Stabilization of the general two-trailer system

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Abstract

Existing methods for nonholonomic feedback stabilization can only be applied to exactly nilpotentizable or flat systems. In this paper, a car towing two off-hooked trailers is considered as a canonical example of robot that does not fall into the above class. We show that exponential convergence to arbitrary configurations can be obtained by means of an iterative steering technique based on a nonhomogeneous nilpotent approximation of the system. Simulation results illustrate the performance of the method.

1 Introduction

Nonholonomic motion planning and control are problems of particular interest in mobile robotics. Substantial progresses have been achieved: for exactly nilpotentizable [1, 2] or, equivalently, flat [3] systems both the steering and the stabilization problem have been solved, see [4] for a review of relevant techniques.

However, the existing methods cannot be used for nonholonomic systems that do not possess the above properties. A first step towards the solution of both planning and stabilization problems for *general* nonholonomic systems is the synthesis of approximate steering techniques. In fact, if the approximation is accurate, this approach may already be a satisfactory planning solution. Furthermore, feedback stabilization to a point can often be obtained by iterated application of approximate steering controls [5]; see [6] for an application to underactuated manipulators.

Approximate steering methods can be devised using nilpotent approximations [7–10]. These are high-order local approximations that are useful when tangent linearization does not retain controllability, as in nonholonomic systems. At the same time, closed-form integration of the nilpotent system (which is polynomial and triangular) under parameterized inputs is possible, thereby allowing the computation of steering controls.

The canonical example of non-nilpotentizable, non-flat nonholonomic system is the general N-trailer system, i.e., a vehicle in which N off-hooked trailers are attached to a tractor. It is well known that this system is non-flat if $N \geq 2$ (see [3] for a proof in the case $N = 2$). To our knowledge, the problem of point stabilization for this system has only been addressed so far in [11], where it is shown that at particular configurations the system can be approximated by a chained-form system; hence, it is possible to devise time-varying feedback laws providing local stability at these configurations. However, the latter are not dense in the state space, so that the method does not apply for stabilizing generic configurations.

In this paper, we consider a particular instance of the above system, i.e., the general two-trailer system. We already considered this system in [12], showing how to build a globally valid nonhomogeneous nilpotent approximation in the presence of singular points. Here, we complete our investigation, proving that stabilization for arbitrary configurations can be achieved by means of an iterative steering technique based on the above mentioned nilpotent approximation.

The paper is organized as follows. In Sect. 2, the model of the general two-trailer system is given together with its nilpotent approximation. Section 3 describes our stabilization strategy, which makes use of a contracting open-loop control (Sect. 3.1) and an iterative steering scheme (Sect. 3.2). The performance of the method is illustrated by simulations in Sect. 4.

2 A general two-trailer system

Consider the system shown in Fig. 1, consisting of a car towing two identical trailers, each hooked at a distance d from the preceding wheel axle (*off-hooking*). The distance between the hooking point and the wheel axle midpoint of each trailer is ℓ. For simplicity, we assume $d = 1$ and $\ell = 1$. However, a similar analysis can be developed for the case $d \neq \ell$.

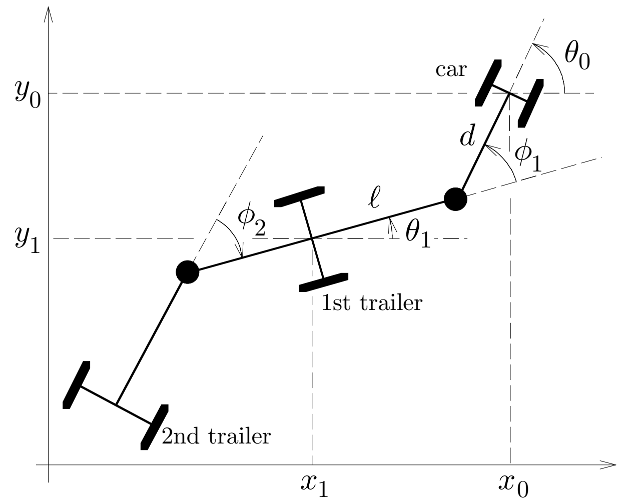

Figure 1: A general two-trailer system

2.1 Kinematic model

Let x_0, y_0 be the cartesian coordinates of the car reference point, θ_0 the car orientation with respect to the x axis, and ϕ_1, ϕ_2 the angles formed by the car and the first trailer respectively with the first and the second trailer (see Fig. 1). Denoting by v_0 and ω_0 the driving and steering velocities of the car, the kinematic model of the vehicle can be written as

