*Robotics 2*

# Robot Interaction with the Environment

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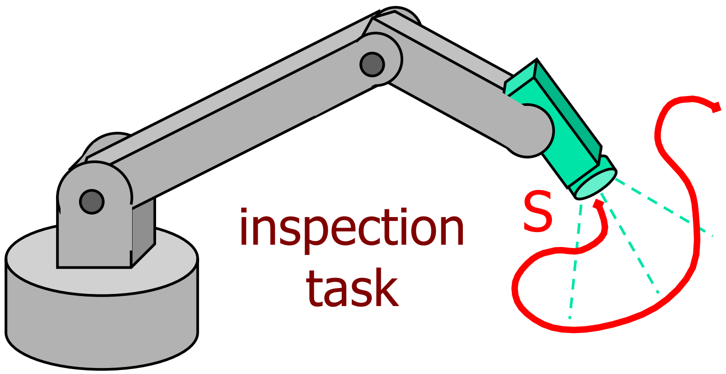
# Robot-environment interaction

a robot (end-effector) may interact with the environment

- modifying the state of the environment (e.g., pick-and-place operations)
- exchanging forces (e.g., assembly or surface finishing tasks)

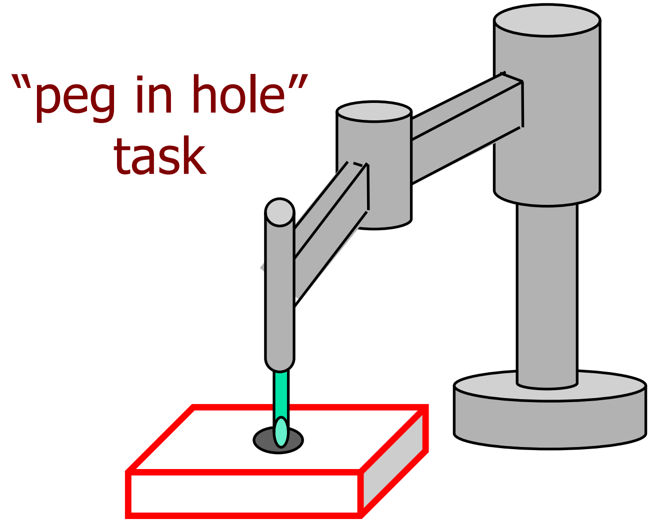
control of free motion




sensors: position (encoders)
at the joints* or
vision at the Cartesian level

*and velocity (by numerical differentiation
or, more rarely, with tachos)

control of compliant motion




sensors: as before +
6D force/torque
(at the robot wrist)

# Robot compliance

## PASSIVE ⟺ ACTIVE

**PASSIVE**

robot end-effector equipped with mechatronic devices that "comply" with the generalized forces applied at the TCP = Tool Center Point

