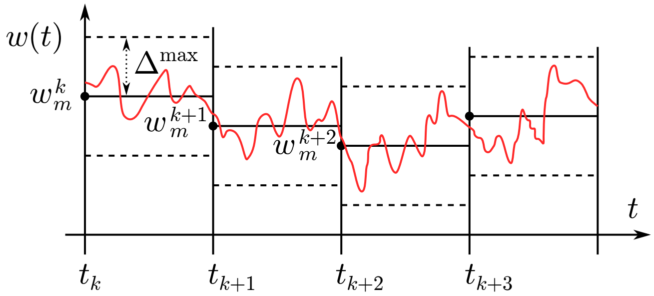

Fig. 1. A typical disturbance (in red) in the considered class.

III. IS-MPC FOR THE PERTURBED CASE

We now present a modification of the previously described IS-MPC algorithm for achieving robustness.

Consider an additive disturbance w acting on the LIP dynamics

$$\ddot{x}_c = \eta^2(x_c - x_z) + w. \tag{9}$$

This disturbance can represent external forces acting on the humanoid as well as dynamics that are unmodeled in the LIP [23]. Applying the same change of variables (2–3), the dynamics of the unstable component x_u becomes:

$$\dot{x}_u = \eta\,(x_u - x_z) + w/\eta. \tag{10}$$

A. Disturbance model

We now define precisely the disturbance model considered in this paper. In particular, we shall deal with disturbances of the form

$$w(t) = w_m^k + \Delta w(t) \qquad t \in [t_k, t_{k+1}), \tag{11}$$

where $|\Delta w(t)| \le \Delta^{\max}$ (see Fig. 1). The value w_m^k is the *mid-range disturbance* in $[t_k, t_{k+1})$ and satisfies the additional requirement

$$|w_m^{k+1} - w_m^k| \le \Delta^{\max}. \tag{12}$$

We assume that the maximum and minimum disturbance value — and thus w_m^k — are known in each interval. The simplifying assumption that the *range amplitude* $\Delta^{\max}$ is constant can always be met by taking the maximum $\Delta w(t)$ over the sampling intervals.

The above model encompasses a large variety of possible disturbances. For example, it can represent the reaction force that arises on the CoM when the humanoid is pushing an object, say, a cart. In general, such force will not be constant, but it can be kept bounded by an arm compliance controller [33].

A special case occurs when $w_m^i = w_m$ for all i, i.e., the mid-range disturbance is the same in all intervals. This could represent the situation in which a humanoid walks on an inclined plane. In fact, the total disturbance in this case will consist of the constant push/pull plus the unmodeled dynamics. If $w_m = 0$, the disturbance w can represent the effect of unmodeled dynamics during an otherwise unperturbed gait.

B. Robust stability constraint

To guarantee internal stability for the perturbed LIP model (9), condition (6) must be modified as follows:

$$x_u^k = \eta \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau - \frac{1}{\eta}\int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} w(\tau) d\tau. \tag{13}$$

This condition requires the knowledge of w after t_k. To obtain a causal version of (13), we should use only the available knowledge of w at t_k. One possibility[1] is to replace $w(\cdot)$ with the mid-range disturbance w_m^k in t_k, obtaining

$$\eta\int_{t_k}^{t_{k+C}} e^{-\eta(\tau - t_k)} x_z(\tau) d\tau = -\eta\int_{t_{k+C}}^{\infty} e^{-\omega(\tau - t_k)} \tilde{x}_z(\tau) d\tau + x_u^k + \frac{w_m^k}{\eta^2} \tag{14}$$

which, by evaluating integrals for our piecewise-linear ZMP trajectory, leads to the *robust stability constraint*

$$\sum_{i=0}^{C-1} e^{-i\eta\delta} \dot{x}_z^{k+i} = -\sum_{i=C}^{P-1} e^{-i\eta\delta} \dot{\tilde{x}}_z^{k+i} + \frac{\eta}{1 - e^{-\eta\delta}}\left(x_u^k - x_z^k + \frac{w_m^k}{\eta^2}\right). \tag{15}$$

Including the known part w_m^k of the disturbance in the stability constraint leads to an IS-MPC scheme where the control inputs (the ZMP velocities within the control horizon) are modified by w, realizing a form of *indirect disturbance compensation* [28].

