*Robotics 2*

# Hybrid Force/Motion Control

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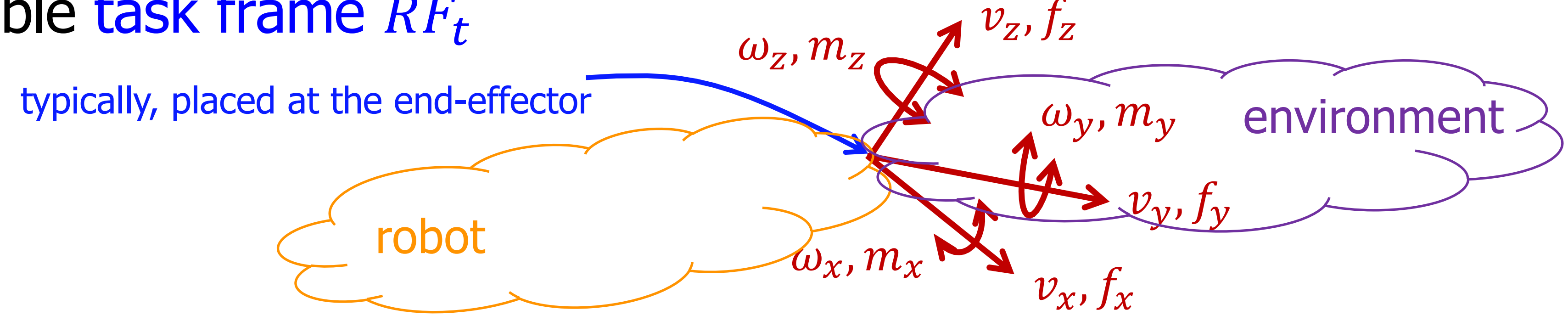
# Hybrid force/motion control

- we consider contacts/interactions between a robot and a stiff environment that naturally constrains the end-effector motion
- compared to an approach using the constrained/reduced robot dynamics with (bilateral) geometric constraints, the differences are
  - the hybrid control law is designed in ideal conditions, but now unconstrained directions of motion and constrained force directions are defined in a more direct way using a task frame formalism
  - all non-ideal conditions (compliant surfaces, friction at the contact, errors in contact surface orientation) are handled explicitly in the control scheme by a geometric filtering of the measured quantities
    - considering only signal components that should appear in certain directions based on the nominal task model, and treating those that should not be there as disturbances to be rejected
- the hybrid control law avoids to introduce conflicting behaviors (force control vs. motion control) in all task space directions!!

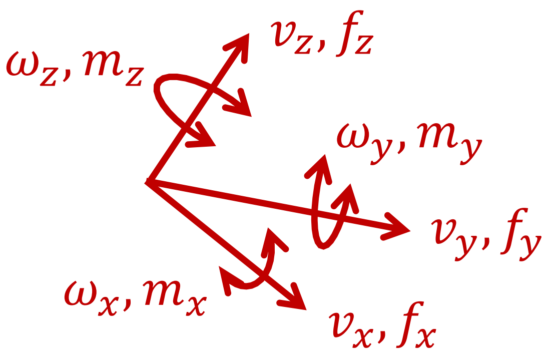
# Natural constraints

- in ideal conditions (robot and environment are perfectly rigid, contact is frictionless), two sets of generalized directions can be defined in the task space
  - end-effector motion $(v/\omega)$ is prohibited along/around $6-k$ directions (since the environment reacts there with forces/torques)
  - reaction forces/torques $(f/m)$ are absent along/around $k$ directions (where the environment does not prevent end-effector motions)
- these constraints have been called the natural constraints on force and motion associated to the task geometry
- the two sets of directions are characterized through the axes of a suitable task frame $RF_t$

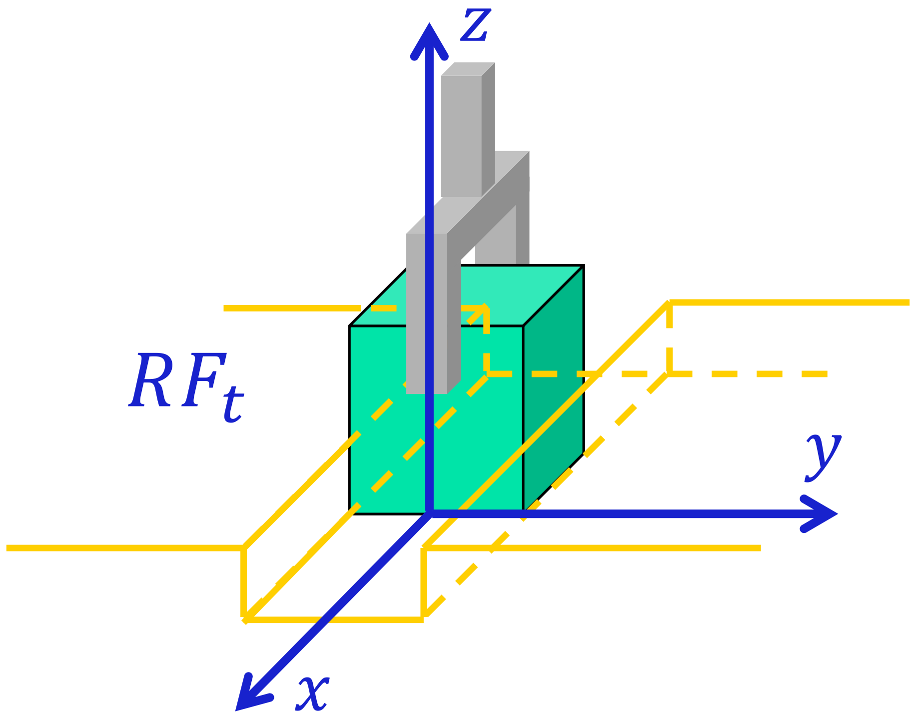


# Artificial constraints

- the way task execution should be performed can be expressed in terms of so-called artificial constraints that specify the desired values (to be imposed by the control law)
  - for the end-effector velocities $(v/\omega)$ along/around $k$ directions where feasible motions can occur
  - for the contact forces/torques $(f/m)$ along/around $6-k$ directions where admissible reactions of the environment can occur
- the two sets of directions are complementary (they cover the 6D generalized task space) and mutually orthogonal, while the task frame can be time-varying ("moves with task progress")
  - directions are intended as 6D screws: twists $V = (v^T\ \omega^T)^T$ and wrenches $F = (f^T\ m^T)^T$



# Task frame and constraints - example 1




task: slide the cube along a guide

**natural (geometric) constraints**

$$\left.\begin{aligned} v_y &= v_z = 0 \\ \omega_x &= \omega_z = 0 \end{aligned}\right\} \; 6-k=4$$

$$\left. f_x = m_y = 0 \right\} \; k=2$$

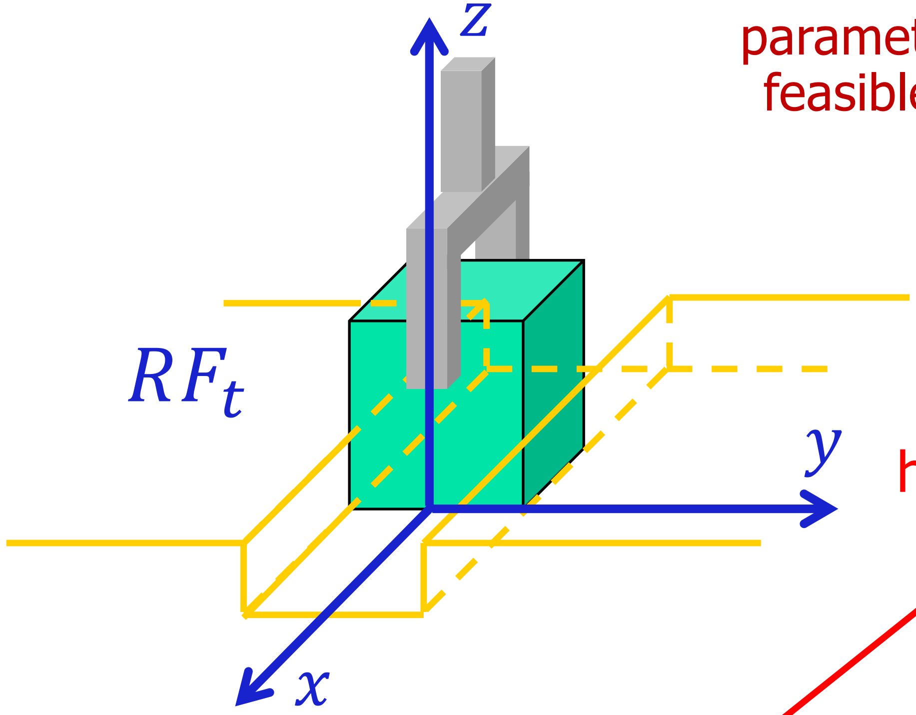
**artificial constraints (to be imposed by the control law)**

$$6-k=4 \; \left\{ \begin{aligned} & f_y = f_{y,des} \; (=0) \text{ (to avoid internal stress)} \\ & m_x = m_{x,des} \; (=0), m_z = m_{z,des} (=0) \\ & f_z = f_{z,des} \end{aligned} \right.$$

$$k=2 \; \left\{ \begin{aligned} & \omega_y = \omega_{y,des} = 0 \text{ (to slide and not to roll !!)} \\ & v_x = v_{x,des} \end{aligned} \right.$$