$$\begin{aligned} \dot{x}_0 &= \cos\theta_0\, v_0 \\ \dot{y}_0 &= \sin\theta_0\, v_0 \\ \dot{\theta}_0 &= \omega_0 \\ \dot{\phi}_1 &= -s_1 v_0 + (1+c_1)\omega_0 \\ \dot{\phi}_2 &= (s_{12}+s_1)v_0 - (c_{12}+c_1)\omega_0, \end{aligned} \tag{1}$$

having set $s_i = \sin\phi_i$, $c_i = \cos\phi_i$, $s_{ij} = \sin(\phi_i - \phi_j)$ and $c_{ij} = \cos(\phi_i - \phi_j)$ for $i, j = 1, 2$.

With an eye to the nilpotent approximation procedure, it is convenient to choose a different set of generalized coordinates as a state vector. In particular, let $q = (x_1, y_1, \theta_1, \phi_1, \phi_2)$, where x_1, y_1 are the cartesian coordinates of the first trailer reference point and θ_1 is the trailer orientation with respect to the x axis. The kinematic model of the vehicle assumes the form

$$\dot{q} = g_1(q)v_1 + g_2(q)\omega_1, \tag{2}$$

where v_1 and ω_1 are the driving and steering velocities of the first trailer. The input vector fields are

$$g_1(q) = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \\ 0 \\ s_1 \\ -s_2 \end{pmatrix} \quad g_2(q) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1-c_1 \\ 1+c_2 \end{pmatrix}. \tag{3}$$

The actual control inputs are related to v_1 and ω_1 through the input transformation

$$\begin{aligned} v_0 &= v_1\cos\phi_1 + \omega_1\sin\phi_1 \\ \omega_0 &= v_1\sin\phi_1 - \omega_1\cos\phi_1, \end{aligned}$$

which is always defined.

If $\phi_1 = \pi$ or $\phi_2 = \pi$, the system is not controllable. In the following, we consider points of the state space defined as $\mathcal{M} = I\!R^2 \times S^1 \times (S^1 - \{\pi\})^2$.

Consider the first 6 elements of the P. Hall [2] family $g_1, g_2, g_3 = [g_1, g_2]$, $g_4 = [g_1, [g_1, g_2]]$, $g_5 = [g_2, [g_1, g_2]]$, $g_6 = [g_1, [g_1, [g_1, g_2]]]$. Vector fields g_1, g_2, g_3, g_4, g_5 span the tangent space of $\mathcal{M}$ at points such that $\phi_1 \neq \phi_2$ (*regular* points), while g_1, g_2, g_3, g_4, g_6 span the tangent space everywhere, including points such that $\phi_1 = \phi_2$ (*singular* points). Hence, the system is controllable and the degree of nonholonomy is 3 at regular points and 4 at singular points.

2.2 Nilpotent approximation

It is well known that system (2–3) cannot be put in chained form [13], nor does it satisfy the necessary and sufficient condition for flatness [3]. However, we shall prove that the use of a nilpotent approximation of the system allows to solve the stabilization problem.