RCC = Remote Center of Compliance device

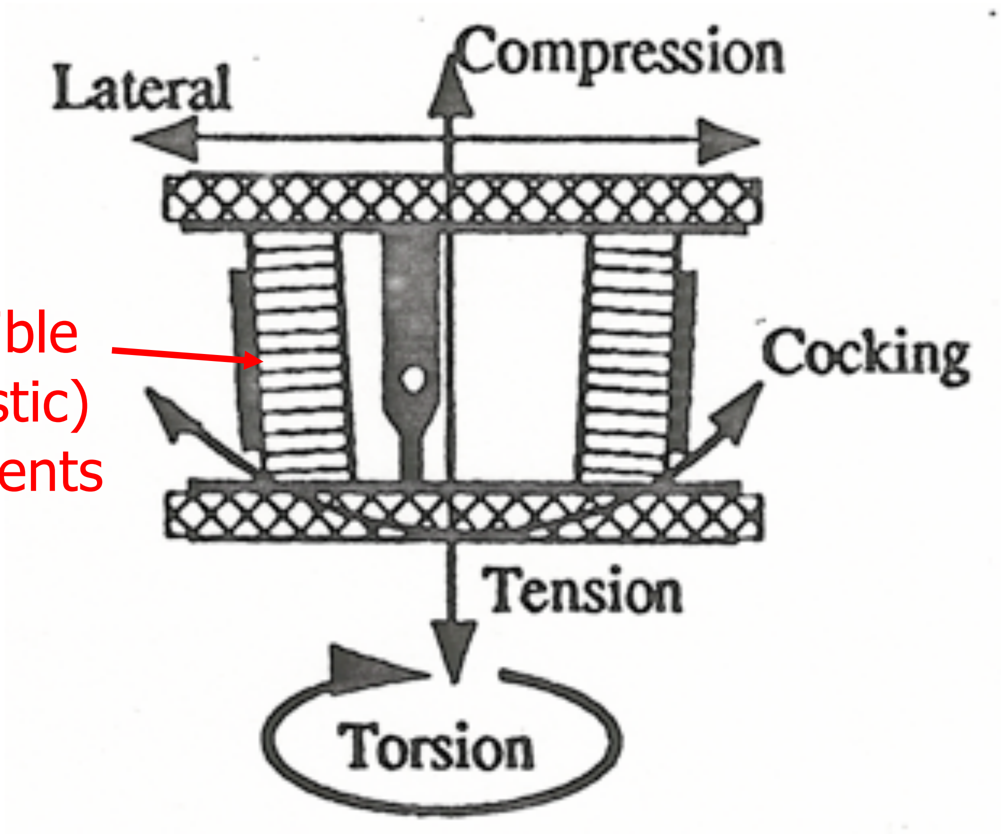
**ACTIVE**

robot is moved by a control law so as to react in a desired way to generalized forces applied at the TCP (typically measured by a F/T sensor)

- admittance control
  contact forces ⇒ velocity commands
- stiffness/compliance control
  contact displacements ⇒ force commands
- impedance control
  contact displacements ⇔ contact forces