Consistently with the above discussion, also the prediction model (4) is modified to include the mid-range disturbance w_m^k

$$\begin{pmatrix} \dot{x}_c \\ \ddot{x}_c \\ \dot{x}_z \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \eta^2 & 0 & -\eta^2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_c \\ \dot{x}_c \\ x_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \dot{x}_z + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} w_m^k \tag{16}$$

and will be used to propagate at time t_{k+1} the effect of the first sample $\dot{x}_z^k$ of the QP solution.

C. Restricted ZMP constraint

The main tool introduced in this paper to guarantee robustness with respect to bounded disturbances is the restriction of the ZMP constraint. Intuitively, enforcing a tighter constraint on the ZMP creates a safety margin for absorbing the effect of disturbances. In the following, we formalize this idea and rigorously prove its effectiveness.

Define a *restriction function* $R(t)$ as a non-decreasing[2] function over $[0, T_c]$ such that

$$|R(t)| \le d/2, \tag{17}$$

with d the size of the ZMP constraint. Accordingly, the restricted ZMP constraint is

$$x_z^m(t) + R(t - t_k) \le x_z(t) \le x_z^M(t) - R(t - t_k) \tag{18}$$

for $t \in [t_k, t_{k+C}]$.

[1] This corresponds to replacing all future values of w in the stability condition (13) with the current mid-range disturbance w_m^k, rather than using, if available, also the future values $w_m^{k+1}, w_m^{k+2}, \ldots$.

[2] This property is instrumental for simplifying the proof of Prop. 1 but not strictly necessary.

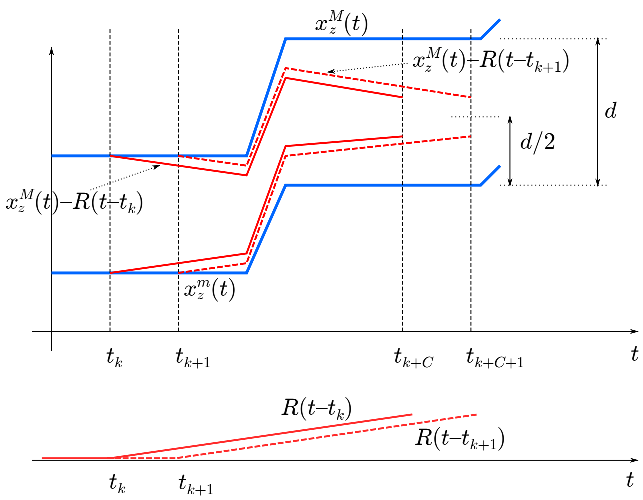

Fig. 2. Using a linear restriction function $R(t-t_k) = r(t-t_k)$ to modify the original ZMP constraint (blue). Note the restricted constraint for the QP problem at t_k (solid red) and for the QP problem at t_{k+1} (dashed red).

For illustration, we will use a linear restriction function

$$R(t) = r\,t \tag{19}$$

where the slope r is a design parameter. Note that it must be

$$r \leq \frac{d}{2T_c} \tag{20}$$

in order to guarantee (17). The effect of a linear restriction on the ZMP constraint is shown in Fig. 2.

It is important to understand that the restriction procedure only affects the constraint *inside* the control horizon, leaving the ZMP bounds at the current time t_k unchanged[3]. This means that the actual ZMP will always use the full unrestricted support polygon.

D. Recursive feasibility for the perturbed case

We now establish the main result of the paper, i.e., a sufficient condition for recursive feasibility of IS-MPC with modified constraints (both stability and ZMP) in the presence of additive disturbances of the form (11).