In the presence of singular points, homogeneous nilpotent approximations [8] do not provide globally valid representations. In a previous work, we have shown that nonhomogeneous nilpotent forms can be adopted to this end [12]. Applying the procedure therein proposed to system (2–3), we obtain the following global nilpotent approximation:

$$\begin{aligned} \dot{z}_1 &= u_1 \\ \dot{z}_2 &= u_2 \\ \dot{z}_3 &= -z_2 u_1 \\ \dot{z}_4 &= \sum_{j=1}^{2} h_{j4}(z_1, \ldots, z_3)u_j \\ \dot{z}_5 &= \sum_{j=1}^{2} h_{j5}(z_1, \ldots, z_4)u_j, \end{aligned} \tag{4}$$

in which

$$\begin{aligned} h_{j4}(z_1, \ldots, z_3) &= a_{j4}^2 z_1^2 + b_{j4}^2 z_1 z_2 + c_{j4}^2 z_2^2 + d_{j4}^2 z_3 \\ h_{15}(z_1, \ldots, z_4) &= c_{15}^2 z_2^2 + a_{15}^3 z_1^3 + b_{15}^3 z_1 z_3 + c_{15}^3 z_1^2 z_2 \\ &+ d_{15}^3 z_2 z_3 + e_{15}^3 z_2^3 + f_{15}^3 z_1 z_2^2 + g_{15}^3 z_4 \\ h_{25}(z_1, \ldots, z_4) &= d_{25}^2 z_3 + a_{25}^3 z_1^3 + b_{25}^3 z_1 z_3 + c_{25}^3 z_1^2 z_2 \\ &+ d_{25}^3 z_2 z_3 + e_{25}^3 z_2^3 + f_{25}^3 z_1 z_2^2 + g_{25}^3 z_4. \end{aligned}$$

The coefficients $a_{j4}^2, \ldots, d_{j4}^2$, c_{15}^2, d_{25}^2 and $a_{j5}^3, \ldots, g_{j5}^3$ $(j = 1, 2)$ are functions of the point $\bar{q}$ around which the approximation is computed. Their expressions are quite complicated and can be found in [14]. However, as will be shown later, they are not needed for implementing the stabilization method, thanks to the structure of the chosen control input.

As for the transformation between the system coordinates q and the nilpotent approximation state z, our control strategy (see Sect. 3) only requires its expression on the manifold defined by $z_1 = 0$, $z_2 = 0$, $z_3 = 0$:

$$\begin{aligned} x_1 &= \bar{x}_1 \\ y_1 &= \bar{y}_1 \\ \theta_1 &= \bar{\theta}_1 \\ \phi_1 &= \bar{\phi}_1 + (1 + \cos\bar{\phi}_1)(z_5 - z_4) \\ \phi_2 &= \bar{\phi}_2 + (1 + \cos\bar{\phi}_2)(z_5 + z_4), \end{aligned} \tag{5}$$

in which $(\bar{x}_1, \ldots, \bar{\phi}_2)$ are the coordinates of the approximation point $\bar{q}$. In particular, at $\bar{q}$ one obtains $z = 0$. The complete coordinate change is given in [14].

3 The stabilization strategy

We now address the problem of finding a feedback controller (necessarily time-varying and/or discontinuous [15]) that transfers the general two-trailer system from an initial point q^0 to a desired point q^d. Without loss of generality, we assume that $q^d = (0, 0, 0, \phi_1^d, \phi_2^d)$; this can always be achieved by translating and rotating the world reference frame so as to align with the desired configuration of the first trailer.

Our solution prescribes the execution of two phases:

I. Drive in finite time the first three variables x_1, y_1 and θ_1 to zero. This amounts to steering the first trailer to its desired configuration regardless of the variables ϕ_1 and ϕ_2, which will converge to generic values ϕ_1^I, ϕ_2^I.

II. Obtain exponential convergence of ϕ_1 and ϕ_2 to ϕ_1^d and ϕ_2^d, while guaranteeing that x_1, y_1 and θ_1 return to their desired zero value.

In view of the first three equations of system (2–3), the first phase may be performed using any controller capable of stabilizing a unicycle (see [4]). However, an (arbitrarily small) error must be accepted in order to terminate this phase.

For the second phase, we adopt the *iterative steering* (IS) approach [5]. The essential tool is a contracting open-loop control law, capable of steering the system closer to the desired state q^d in a finite time T. Its iterated application (i.e., from the state reached at the end of the previous iteration), guarantees asymptotic stability and exponential convergence to q^d, provided that the open-loop control is Hölder-continuous with respect to the initial conditions. The resulting control is given by a time-varying law whose expression depends on a sampled feedback action. A certain degree of robustness is also achieved, as non-persistent perturbations are rejected, while the error is ultimately bounded in the presence of persistent perturbations.