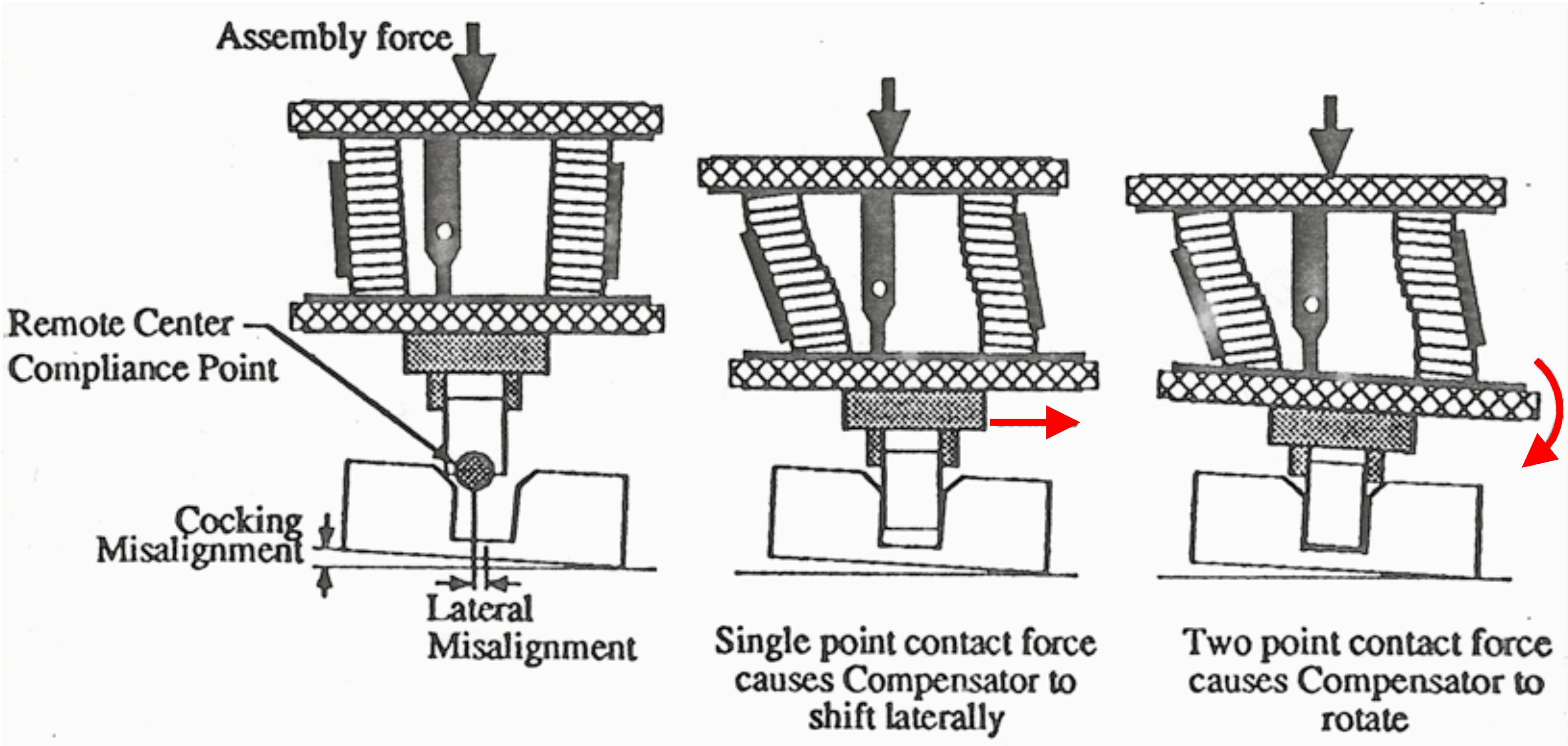
# RCC device

flexible
(elastic)
elements

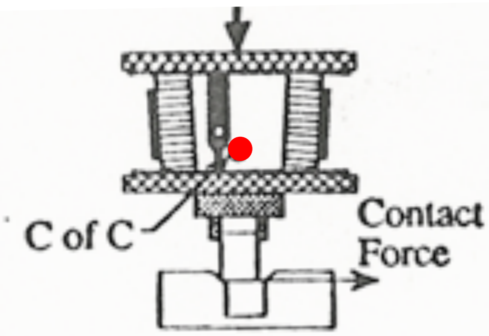



RCC models of
different size
by ATI

# RCC behavior
## in case of misalignment errors in assembly tasks

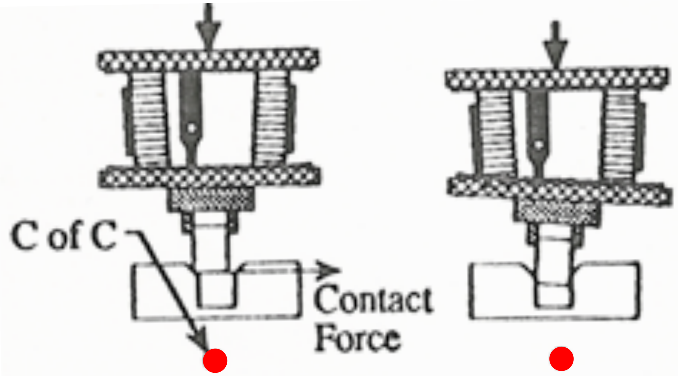


# Effects of RCC positioning



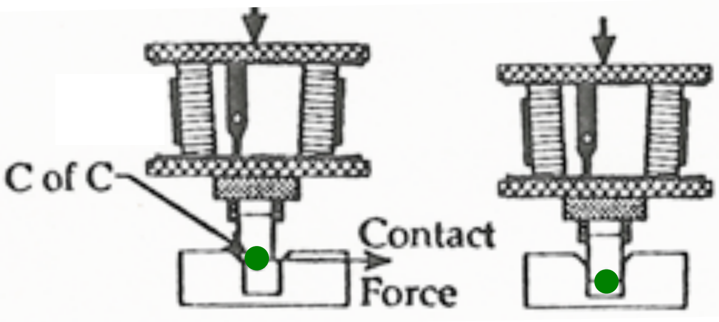

With the C of C far above the point of contact a lateral contact force causes the part to enter at an angle, causing a two point contact.

too high...