$v$ = linear velocity
$\omega$ = angular velocity
$f$ = force
$m$ = moment

# Selection of directions - example 1




parametrization of feasible motions

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ \omega_y \end{pmatrix} = T \begin{pmatrix} v_x \\ \omega_y \end{pmatrix}$$

here, constant and unitary
("selection" of columns from
the $6 \times 6$ identity matrix)

parametrization of feasible reactions

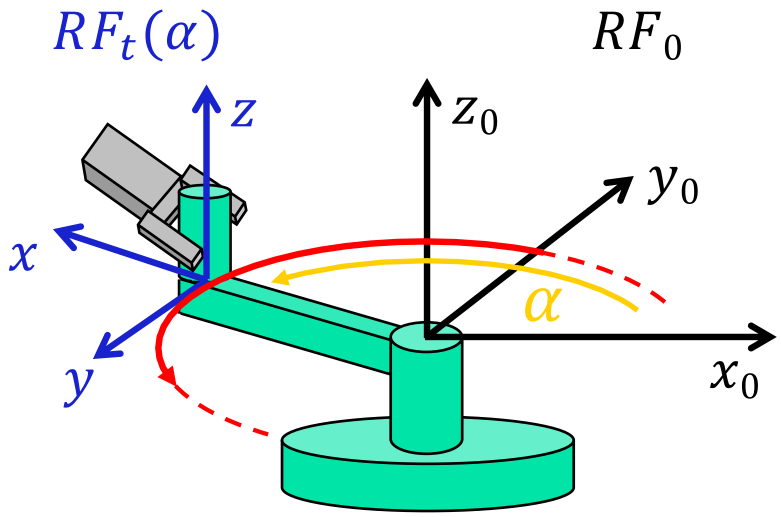
$$\begin{pmatrix} f \\ m \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_y \\ f_z \\ m_x \\ m_z \end{pmatrix} = Y \begin{pmatrix} f_y \\ f_z \\ m_x \\ m_z \end{pmatrix}$$

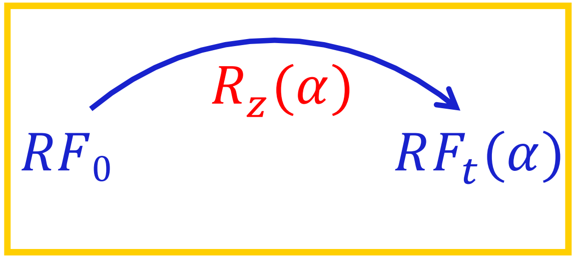
$$T^T Y = 0$$

reaction forces/torques do **not** perform work on feasible motions

$$\begin{pmatrix} f^T & m^T \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} = 0$$

# Task frame and constraints - example 2




task: turning a crank
(free handle)

**natural constraints**

$v_x = v_z = 0$

$\omega_x = \omega_y = 0$

$f_y = m_z = 0$

**artificial constraints**

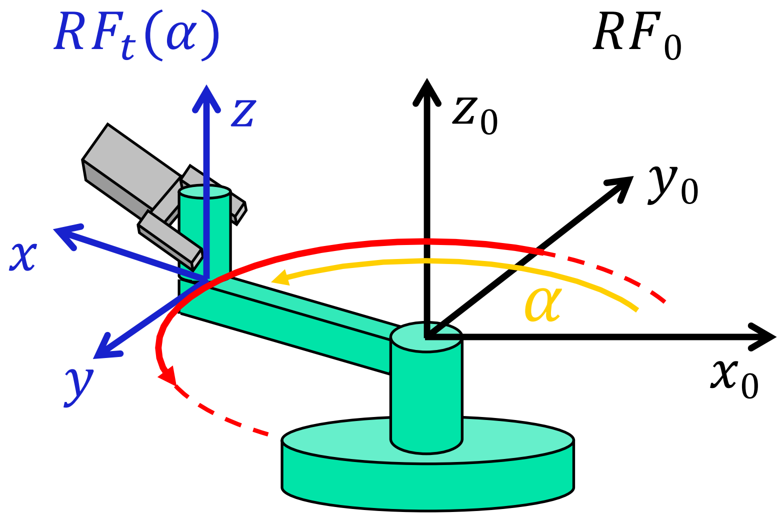
$f_x = f_{x,des}\ (= 0), f_z = f_{z,des}\ (= 0)$

$m_x = m_{x,des}\ (= 0), m_y = m_{y,des}\ (= 0)$

$v_y = v_{y,des}$ (the tangent speed of rotation)

$\omega_z = \omega_{z,des}$ $(= 0$ if handle should not spin)

# Selection of directions – example 2




parametrization of feasible motions

$$\begin{pmatrix} {}^0v \\ {}^0\omega \end{pmatrix} = \begin{pmatrix} R^T(\alpha) & 0 \\ 0 & R^T(\alpha) \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_y \\ \omega_z \end{pmatrix}$$

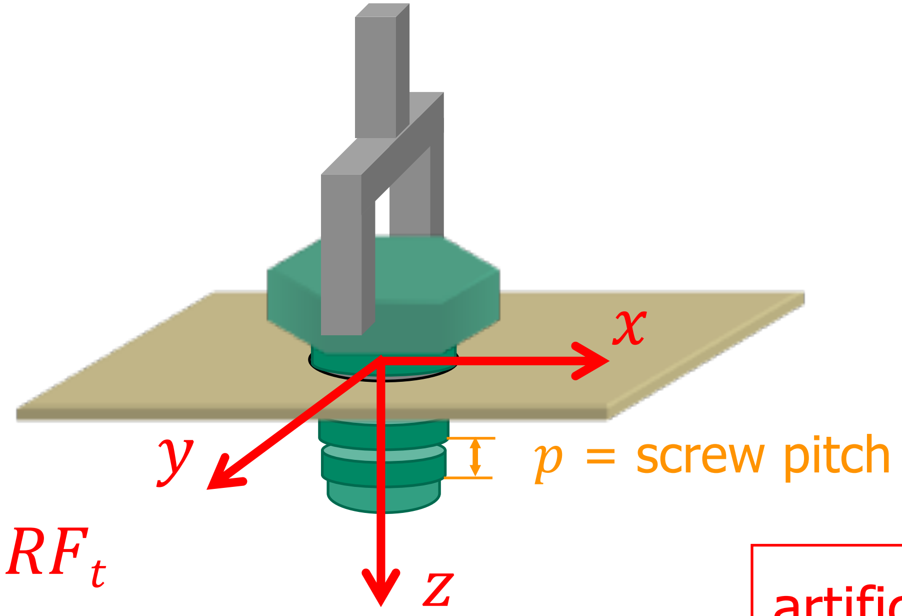
$$= T(\alpha) \begin{pmatrix} v_y \\ \omega_z \end{pmatrix}$$

$$\boxed{T^T(\alpha) Y(\alpha) = 0}$$

parametrization of feasible reactions

$$\begin{pmatrix} {}^0f \\ {}^0m \end{pmatrix} = \begin{pmatrix} R^T(\alpha) & 0 \\ 0 & R^T(\alpha) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_x \\ f_z \\ m_x \\ m_y \end{pmatrix} = Y(\alpha) \begin{pmatrix} f_x \\ f_z \\ m_x \\ m_y \end{pmatrix}$$

# Task frame and constraints - example 3




task: insert a screw in a bolt

natural constraints (partial…)

$v_x = v_y = 0$

$\omega_x = \omega_y = 0$

the screw proceeds along and around the $z$-axis, but **not** in an independent way! (1 dof)

accordingly, $f_z$ and $m_z$ **cannot** be independent

artificial constraints (abundant…)

$f_x = f_{x,des} = 0, f_y = f_{y,des} = 0$

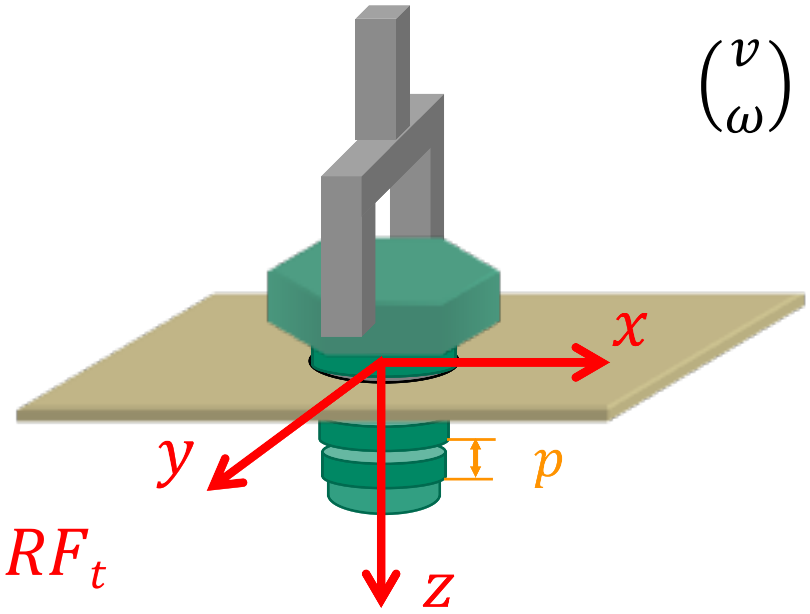
$m_x = m_{x,des} = 0, m_y = m_{y,des} = 0$

$\boxed{v_z = v_{z,des}, \omega_z = \omega_{z,des} = (2\pi/p) v_{z,des}}$

$\boxed{f_z = f_{z,des}, m_z = m_{z,des} \text{ (a function of } f_{z,des})}$

wrench (force/torque) direction should be orthogonal to motion twist!