Considering the robust stability constraint in the integral form (14) and using the restricted ZMP constraint (18), the feasibility region (8) at t_k is modified as

$$\bar{x}_u^{k,m} \leq x_u^k \leq \bar{x}_u^{k,M} \tag{21}$$

with

$$\begin{aligned}\bar{x}_u^{k,m} &= x_u^{k,m} + \int_{t_k}^{t_{k+C}} e^{-\eta(\tau-t_k)} R(\tau-t_k)d\tau - \frac{w_m^k}{\eta^2}\\ \bar{x}_u^{k,M} &= x_u^{k,M} - \int_{t_k}^{t_{k+C}} e^{-\eta(\tau-t_k)} R(\tau-t_k)d\tau - \frac{w_m^k}{\eta^2}.\end{aligned}$$

The following proposition clarifies how robust recursive feasibility can be achieved by properly choosing the slope r of the linear restriction function (19).

[3]This is true as long as $R(0) = 0$, which certainly holds for linear restriction functions

Proposition 1: Assume an additive disturbance of the form (11–12) is present. Then, IS-MPC with the robust stability constraint (15) and the linearly restricted ZMP constraint (18–19) is recursively feasible if

$$\frac{1}{\delta(1-e^{-\eta T_c})}\left(\frac{\Delta^{\max} e^{\eta\delta}}{\eta^2} + \mu^{\max}\right) \leq r \leq \frac{d}{2T_c}. \tag{22}$$

Proof. The upper bound comes directly from (20). For the lower bound, see the Appendix.

In the left-hand side of (22), $\mu^{\max}$ is a bound on the *tail mismatch*, a term arising from the variation over time of the ZMP velocity tail due to the preview horizon T_p being finite. This mismatch exists independently of the disturbance, which instead affects the first term in the left-hand side through its range amplitude $\Delta^{\max}$. See the proof for an explicit expression of $\mu^{\max}$.

In the special case of constant mid-range disturbance it is possible to tighten the previous result.

Proposition 2: Assume an additive disturbance of the form (11–12) is present, with $w_m^i = w_m$ for all i. Then, IS-MPC with the robust stability constraint (15) and the linearly restricted ZMP constraint (18–19) is recursively feasible and

$$\frac{1}{\delta(1-e^{-\eta T_c})}\left(\frac{\Delta^{\max}(e^{\eta\delta}-1)}{\eta^2} + \mu^{\max}\right) \leq r \leq \frac{d}{2T_c}. \tag{23}$$

Proof. See the Appendix.

A few remarks are in order.

- The lower bound in (23) is smaller (hence, less restrictive) than in (22) because the assumption $w_m = w_m^i$ for all i means that the robust stability constraint (15) actually takes into account all the future mid-range disturbances rather than only the current.
- By equating the right- and left-hand side of (22) (or (23)) one obtains an upper bound on the disturbance range amplitude $\Delta^{\max}$ which can be tolerated in order to maintain recursive feasibility of IS-MPC algorithm.
- If the whole footstep plan is known a priori (i.e, $T_p = \infty$), the tail mismatch $\mu^{\max}$ is zero.
- Increasing the control horizon T_c decreases the lower as well as the upper bound on the slope r. This is due to the choice of a linear restriction function $R(t)$. Other choices are possible, such as linear/saturated or exponential.

Wrapping up, the proposed approach may be paraphrased as follows: the robust stability constraint indirectly compensates for the known part w_m^k of the disturbance, while the ZMP restriction takes care of its uncertain part in a preventive way based on the range amplitude $\Delta^{\max}$.

To conclude this section, we claim that robust recursive feasibility guarantees robust internal stability, i.e., the boundedness of the CoM trajectory with respect to the ZMP trajectory. In fact, since the disturbance is bounded and the restriction function is linear (and thus exponential of order 0), Prop. 6 of [32] still holds.