With the C of C far below the point of contact the part enters at an angle causing two point contact

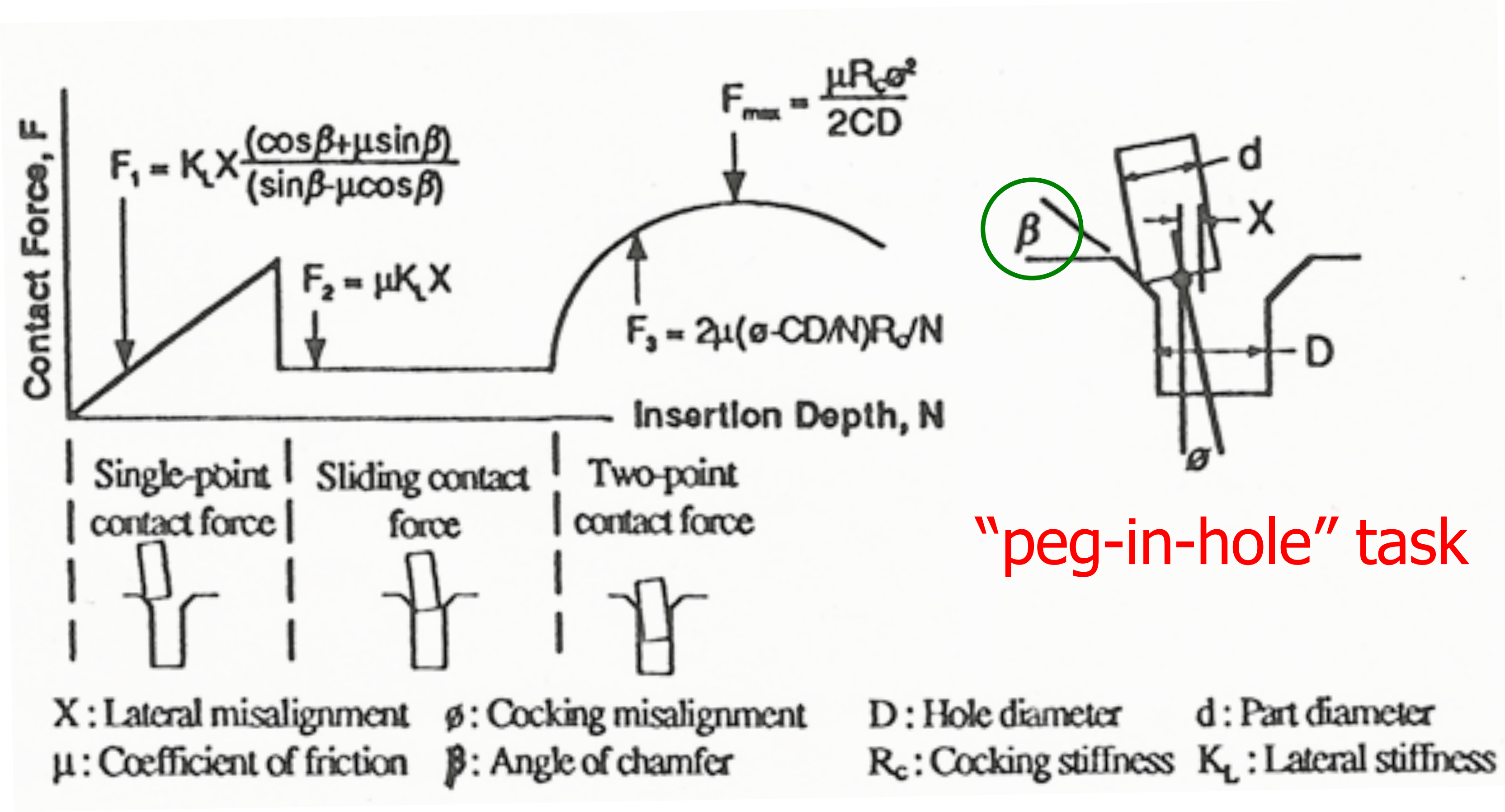
too low...