To apply the IS technique, we must design contracting open-loop control inputs v_1, ω_1 for system (2–3). One possible choice is to perform a cyclic motion of period T on x_1, y_1 and θ_1, giving final values $\phi_1(T)$, $\phi_2(T)$ closer to ϕ_1^d, ϕ_2^d than the initial condition ϕ_1^I, ϕ_2^I. If such a cycle can be produced by v_1, ω_1 that are Hölder-continuous with respect to the initial conditions, exponential stabilization will be achieved.

3.1 A contracting open-loop control

The search for a contracting open-loop control may be conveniently performed within a parameterized class of inputs. As the general two-trailer system is not exactly nilpotentizable, the steering parameters cannot be computed by forward integration of the dynamic equations. Therefore, we resort to the nilpotent approximation (4), that is polynomial and triangular, and thus closed-form integrable under appropriate controls. The use of an approximate representation is consistent with the iterative steering paradigm, which only requires the error to contract at each iteration.

In view of the stabilization strategy outlined above, we must design an open-loop control which can steer system (2–3) from $q^I = (0, 0, 0, \phi_1^I, \phi_2^I)$ to a point $q^{II} = (0, 0, 0, \phi_1^{II}, \phi_2^{II})$ closer in norm to $q^d = (0, 0, 0, \phi_1^d, \phi_2^d)$. The first step is to compute the nilpotent approximation (4) at q^I. We emphasize that, in view of the globality of the representation (4), q^I may be a regular or singular point. The synthesis of a control law that transfers the state of system (4) from $z^I = 0$ (the image of q^I) exactly to z^{II} (the image of q^{II}, computed through eqs. (5)) is relatively straightforward.

Choose the open-loop control inputs as

$$v_1 = a_1 \cos\omega t + a_2 \sin\omega t \tag{6}$$
$$\omega_1 = a_3 \cos 2\omega t, \tag{7}$$

with $a_1, a_2, a_3 \in I\!R$, $\omega = 2\pi/T$ and T the duration of the control interval. Integration of eqs. (4) gives

$$\begin{aligned} z_1(T) &= 0 \\ z_2(T) &= 0 \\ z_3(T) &= 0 \\ z_4(T) &= k_1 a_3 (a_1^2 - a_2^2) \\ z_5(T) &= k_2 a_2 a_3 (a_1^2 - a_2^2), \end{aligned} \tag{8}$$

having set $k_1 = -T^3/32\pi^2$ and $k_2 = -T^4/64\pi^3$. The required computations are tedious but simple; details are given in [14].

In order to obtain $z_4(T) = z_4^{II}$ and $z_5(T) = z_5^{II}$, parameters a_1 and a_2 in (6–7) can be chosen as

$$a_1 = \sqrt{a_2^2 + \frac{z_4^{II}}{k_1 a_3}} \tag{9}$$

$$a_2 = \frac{2\pi}{T}\frac{z_5^{II}}{z_4^{II}} \tag{10}$$

provided that $z_4^{II} \neq 0$. As can be proven by direct substitution of eqs. (9–10) in the last two of (8), the value of a_3 is immaterial for the steering task, as long as $a_3 \neq 0$ and $\mathrm{sign}(a_3) = -\mathrm{sign}(z_4^{II})$ (so that a_1 is always well defined). In particular, we can let

$$a_3 = -\mathrm{sign}(z_4^{II}) \cdot \left\| \begin{pmatrix} z_4^{II} \\ z_5^{II} \end{pmatrix} \right\|^{1/r} \qquad r > 1, \tag{11}$$

where $||\cdot||$ denotes the euclidean norm. The rationale for this choice will be clarified in the next subsection.

3.2 Stabilization via iterative steering

Having identified a steering control for the nilpotent approximation (4), we must now clarify its use within the iterative steering framework.