# Selection of directions – example 3




$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & \frac{2\pi}{p} \end{pmatrix}^T v_z = T v_z \quad (k = 1)$$

or $\boxed{\omega_z = 2\pi \frac{v_z}{p}}$

$Y$: such that $T^T Y = 0$

$$\boxed{f_z = -\frac{2\pi}{p} m_z}$$

$(6 - k = 5)$

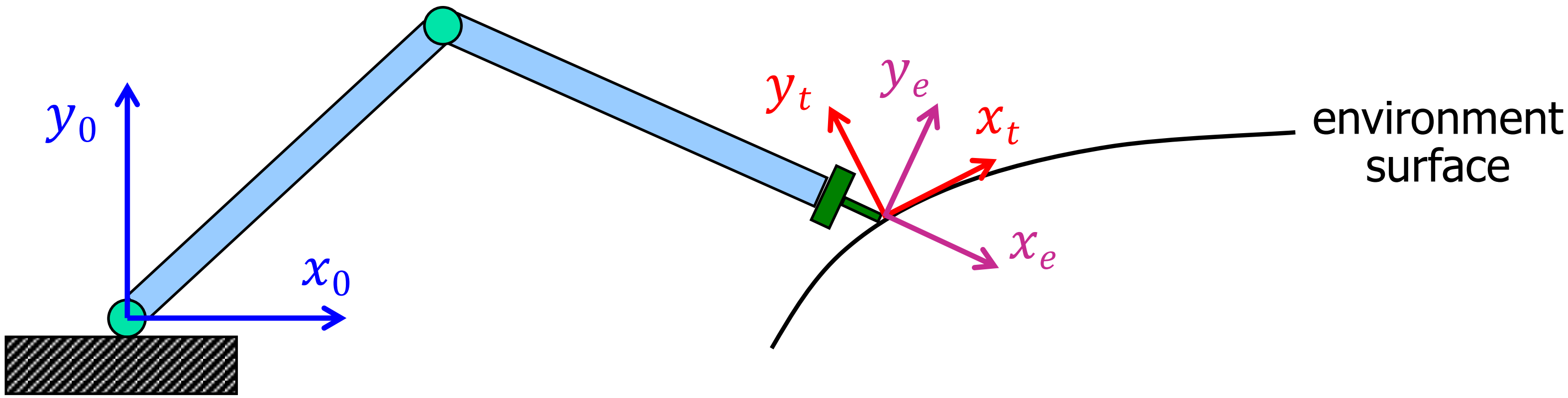
$$\begin{pmatrix} f \\ m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\pi/p \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ m_x \\ m_y \\ m_z \end{pmatrix} = Y \begin{pmatrix} f_x \\ f_y \\ m_x \\ m_y \\ m_z \end{pmatrix}$$

the columns of $T$ and $Y$ do not necessarily coincide with selected columns of the $6 \times 6$ identity matrix

⇒ generalized (screw) directions

# Frames of interest – example 4

planar motion of a 2R robot in pointwise contact with a surface ($M = 2$)




- task frame $\boldsymbol{RF_t}$ used for an independent definition of the hybrid reference values (here: ${}^t v_{x,des}$ $[k = 1]$ and ${}^t f_{y,des}$ $[M - k = 1]$) and for computing the errors that drive the feedback control law
- sensor frame $\boldsymbol{RF_e}$ (here: $RF_2$) where the force ${}^e f = ({}^e f_x, {}^e f_y)$ is measured
- base frame $\boldsymbol{RF_0}$ in which the end-effector velocity is expressed (here: ${}^0 v = \left({}^0 v_x, {}^0 v_y\right)$ of $O_2$), computed using robot Jacobian and joint velocities

all quantities (and errors!) should be expressed ("rotated") in the same reference frame ⇒ the task frame!

# General parametrization of hybrid tasks

a "description" of robot-environment contact type: it implicitly defines the task frame

$$\begin{cases} \begin{pmatrix} v \\ \omega \end{pmatrix} = T(s)\dot{s} & \boxed{s \in \mathbb{R}^k} \\ \begin{pmatrix} f \\ m \end{pmatrix} = Y(s)\lambda & \boxed{\lambda \in \mathbb{R}^{M-k}} \end{cases}$$

parametrizes robot E-E free motion

parametrizes reaction forces/torques

reaction forces/torques do not perform work on E-E displacements

$$T^T(s)Y(s) = 0$$

axes directions of task frame depend in general on $s$ (i.e., on robot E-E pose in the environment)

in the previous examples, and in general, it is $M = 6$

+

robot dynamics

$$\boxed{M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = u + J^T(q)\begin{pmatrix} f \\ m \end{pmatrix}}$$

robot kinematics

$$\boxed{\begin{pmatrix} v \\ \omega \end{pmatrix} = J(q)\dot{q}}$$

# Hybrid force/velocity control

- control objective: to impose desired task evolution to the parameters $s$ of motion and to the parameters $\lambda$ of force

$$s \longrightarrow s_d(t) \qquad \lambda \longrightarrow \lambda_d(t)$$

- the control law is designed in two steps
  1. exact linearization and decoupling in the task frame by feedback

     closed-loop model $\rightarrow$ $\begin{pmatrix} \ddot{s} \\ \lambda \end{pmatrix} = \begin{pmatrix} a_s \\ a_\lambda \end{pmatrix}$

  2. (linear) design of $a_s$ and $a_\lambda$ so as to impose the desired dynamic behavior to the errors $e_s = s_d - s$ and $e_\lambda = \lambda_d - \lambda$

- assumptions: $N = M$ ($= 6$ usually), $J(q)$ out of singularity

Note: in "simple" cases, $\dot{s}$ and $\lambda$ are just single components of $v$ or $\omega$ and of $f$ or $m$; accordingly, $T$ and $Y$ will be simple 0/1 selection matrices

# Feedback linearization in task space

$$J(q)\dot{q} = \begin{pmatrix} v \\ \omega \end{pmatrix} = T(s)\dot{s} \Rightarrow J\ddot{q} + \dot{J}\dot{q} = T\ddot{s} + \dot{T}\dot{s} \Rightarrow \ddot{q} = J^{-1}(T\ddot{s} + \dot{T}\dot{s} - \dot{J}\dot{q})$$

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = u + J^T(q)\begin{pmatrix} f \\ m \end{pmatrix} = u + J^T(q)Y(s)\lambda$$

nonsingular $N \times N$ matrix
(under the assumptions made)

$$\left(M(q)J^{-1}(q)T(s) \;\vdots\; -J^T(q)Y(s)\right)\begin{pmatrix} \ddot{s} \\ \lambda \end{pmatrix}$$

$$+ M(q)J^{-1}(q)\left(\dot{T}(s)\dot{s} - \dot{J}(q)\dot{q}\right) + S(q,\dot{q})\dot{q} + g(q) = u$$

$$u = (MJ^{-1}T \;\vdots\; -J^TY)\begin{pmatrix} a_s \\ a_\lambda \end{pmatrix} + MJ^{-1}\left(\dot{T}\dot{s} - \dot{J}\dot{q}\right) + S\dot{q} + g$$

linearizing and decoupling control law

$$\Rightarrow \begin{pmatrix} \ddot{s} \\ \lambda \end{pmatrix} = \begin{pmatrix} a_s \\ a_\lambda \end{pmatrix} \begin{matrix} \}\, k \\ \}\, M - k \end{matrix}$$

$s$ has "relative degree" = 2

$\lambda$ has "relative degree" = 0

# Stabilization with $a_s$ and $a_\lambda$

as usual, it is sufficient to apply linear control techniques for the exponential stabilization of tracking errors (on each single, input-output decoupled channel)

$$\boxed{a_s = \ddot{s}_d + K_D(\dot{s}_d - \dot{s}) + K_P(s_d - s)}$$

$K_P, K_D > 0$ and diagonal

$$\Rightarrow \quad \ddot{e}_s + K_D \dot{e}_s + K_P e_s = 0 \quad \Rightarrow \quad e_s = s_d - s \longrightarrow 0$$

$K_I \geq 0$ diagonal

$$\boxed{a_\lambda = \lambda_d + K_I \int (\lambda_d - \lambda) dt}$$

$a_\lambda = \lambda_d$ would be enough, but adding an integral with the **force error** gives more robustness to (constant) disturbances

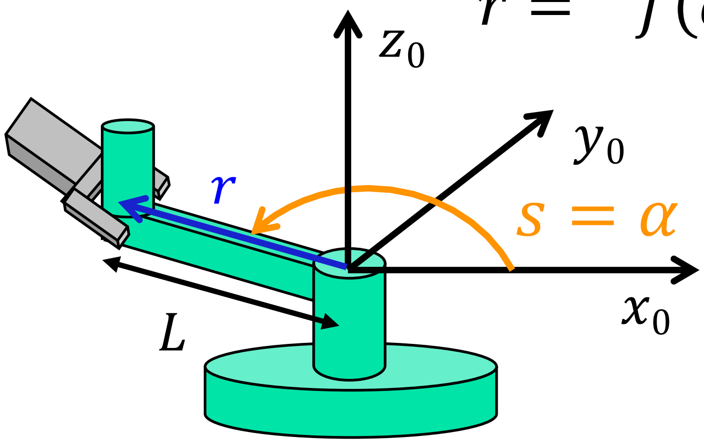
$$\Rightarrow \quad e_\lambda + K_I \int e_\lambda \, dt = 0 \quad \Rightarrow \quad e_\lambda = \lambda_d - \lambda \longrightarrow 0$$

we need "values" for $s$, $\dot{s}$ and $\lambda$ to be extracted from actual measurements !