When the C of C is near the contact point the part enters correctly

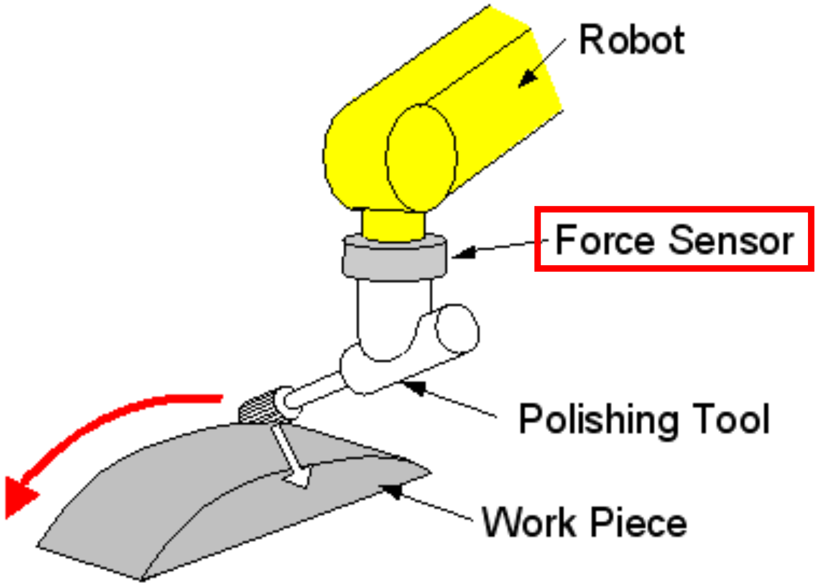
correct!
(TCP = RCC)

# Typical evolution of assembly forces




chamfer angle $\beta$ = to ease the insertion,
related also to the tolerances of the hole