Assume phase I of our stabilization technique has been completed. Given the values ϕ_1^I, ϕ_2^I of ϕ_1, ϕ_2, the desired z_4^{II} and z_5^{II} are generated as

$$z_4^{II} = \beta_1 z_4^d \tag{12}$$

$$z_5^{II} = \beta_2 z_5^d, \tag{13}$$

where $\beta_1 < 1$, $\beta_2 < 1$ are the chosen contraction rates and z_4^d, z_5^d are the images of ϕ_1^d, ϕ_2^d computed inverting eqs. (5), in which $\bar{\phi}_1 = \phi_1^I$, $\bar{\phi}_2 = \phi_2^I$. Conditions (12–13) guarantee error contraction on the z variables. Indeed, the use of the nilpotent dynamics (4) for computing $z_4(T)$ and $z_5(T)$ will induce an approximation error[1] on ϕ_1 and ϕ_2. However, by enforcing sufficient contraction on the approximate system, one can guarantee that the contraction property is preserved for the original one.

At this point, eqs. (9–11) are used to compute the parameters a_i, and the open-loop controls (6–7) are applied to system (2–3). After T seconds, the system state is sampled, the corresponding nilpotent approximation is derived and the procedure is repeated. In particular, the values of z_4^{II} and z_5^{II} are updated at each iteration using eqs. (12–13) (with constant β_1, β_2). As the coordinate transformation (5) depends on the approximation point, the same is true for z_4^d, z_5^d.

To prove asymptotic stability and exponential convergence with this scheme, it is necessary to show that the contracting open-loop controls satisfy the assumption in [5], namely Hölder-continuity with respect to the initial conditions. In practice, we need to establish the same property for the coefficients a_i ($i = 1, \ldots, 3$). While we shall not give a complete proof here (see [14] for details), we point out that:

- According to eq. (10), a_2 is Hölder-continuous if z_5^{II} converges to zero faster than z_4^{II}. To this end, one simply sets $\beta_1 < \beta_2$ in eqs. (12–13).
- The first coefficient a_1 given by eq. (9) is Hölder-continuous in view of the choice (11) for a_3.

A difficulty with the method so far outlined is that the steering controls (6–7) are not defined when $z_4^{II} = 0$. On the other hand, eq. (12) gives $z_4^{II} = 0$ if $z_4^d = 0$, i.e., if no reconfiguration is needed for the nilpotent approximation variable z_4. To circumvent this problem, it is relatively easy to work out a more general rule than eqs. (12–13) for generating z_4^{II} and z_5^{II}. In practice, any contraction on the norm of the error $(z_4 - z_4^d \quad z_5 - z_5^d)$ is admissible as long as $z_4^{II} \neq 0$.

4 Simulation results

We present two simulations of the proposed stabilization strategy. In both cases, it is assumed that phase I has already been executed, so that the first trailer is already at its desired configuration $x_1^d = 0$, $y_1^d = 0$, $\theta_1^d = 0$. Phase II is executed by iterative application of the control inputs (6–7), with $T = 1$ sec and the coefficients a_i ($i = 1, \ldots, 3$) given by eqs. (9–11), with $r = 4$. The contraction rates in eqs. (12–13) have been chosen as $\beta_1 = 0.6$ and $\beta_2 = 0.7$.

In the first simulation, it is $\phi_1^I = \pi/4$ and $\phi_2^I = -\pi/4$, while the desired values are $\phi_1^d = 0$ and $\phi_2^d = 0$ (a singular configuration). Figure 2 shows the cyclic evolution of x_1, y_1, and θ_1. The trajectory of ϕ_1 and ϕ_2 is reported in Fig. 3. Exponential convergence is confirmed by the logarithmic plot of the state error norm $||q - q^d||$ in Fig. 4. The motion of the first trailer is shown in Fig. 5 (with different scale on the two axes). Note how the cyclic path length is reduced over the iterations. Figure 6 depicts the vehicle configurations at the beginning of phase II, at the end of the first and of the 15-th iteration.