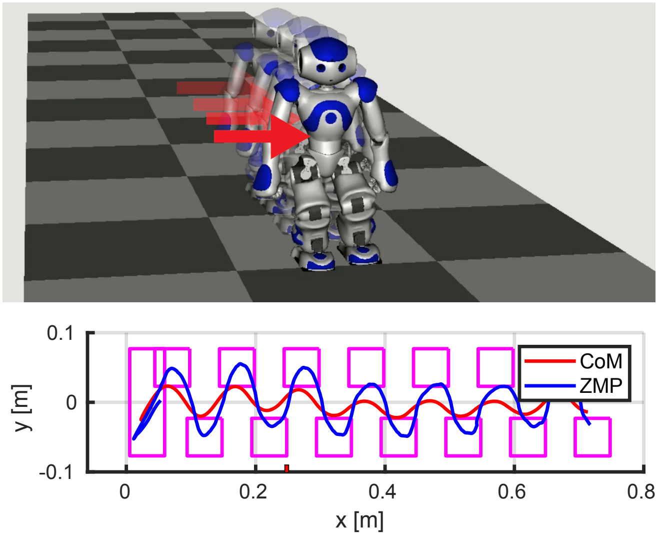

Fig. 3. Simulation 1. NAO walking on horizontal ground and subject to a constant lateral push starting from $t = 2.5$ s (top). CoM and ZMP trajectories (bottom); the red tick identifies the start of the lateral push.

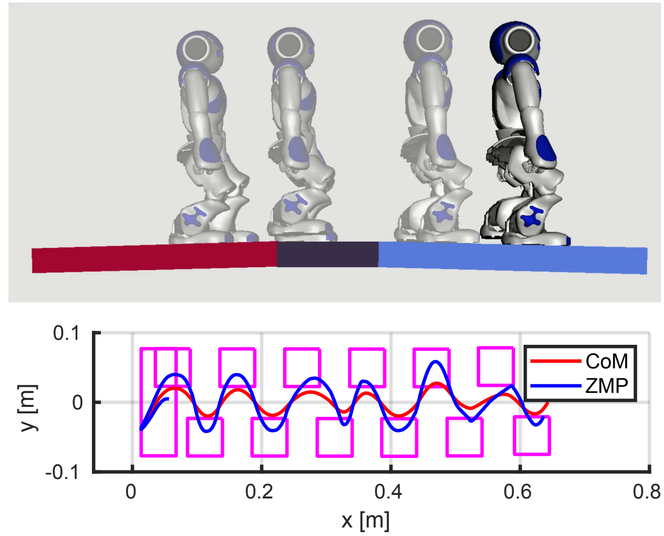

Fig. 4. Simulation 2. NAO walking on a sequence of ramps of unknown slope (top). CoM and ZMP trajectories (bottom).

IV. Dynamic simulations

To validate the proposed approach, we performed some dynamic simulations in DART (Dynamic Animation Robotics Toolkit) for NAO, a small humanoid robot with CoM height $\bar{z}_c = 0.33$ m.

A square footprint with a side of $d = 0.05$ m is used to define the ZMP constraint (5). The duration of the single and double support phase is 0.3 s and 0.2 s, respectively. IS-MPC uses a sampling interval $\delta = 0.05$ s, a control horizon $T_c = 1$ s and a preview horizon $T_p = 2$ s. The controller itself runs at 100 Hz.

In the first simulation, shown in Fig. 3, NAO is walking on horizontal ground. From $t = 2.5$ s on, a lateral constant push of 3.2 N is applied from the right, corresponding to a disturbance $w = 0.71$ m/s^2 in (9). This value is unknown to the robot, which however reconstructs a real-time estimate $\hat{w}(t_k)$ of w using the ZMP-based observer proposed in [34]. The mid-range disturbance is then guessed as $w_m^k = \hat{w}(t_k)$, while the range amplitude $\Delta^{\max} = 0.08$ m/s^2 around w_m^k is assumed to be known. The results show that the proposed IS-MPC method produces an internally stable gait thanks to the use of modified constraints; namely, the robust stability constraint (15) and a linearly restricted ZMP constraint (18–19) with $r = 0.071$ computed from eq. (22). Observe, in particular, how the CoM trajectory is shifted towards the right of the robot, indicating that the robot is actually 'leaning against the disturbance' in order to compensate it (a behavior already observed in [28]). In contrast, nominal IS-MPC (i.e., IS-MPC with unmodified constraints) quickly becomes unfeasible, resulting in a failure. See the accompanying video for a side-by-side clip comparison.