# Active compliance
## for contour following

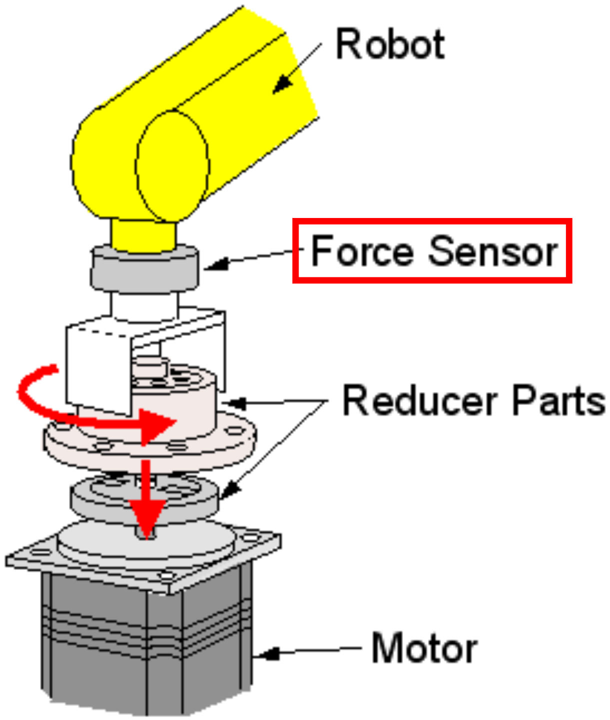



Following with constant pushing force

Washstand

Metal Cabinet

# Active compliance
## "matching" of mechanical parts



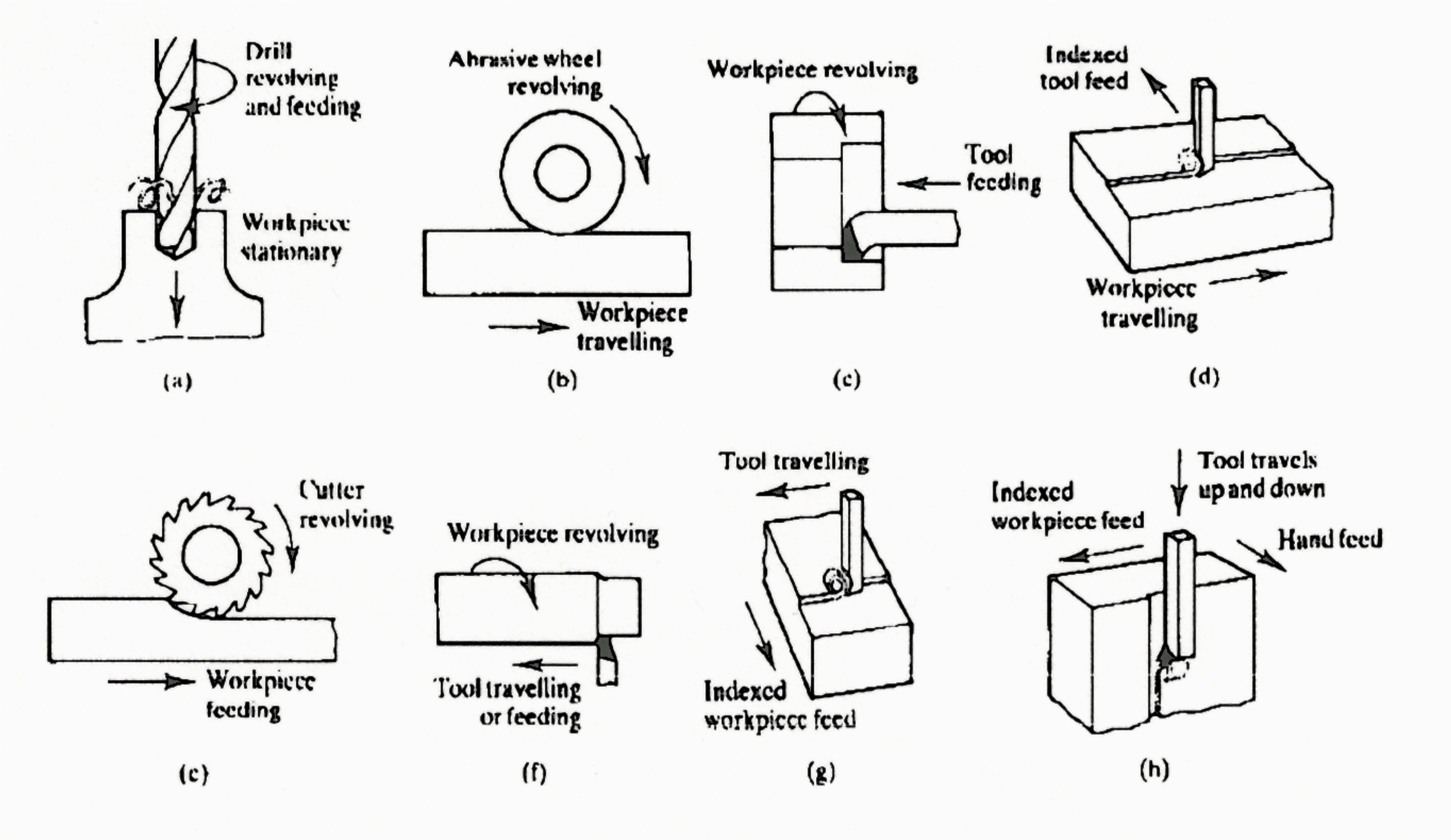

Phase matching by force sensing

Gear Parts

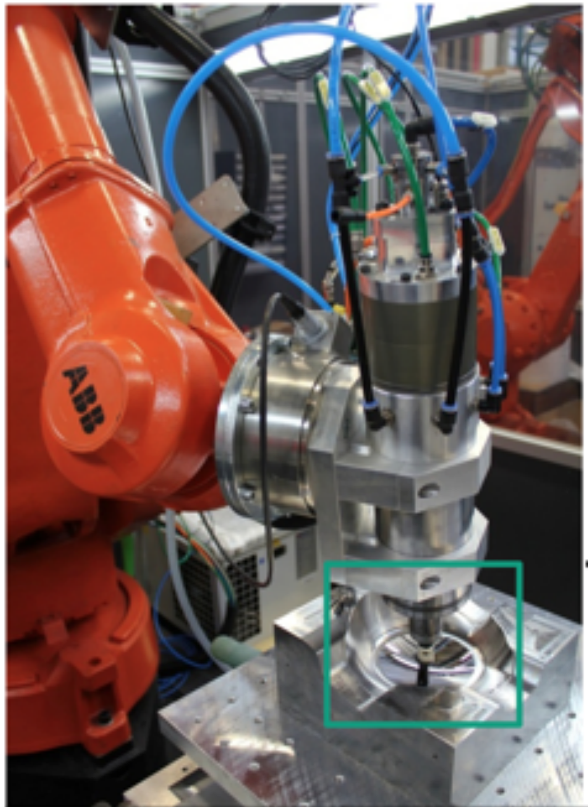
# Tasks with environment interaction

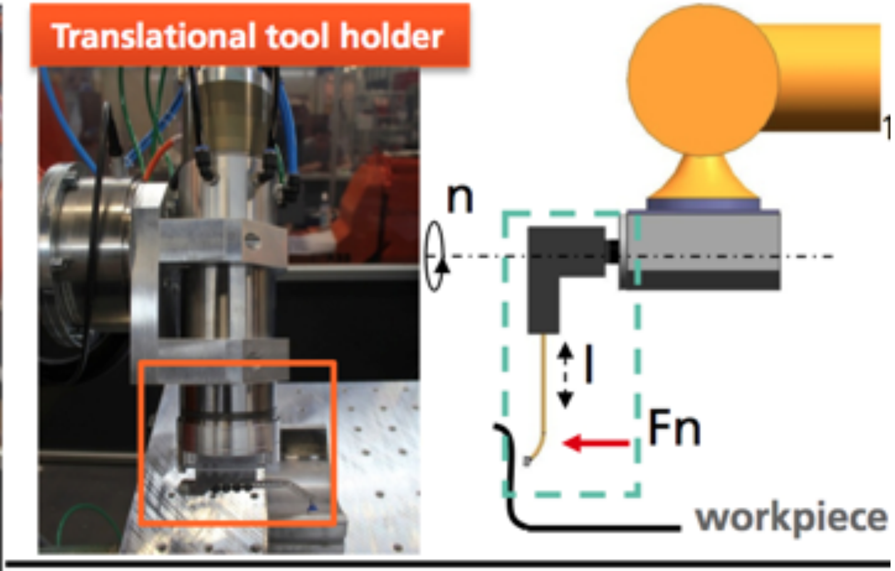
- mechanical machining
  - deburring, surface finishing, polishing, assembly,...
- tele-manipulation
  - force feedback improves performance of human operators in master-slave systems
- contact exploration for shape identification