The second simulation starts from $\phi_1^I = \pi/8$, $\phi_2^I = 0$, with the desired configuration given as $\phi_1^d = -\pi/4$,

[1]Note that x_1, y_1 and θ_1 return to zero, as can be verified by integrating the first three equations of the original system (2–3). Therefore, the open-loop control inputs (6–7) are exactly cyclic in x_1, y_1 and θ_1, as required by our method.

$\phi_2^d = \pi/3$. The stabilization point is now regular; in particular, the method in [11] would not apply in this case. Figure 7 shows the time evolution of x_1, y_1 and θ_1, while Fig. 8 reports the history of ϕ_1 and ϕ_2. Again, Fig. 9 indicates that exponential convergence has been achieved. Figures 10–11 report, respectively, the cartesian motion of the first trailer and the configurations of the vehicle at the beginning of phase II, at the end of the first and of the 15-th iteration.

5 Conclusions

We have presented a method for asymptotic stabilization of the general two-trailer vehicle, a system for which neither an exact planner nor a stabilizing feedback were available. Our solution makes use of two tools: a nonhomogeneous nilpotent approximation, valid both at regular and singular points, and an iterative steering scheme. The satisfactory performance of the control algorithm (in particular, the exponential rate of convergence) has been confirmed by simulations. The above approach is in principle generalizable to the case of N trailers, provided that an appropriate contracting open-loop control is identified.

However, some issues deserve further investigation:

- During phase I, care should be taken to prevent the system assuming configurations such that $\phi_1 = \pi$ or $\phi_2 = \pi$, causing loss of controllability.
- The error induced by the use of the nilpotent approximation may be large if a large reconfiguration is required on ϕ_1, ϕ_2. To preserve the contraction property in this case, one may either use small contraction factors β_1, β_2 or divide the steering task into a series of subtasks, through the definition of via points.
- The possibility of shaping the system trajectories during stabilization through the choice of the open-loop controls could be exploited for achieving fine positioning in cluttered workspaces.

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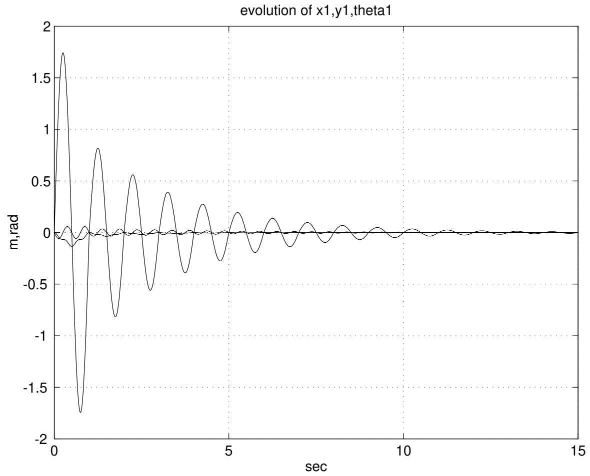