# "Filtering" position and force measures

- $s$ and $\dot{s}$ are obtained from measures of $q$ and $\dot{q}$, equating the descriptions of the end-effector pose and velocity "from the robot side" (direct and differential kinematics) and "from the environment side" (function of $s, \dot{s}$)

example




$$ {}^0r = {}^0f(q) = \begin{pmatrix} L\cos s \\ L\sin s \\ 0 \end{pmatrix} \Rightarrow s = \operatorname{atan2}\{ {}^0f_y(q),\ {}^0f_x(q)\} $$

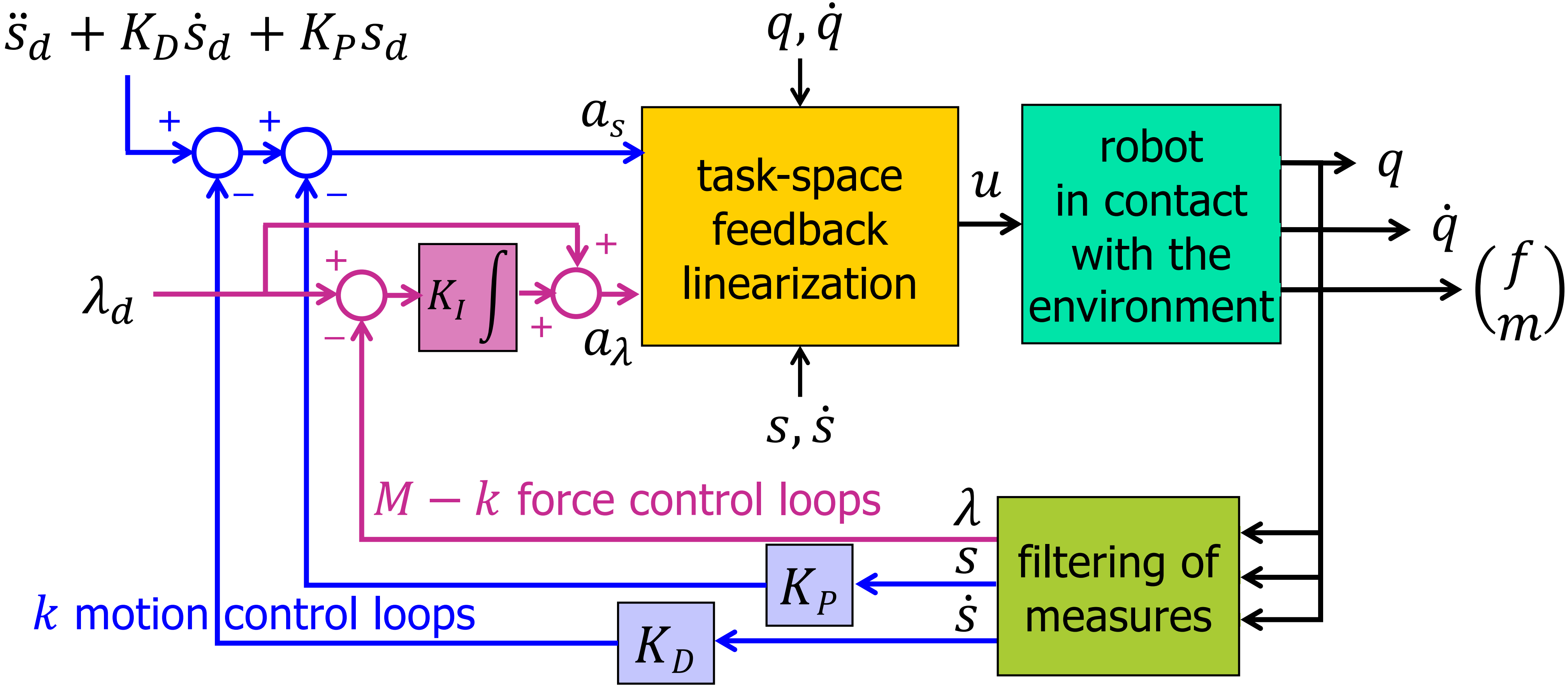
$$ J(q)\dot{q} = T(s)\dot{s} \Rightarrow \dot{s} = T^{\#}(s)J(q)\dot{q} $$

- $\lambda$ is obtained from force/torque measures at end-effector

$$ \begin{pmatrix} f \\ m \end{pmatrix} = Y(s)\lambda \Rightarrow \lambda = Y^{\#}(s)\begin{pmatrix} f \\ m \end{pmatrix} $$

pseudoinverses of "tall" matrices having full column rank, e.g.,
$T^{\#} = (T^T T)^{-1} T^T$
(or weighted)

# Block diagram of hybrid control



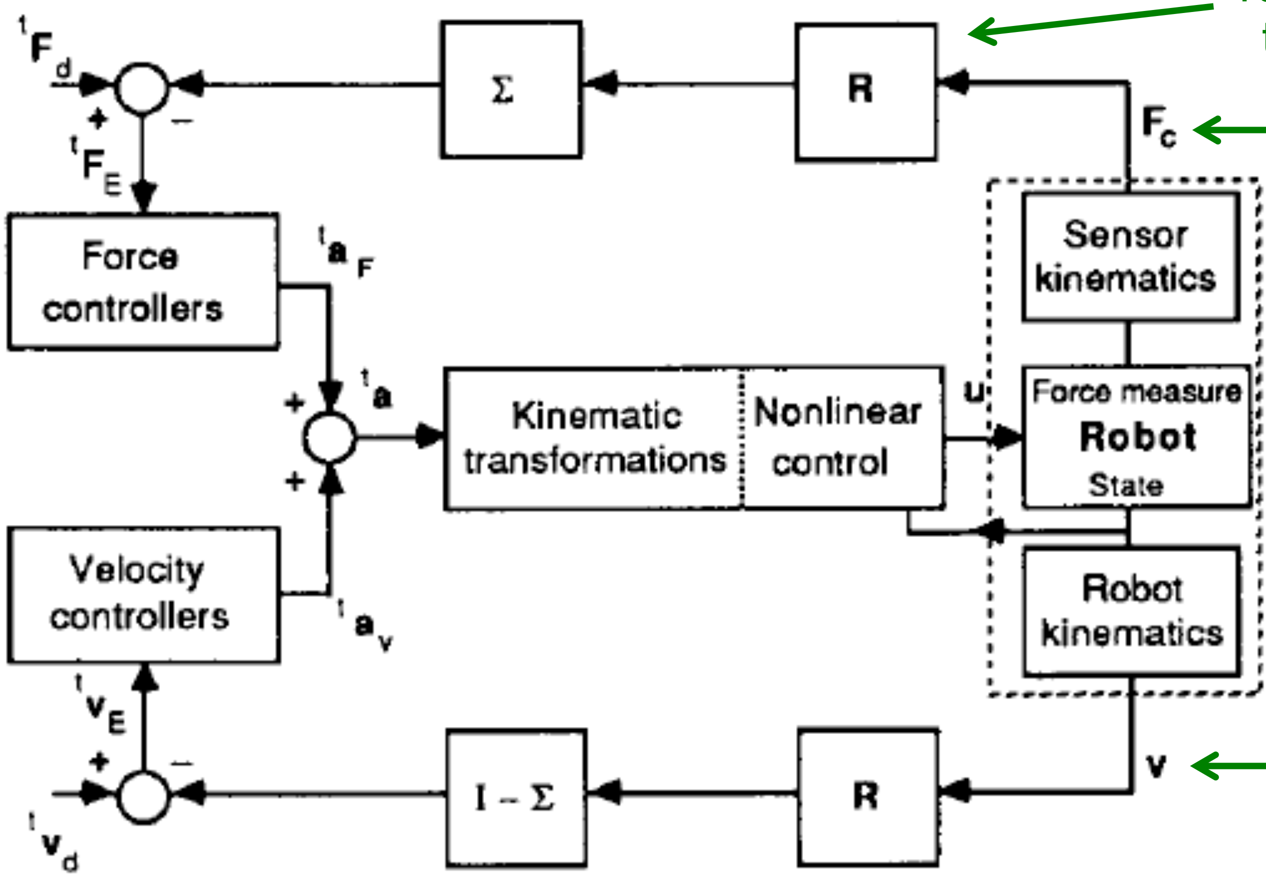

usually $M = 6$ (complete 3D space)

limit cases $k = M$: no force control loops, only motion (free motion)

$k = 0$ : no motion control loops, only force ("frozen" robot end-effector)

# Block diagram of hybrid control

simpler case of 0/1 selection matrices




rotation matrix to task frame $RF_t$

in base frame $RF_0$

in sensor frame $RF_s$

in base frame $RF_0$

compact notation in this slide

$$F = \begin{pmatrix} f \\ m \end{pmatrix}$$

$$V = \begin{pmatrix} v \\ \omega \end{pmatrix}$$

$\dot{s}$ and $\lambda$ are just single components of $v$ (or $\omega$) and $f$ (or $m$)

$T$ and $Y$ are replaced by 0/1 selection matrices: $I - \Sigma$ and $\Sigma$

# Force control via an impedance model

- in a force-controlled direction of the hybrid task space, when the contact stiffness is limited (i.e., far from infinite, as assumed in the ideal case), one may use impedance model ideas to explicitly control the contact force
  - let $x$ be the position of the robot along such a direction, $x_d$ the (constant) contact point, $k_s > 0$ the contact (viz., sensor) stiffness, and $f_d > 0$ the desired contact force
- the impedance model is chosen then as

$$m_m \ddot{x} + d_m \dot{x} + k_s(x - x_d) = f_d$$

where the force sensor measures $f_s = k_s(x - x_d)$, and only $m_m > 0$ and $d_m > 0$ are free model parameters

- after feedback linearization ($\ddot{x} = a_x$), the command $a_x$ is designed as

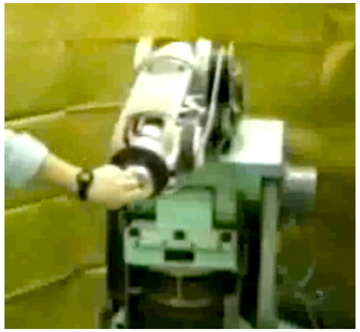
$$a_x = (1/m_m)[(f_d - f_s) - d_m \dot{x}]$$

which is a P-regulator of the desired force, with velocity damping

- the same control law works also before the contact ($f_s = 0$), guaranteeing a steady-state speed $\dot{x}_{ss} = f_d/d_m > 0$ in the approaching phase

# First experiments with hybrid control





video

video

MIMO-CRF robot
(DIS, Laboratorio di Robotica, 1991)

# Sources of inconsistency
## in force and velocity measurements

1. presence of friction at the contact
   - ➜ a reaction force component appears that opposes motion in a "free" motion direction (in case of Coulomb friction, the tangent force intensity depends also on the applied normal force ...)
2. compliance in the robot structure and/or at the contact
   - ➜ a (small) displacement may be present also along directions that are nominally "constrained" by the environment

NOTE: if the environment geometry at the contact is perfectly known, the task inconsistencies due to 1. and 2. on parameters $s$ and $\lambda$ are already filtered out by the pseudo-inversion of matrices $T$ and $Y$

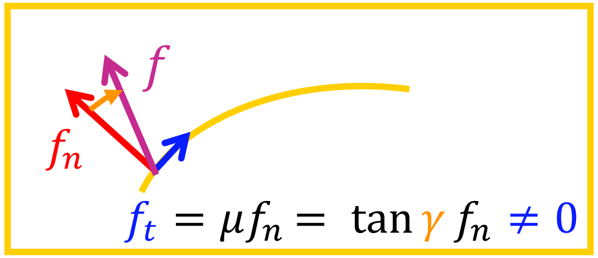
3. uncertainty on environment geometry at the contact
   - ➜ can be reduced/eliminated by real-time estimation processes driven by external sensors (e.g., vision –but also force!)