In the second simulation, shown in Fig. 4, the robot is walking on a sequence of ramps whose slope is unknown but always in the range $-1.6° \div 1.6°$. Any nonzero slope will obviously generate a sagittal disturbance on the robot CoM because the gravity force is not completely compensated by the ground reaction force. Considering the maximal and minimal disturbance $g \sin(\pm 1.6\circ) = \pm 0.28$ m/s^2 leads to a range amplitude $\Delta^{\max} = 0.28$ m/s^2, with mid-range disturbance $w_m^i = 0$ for all i. Equation (23) indicates that a linearly restricted ZMP constraint (18–19) with $r = 0.035$ will guarantee[4] recursive feasibility and, hence, produce an internally stable gait in spite of the disturbance, as confirmed by the simulation. Once again, nominal IS-MPC fails (see the accompanying video).

Note that the bottom plots in Figs. 3 and 4 show the unrestricted ZMP constraints, as the restriction is only active in prediction inside the control horizon. As already mentioned, the actual ZMP is allowed to use the full unrestricted area.

Finally, it is important to note that, while control design is based on the simplified LIP model, the motion of the robot in the above simulations actually obeys the full nonlinear model. This means that dynamics that are neglected in the LIP represent an additional source of persistent perturbation which is unknown to the controller. The resulting stable gaits confirm then that the proposed version of IS-MPC achieves a significant degree of robustness with respect to external disturbances as well as unmodeled dynamics.

V. Conclusions

We have presented an extension of our previously proposed IS-MPC method for humanoid gait generation aimed at obtaining robust performance in the presence of disturbances. The considered disturbance signals vary in a range of known amplitude around a mid-range value that can change at each sampling time, but whose current value is assumed to be available, possibly via estimation. The method consists in:

[4]In this case, the nominal and robust stability constraints, respectively (7) and (15), actually coincide because the mid-range disturbance is zero. Robustness is then the result of ZMP constraint restriction only.

- modifying the stability constraint that is at the core of IS-MPC by incorporating the current mid-range disturbance;
- performing an appropriate linear restriction of the ZMP constraint in the control horizon on the basis of the range amplitude of the disturbance.

We have derived explicit conditions for recursive feasibility and internal stability of the proposed IS-MPC method with constraint modification. Finally, we have illustrated its superior performance with respect to the nominal version by performing dynamic simulations on the NAO robot.

Future work will address several points, such as:

- integrating the disturbance observer of [28] for estimating the mid-range disturbance in the robust stability constraint;
- developing a compliant foot trajectory generation to allow robust stepping when the disturbance is the result of non-horizontal ground;
- performing experiments on a real humanoid robot.

APPENDIX

Proof of Proposition 1

The proof goes through a set of upper and lower inequalities. We omit the lower inequalities for compactness.

Assume that the robust stability constraint in the integral form (14) and restricted ZMP constraint (18) are satisfied at t_k. The first step is to derive a bound on x_u^{k+1}. Integration of (10) gives

$$x_u^{k+1} = e^{\eta\delta} x_u^k - \eta \int_{t_k}^{t_{k+1}} e^{\eta(t_{k+1}-\tau)} x_z(\tau) d\tau + \frac{1}{\eta} \int_{t_k}^{t_{k+1}} e^{\eta(t_{k+1}-\tau)} w(\tau) d\tau.$$

Plugging (14) into this and using (18), we can write

$$\begin{aligned} x_u^{k+1} \leq \; & \eta \int_{t_{k+1}}^{t_{k+C}} e^{-\eta(\tau - t_{k+1})} (x_z^M - R(\tau - t_k)) d\tau + \\ & \eta \int_{t_{k+C}}^{\infty} e^{-\eta(\tau - t_{k+1})} \tilde{x}_z(\tau) d\tau + \\ & \frac{1}{\eta} \int_{t_k}^{t_{k+1}} e^{\eta(t_{k+1}-\tau)} w(\tau) d\tau - \frac{w_m^k e^{\eta\delta}}{\eta^2}. \end{aligned}$$