Figure 2: Simulation 1: Evolution of x_1, y_1 and θ_1

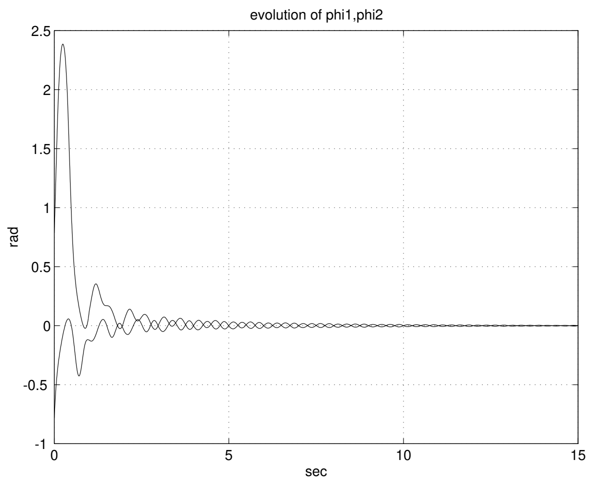

Figure 3: Simulation 1: Evolution of ϕ_1 and ϕ_2

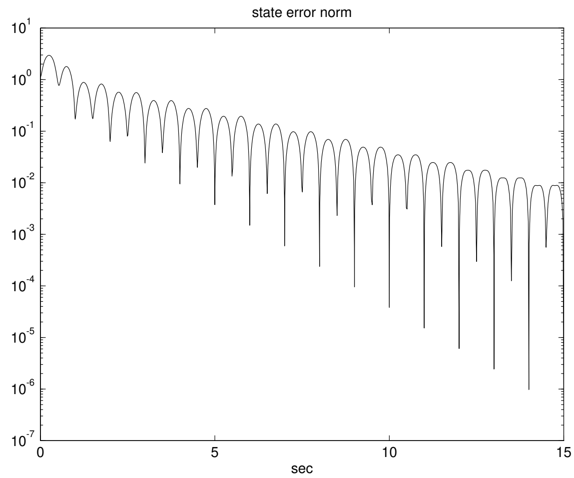

Figure 4: Simulation 1: Logarithmic plot of the state error norm

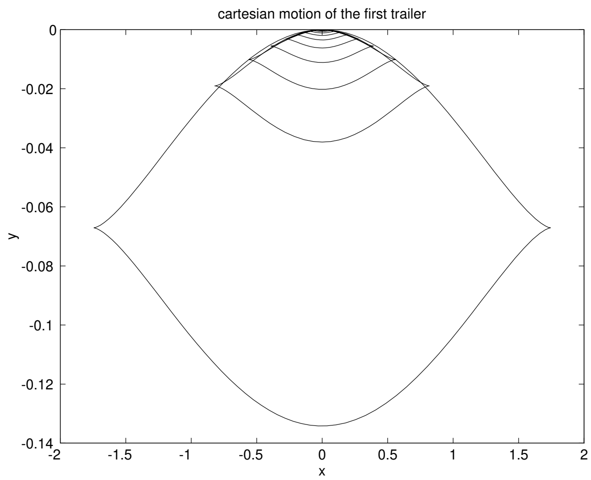

Figure 5: Simulation 1: Motion of the first trailer

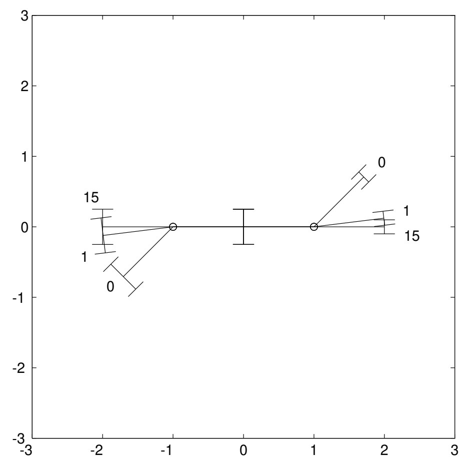

Figure 6: Simulation 1: Configuration of the vehicle at the beginning of phase II (0) after one iteration (1) and after 15 iterations