# Estimation of an unknown surface

how difficult is to estimate the unknown profile of the environment surface, using information from velocity and force measurements at the contact?

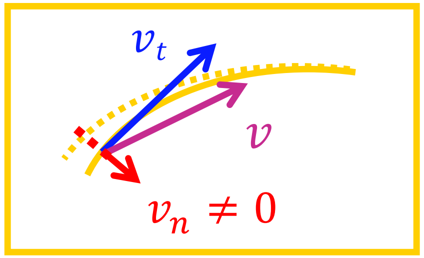
1. normal = nominal direction of measured force

   ... in the presence of contact motion with friction, the measured force $f$ is slightly rotated from the actual normal by an (unknown) angle $\gamma$




2. tangent = nominal direction of measured velocity

   ... compliance in the robot structure (joints) and/or at the contact may lead to a computed velocity $v$ having a small component along the actual normal to the surface

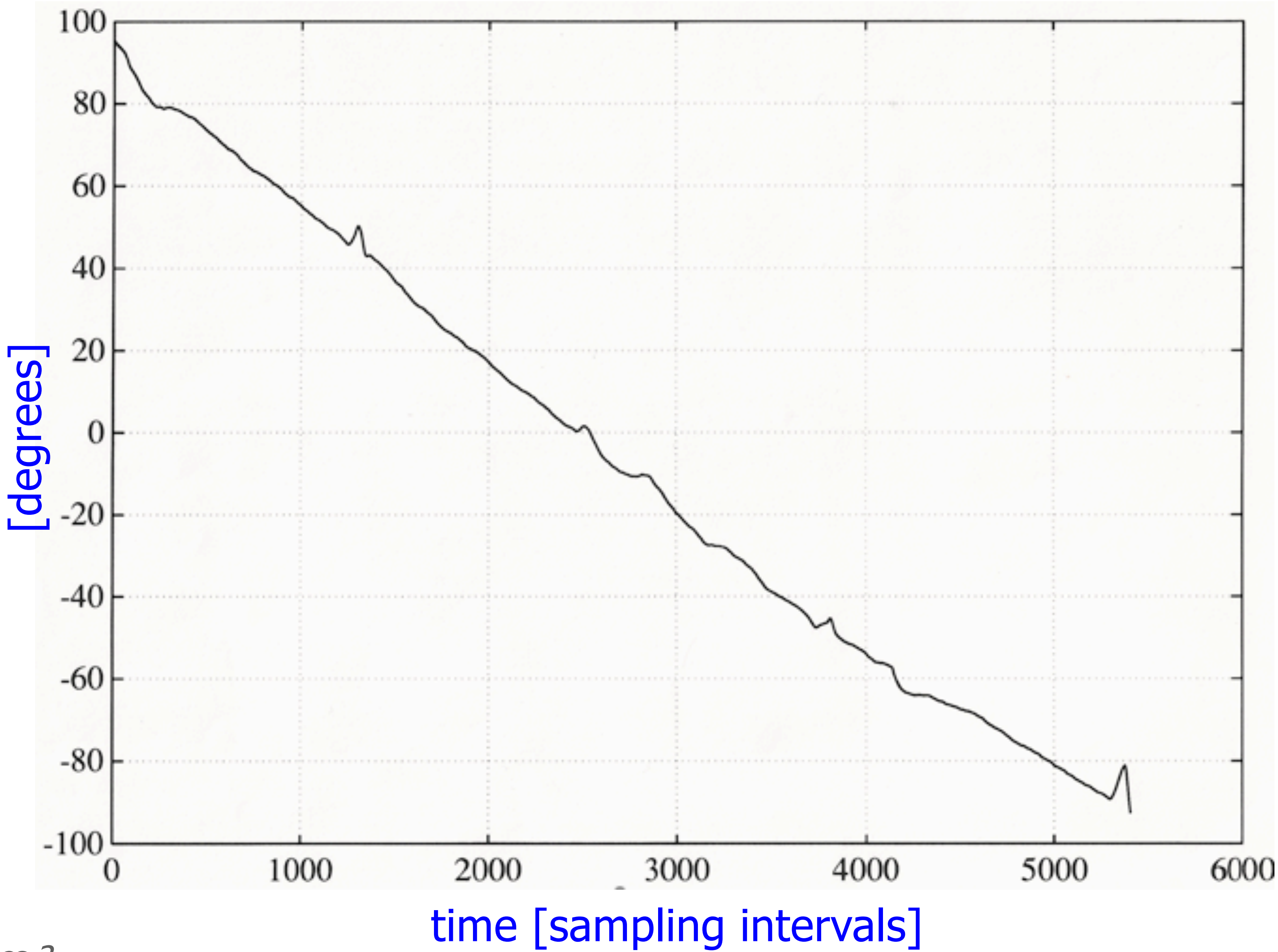



3. mixed method (sensor fusion) with RLS
   a. tangent direction is estimated by a recursive least squares method from position measurements
   b. friction angle is estimated by a recursive least squares method, using the current estimate of the tangent direction and from force measurements

to approach an unknown surface or to recover contact (in case of loss), the robot uses a simple exploratory logic

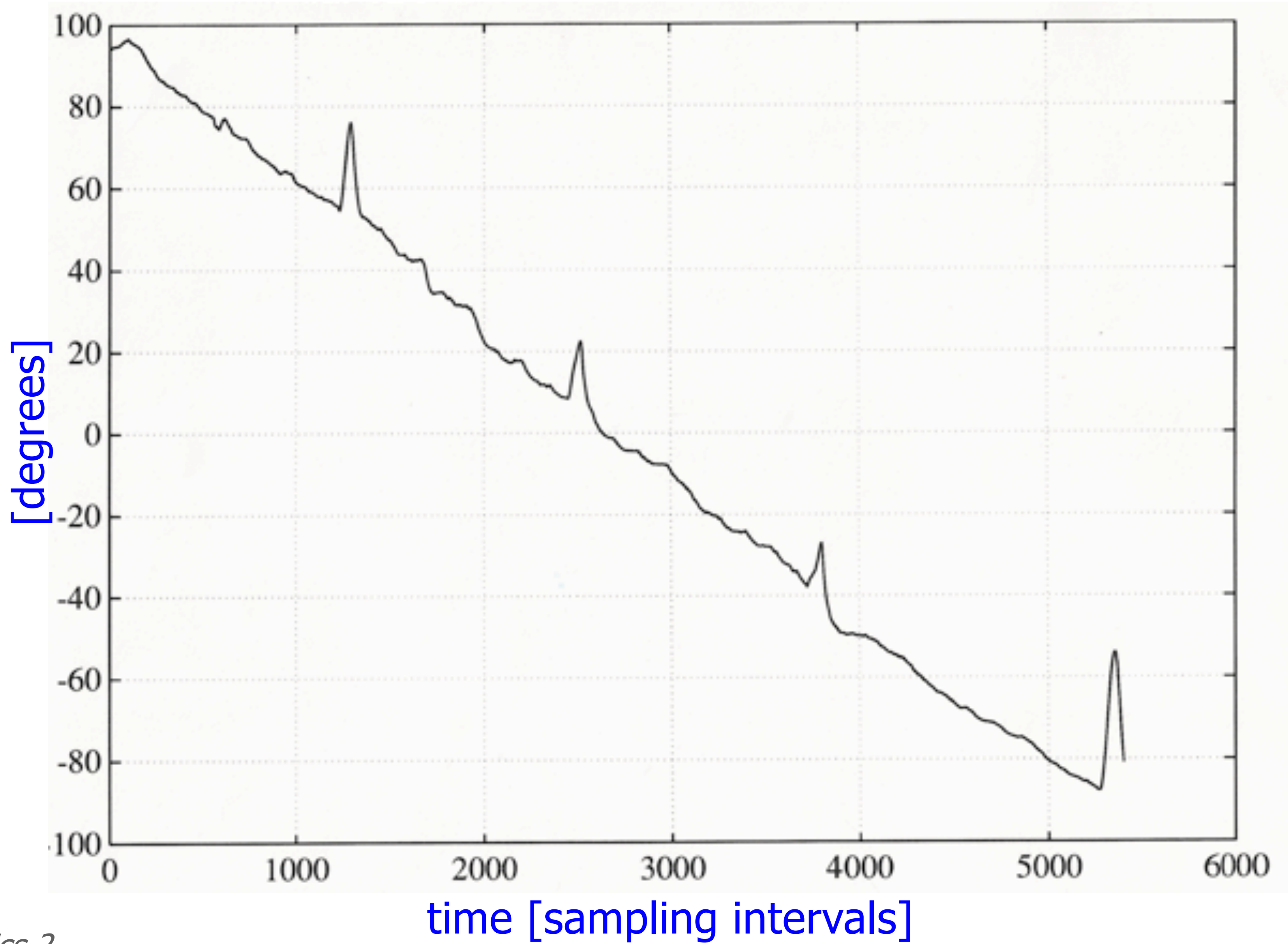
# Position-based estimation of the tangent