On the other hand, x_u belongs to the feasibility region at time t_{k+1} if

$$\begin{aligned} x_u^{k+1} \leq \; & \eta \int_{t_{k+1}}^{t_{k+C+1}} e^{-\eta(\tau - t_{k+1})} (x_z^M - R(\tau - t_{k+1})) d\tau + \\ & \eta \int_{t_{k+C+1}}^{\infty} e^{-\eta(\tau - t_{k+1})} \tilde{x}'_z(\tau) d\tau - \frac{w_m^{k+1}}{\eta^2}, \end{aligned}$$

where $\tilde{x}'_z(\tau)$ is the ZMP position obtained by integration of the tail at t_{k+1}.

A sufficient condition for recursive feasibility is obtained by imposing that the right-hand side of the penultimate inequality is smaller than that of the last one. Rearranging leads to

$$\begin{aligned} & \eta \int_{t_{k+1}}^{t_{k+C}} e^{-\eta(\tau - t_{k+1})} (R(\tau - t_k) - R(\tau - t_{k+1})) d\tau + \\ & \eta \int_{t_{k+C}}^{t_{k+C+1}} e^{-\eta(\tau - t_{k+1})} (x_z^M - R(\tau - t_{k+1}) - \tilde{x}_z(\tau)) d\tau + \\ & \eta \int_{t_{k+C+1}}^{\infty} e^{-\eta(\tau - t_{k+1})} (\tilde{x}'_z(\tau) - \tilde{x}_z(\tau)) d\tau + \\ & -\frac{w_m^{k+1} - w_m^k}{\eta^2} - \frac{1}{\eta} \int_{t_k}^{t_{k+1}} e^{\eta(t_{k+1}-\tau)} (w(\tau) - w_m^k) d\tau \geq 0. \end{aligned}$$

We can neglect the second integral, which is always positive and much smaller than the other terms; this leads to a slightly conservative result, but considerably simplifies the computation. Rewrite the resulting inequality as

$$\beta \leq \mu + \gamma, \tag{24}$$

where we have set

$$\begin{aligned} \beta &= \eta \int_{t_{k+1}}^{t_{k+C}} e^{-\eta(\tau - t_{k+1})} (R(\tau - t_k) - R(\tau - t_{k+1})) d\tau \\ \mu &= \eta \int_{t_{k+C+1}}^{\infty} e^{-\eta(\tau - t_{k+1})} (\tilde{x}_z(\tau) - \tilde{x}'_z(\tau)) d\tau \\ \gamma &= \frac{w_m^{k+1} - w_m^k}{\eta^2} + \frac{1}{\eta} \int_{t_k}^{t_{k+1}} e^{\eta(t_{k+1}-\tau)} (w(\tau) - w_m^k) d\tau. \end{aligned}$$

For the considered linear restriction (19), β is computed as

$$\beta = r\delta(1 - e^{-\eta T_c}).$$

As for μ, this term is the *tail mismatch*, which arises from the variation over time of the ZMP velocity tail due to the preview horizon T_p being finite. As shown in [32], μ can be bounded as

$$\mu \leq e^{-\eta T_p} \frac{1 - e^{-\eta\delta}}{\eta} v_z^{\max} = \mu^{\max},$$

where $v_z^{\max}$ is the maximum velocity of the ZMP in the anticipative tail. Finally, γ can be bounded as

$$\gamma \leq \frac{\Delta^{\max} e^{\eta\delta}}{\eta^2},$$

having used (12) and the fact that $|\Delta w(t)| \leq \Delta^{\max}$. By plugging the expression of β and the bounds on μ and γ into (24), the thesis follows. ■

Proof of Proposition 2

Repeating the proof of Prop. 1 with $w_m^{k+1} = w_m^k = w_m$ the bound on γ is modified as

$$\gamma \leq \frac{1}{\eta} \int_{t_k}^{t^{k+1}} e^{\eta(t_{k+1}-\tau)} \Delta w(\tau) d\tau \leq \frac{\Delta^{\max} e^{\eta\delta}}{\eta^2} (e^{\eta\delta} - 1)$$

which proves the thesis. ■

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