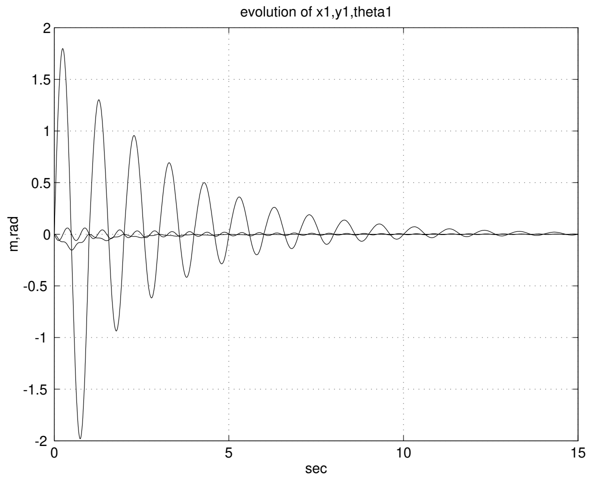

Figure 7: Simulation 2: Evolution of x_1, y_1 and θ_1

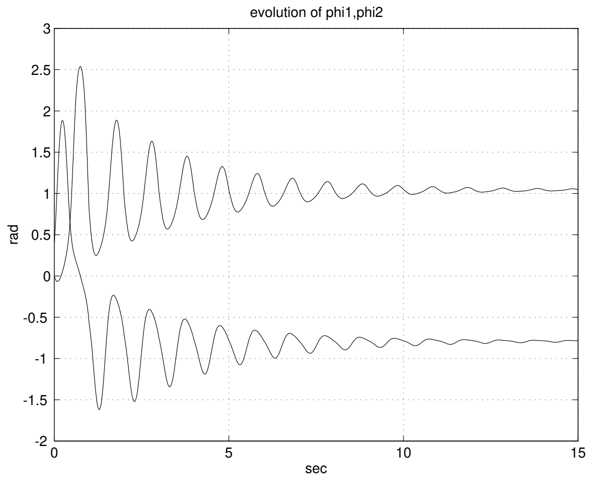

Figure 8: Simulation 2: Evolution of ϕ_1 and ϕ_2

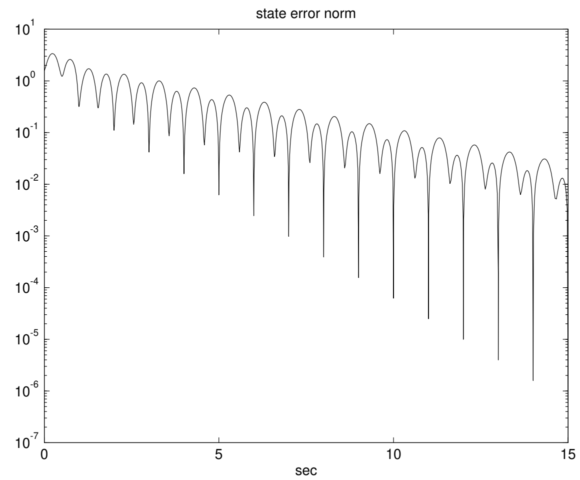

Figure 9: Simulation 1: Logarithmic plot of the state error norm

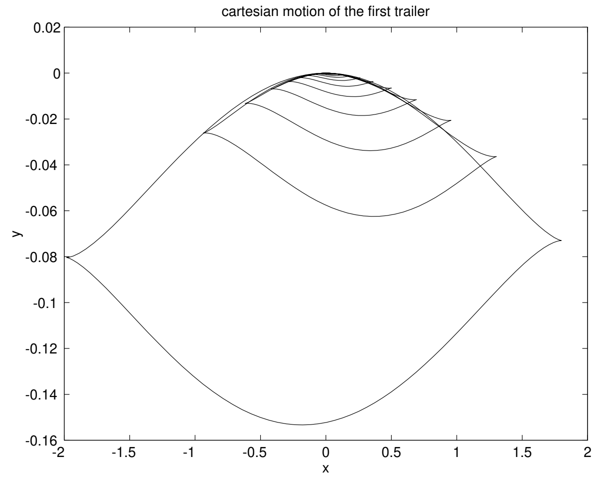

Figure 10: Simulation 2: Motion of the first trailer

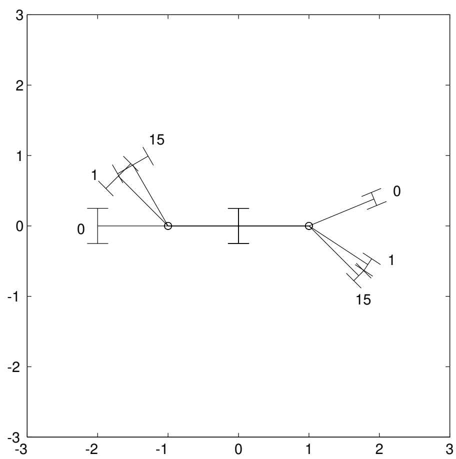

Figure 11: Simulation 2: Configuration of the vehicle at the beginning of phase II (0) after one iteration (1) and after 15 iterations