(for a circular surface traced at constant speed)

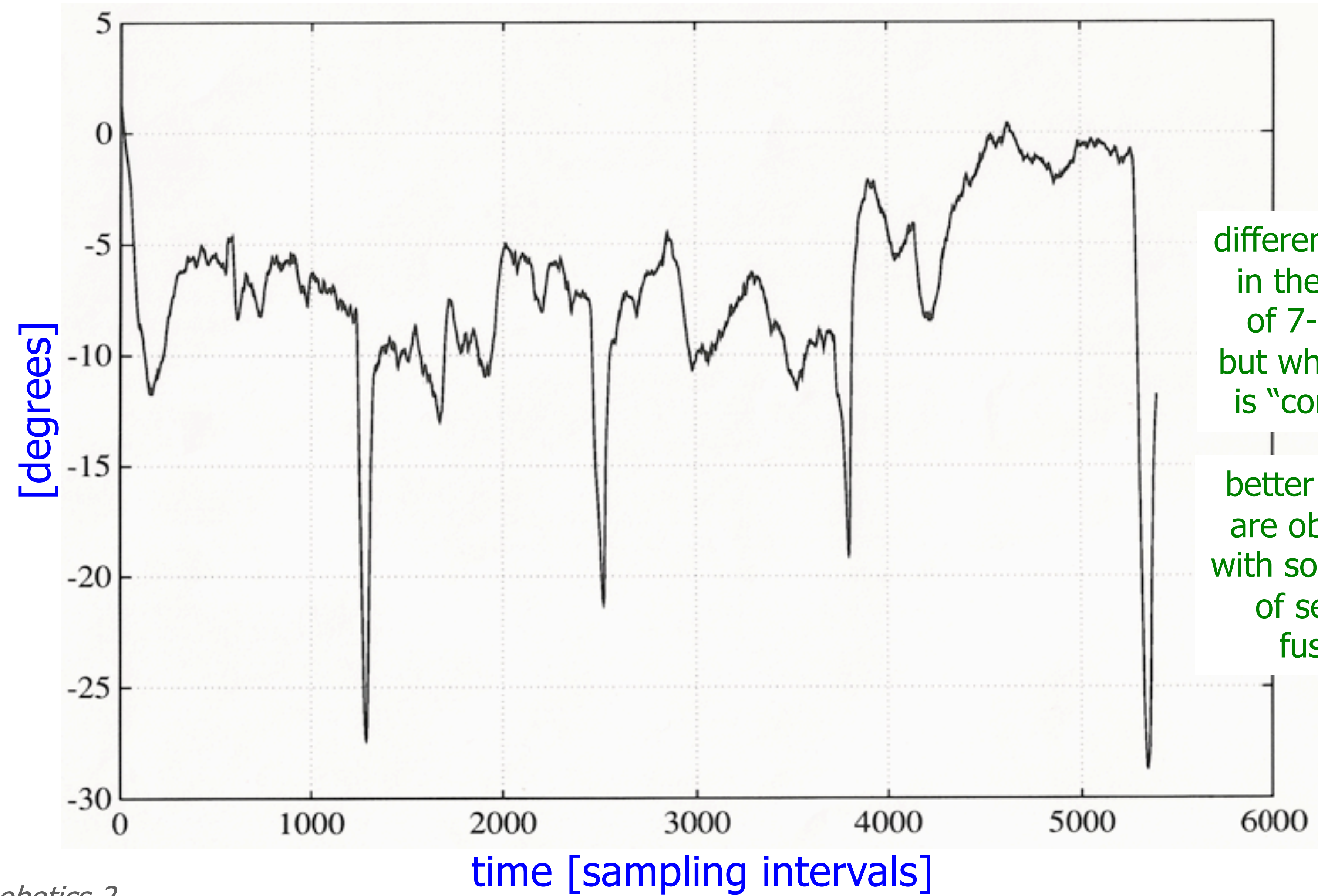


# Force-based estimation of the tangent

(for the same circular surface traced at constant speed)



# Difference between estimated tangents

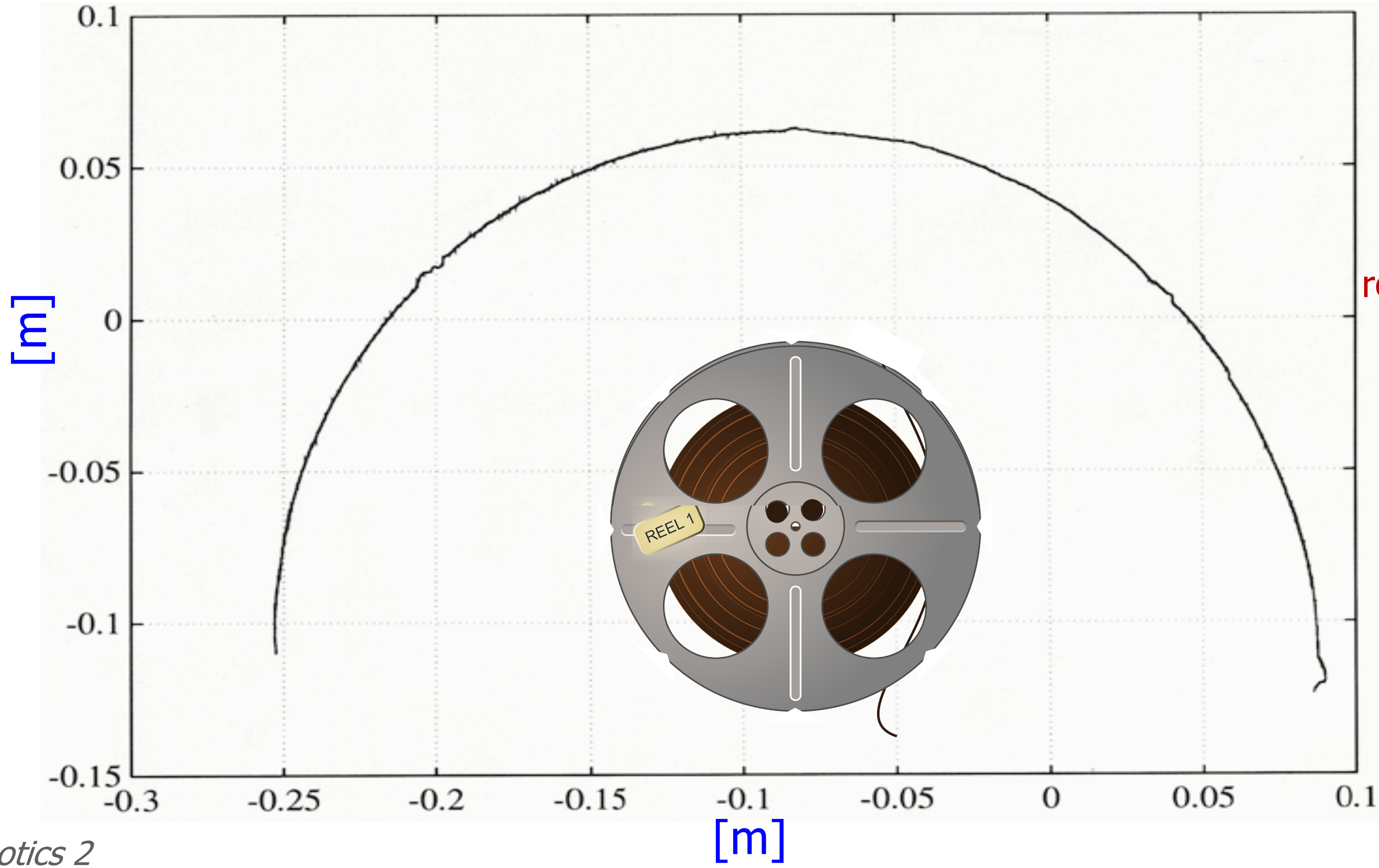



differences are in the order of 7-8° ... but which one is "correct"?

better results are obtained with some kind of sensor fusion

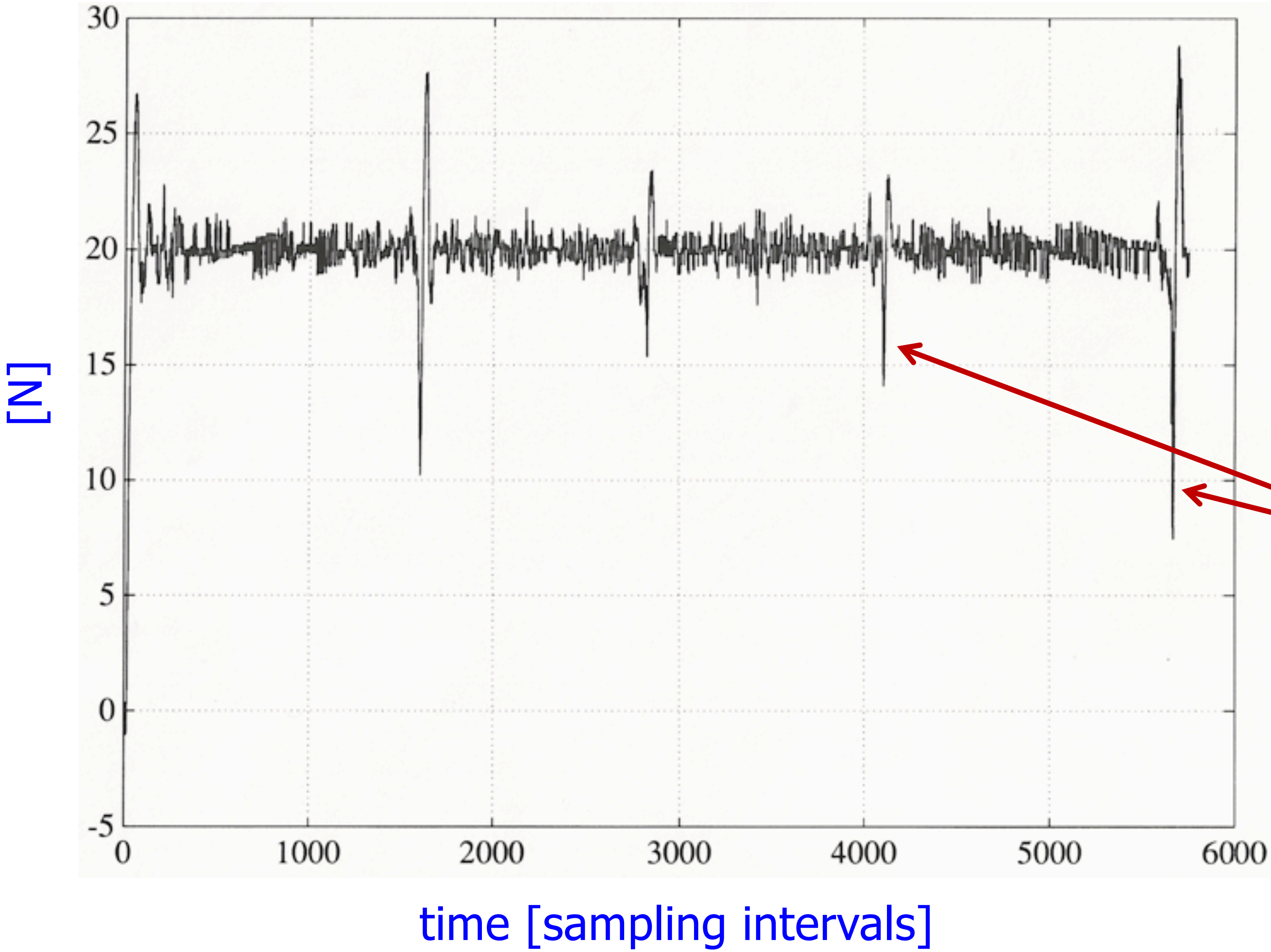
# Reconstructed surface profile

estimation by a RLS (Recursive Least Squares) method: we continuously update the coefficients of two quadratic polynomials that fit locally the unknown contour, using data fusion from both force and position/velocity measurements




this is the reconstructed contour of a cinema "film reel" (of radius = 17 cm)

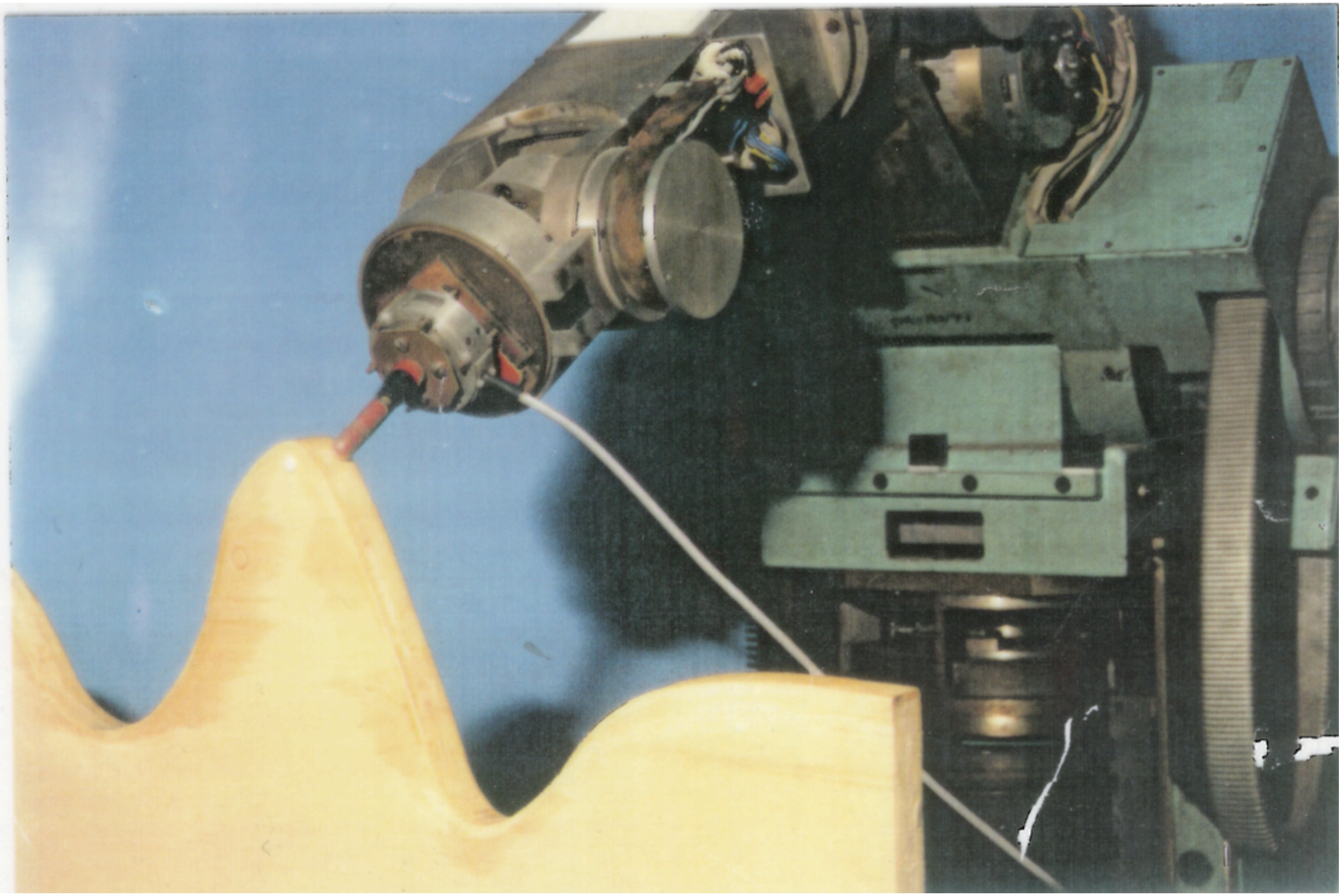
# Normal force




regulated to 20 N during simultaneous motion and estimation

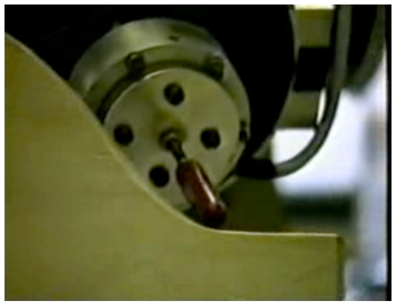
force peaks correspond to the grooves on the surface contour

# Contour estimation and hybrid control
## performed simultaneously



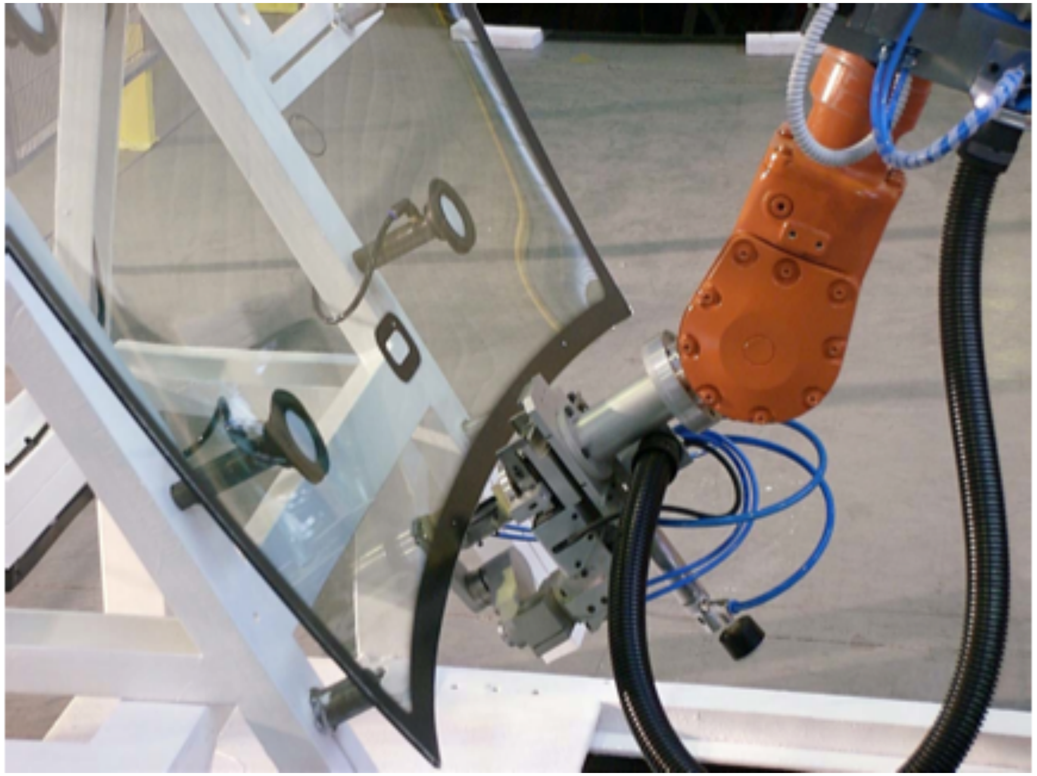
MIMO-CRF robot (DIS, Laboratorio di Robotica, 1992)

# Contour estimation and hybrid control

video

# Robotized deburring of car windshields



- car windshield with sharp edges and fabrication tolerances, with excess of material (PVB = Polyvinyl butyral for glueing glass layers) on the contour
- robot end-effector follows the pre-programmed path, despite the small errors w.r.t. the nominal windshield profile, thanks to the passive compliance of the deburring tool
- contact force between tool blades and workpiece can be independently controlled by a pneumatic actuator in the tool

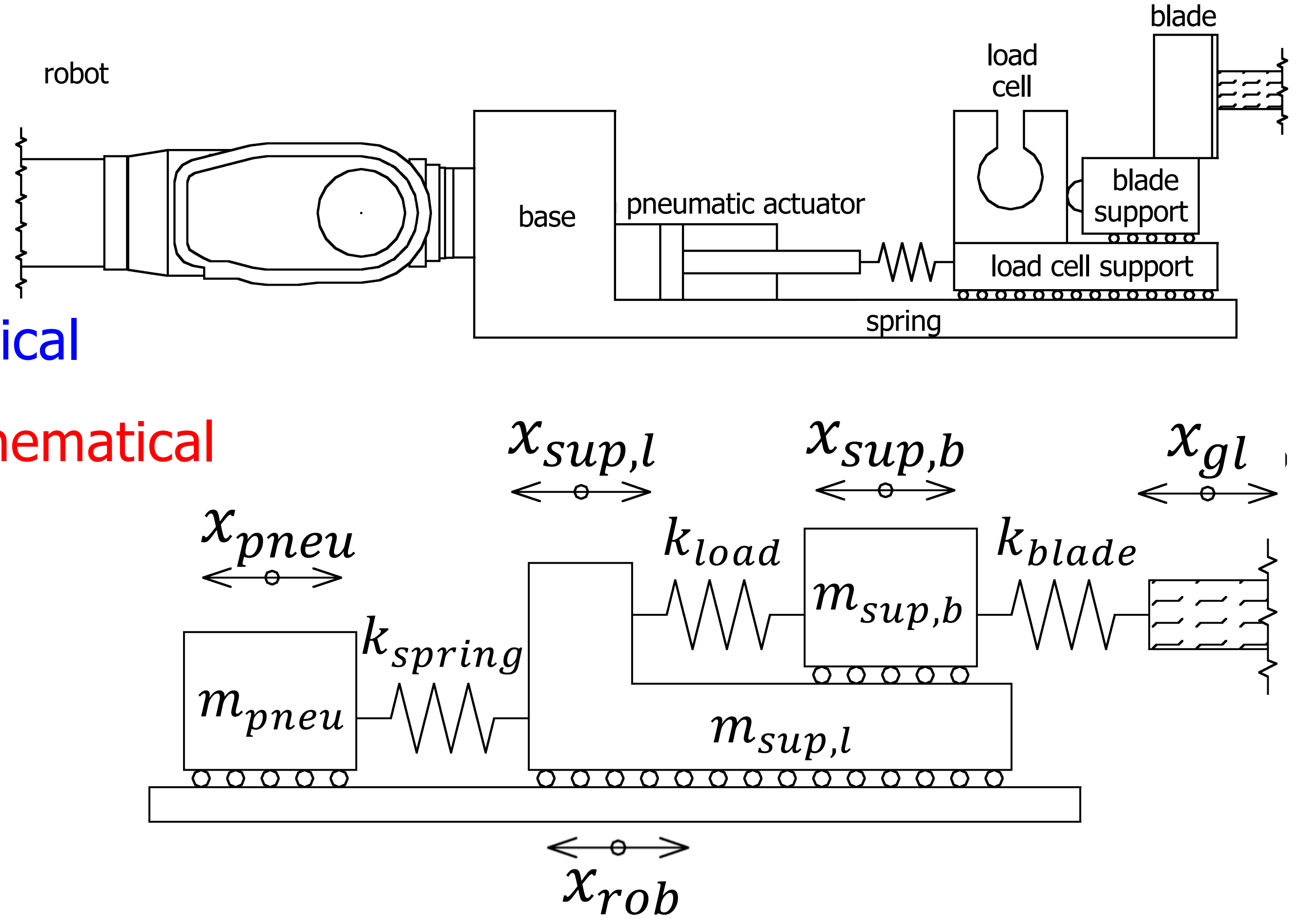
the robotic deburring tool contains in particular

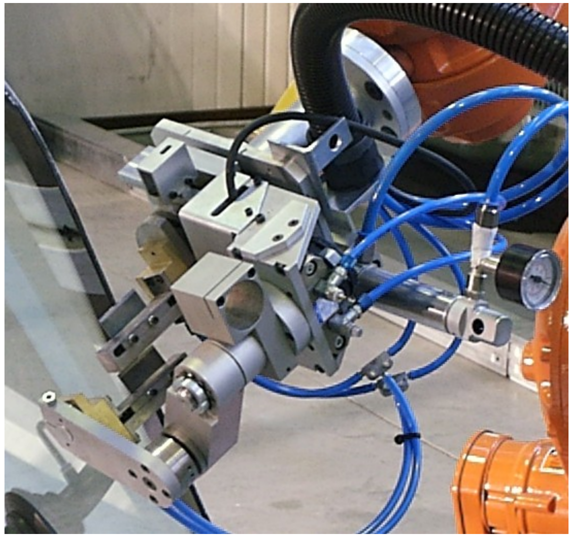
- two blades for cutting the exceeding plastic material (PVB), the first one actuated, the second passively pushed against the surface by a spring
- a load cell for measuring the 1D applied normal force at the contact
- on-board control system, exchanging data with the ABB robot controller

# Model of the deburring work tool

physical

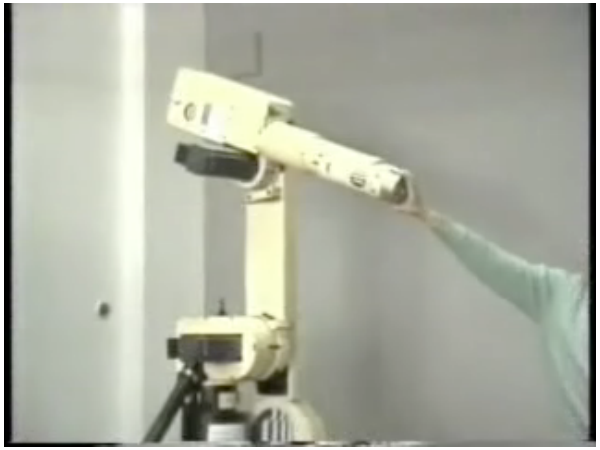
mathematical



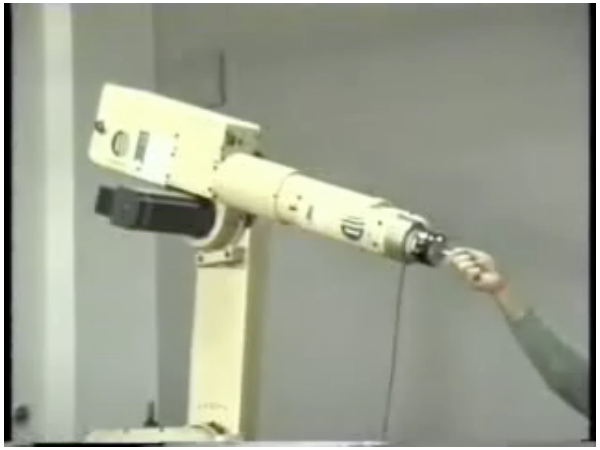



for a stability analysis (based on linear models and root locus techniques) of force control in a single direction and in presence of multiple masses/springs, see again Eppinger & Seering, IEEE CSM, 1987 (material in the course web site)

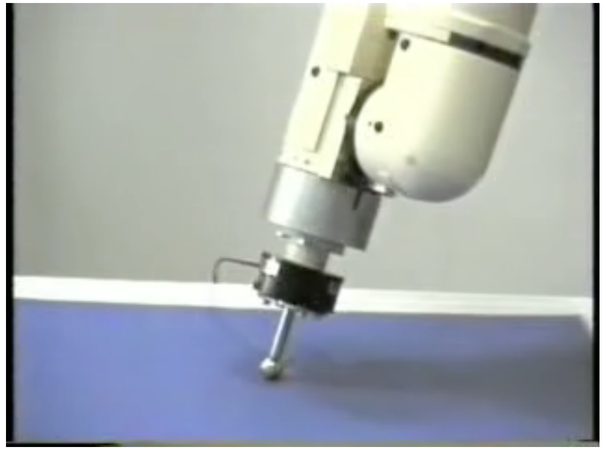
# Summary through video segments



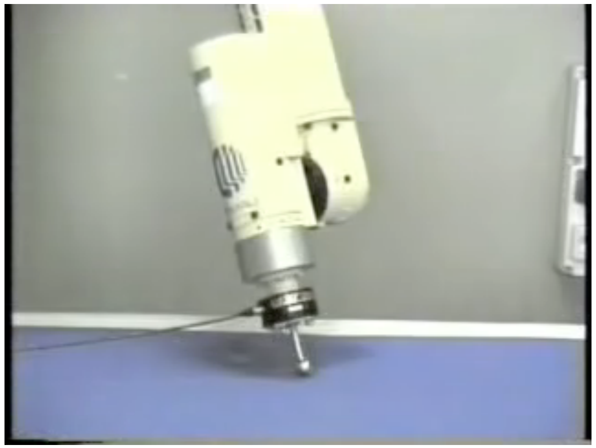
compliance control
(active Cartesian stiffness control without F/T sensor)



impedance control
(with F/T sensor)



force control
(realized as external loop providing the reference to an internal position loop –see Appendix)



hybrid force/position control

COMAU Smart robot
c/o Università di Napoli, 1994
(full video on course web site)

# Appendix

- force control can also be realized as an external loop providing reference values to an internal motion loop (see video in slide #32)
- inner-outer (or cascaded) control scheme
  - angular position quantities (E-E orientation, errors, commands) can be expressed in different ways (Euler angles $\boldsymbol{\phi}$, rotation matrices $\boldsymbol{R}$, ...)