  - force and velocity/vision sensor fusion allow 2D/3D geometric identification of unknown objects and their contour following
- dexterous robot hands
  - power grasp and fine in-hand manipulation require force/motion cooperation and coordinated control of the multiple fingers
- cooperation of multi-manipulator systems
  - the environment includes one of more other robots with their own dynamic behaviors
- physical human-robot interaction
  - humans as active, dynamic environments that need to be handled under full safety premises …

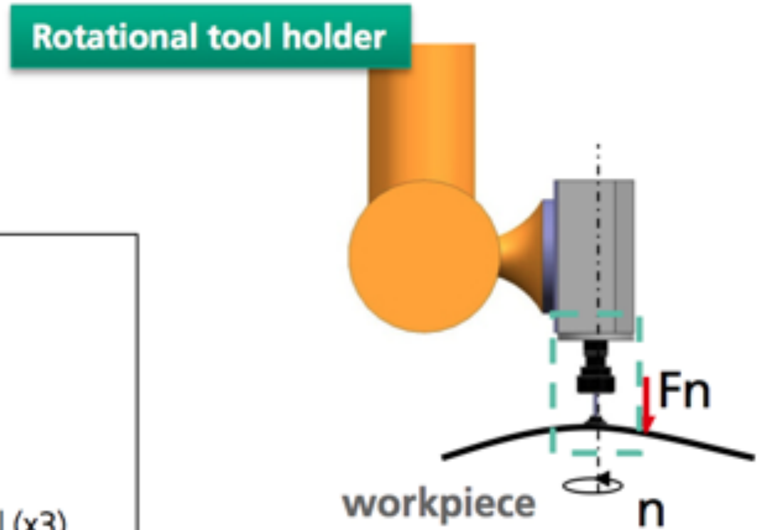
# Examples of mechanical machining



# Abrasive finishing of surfaces



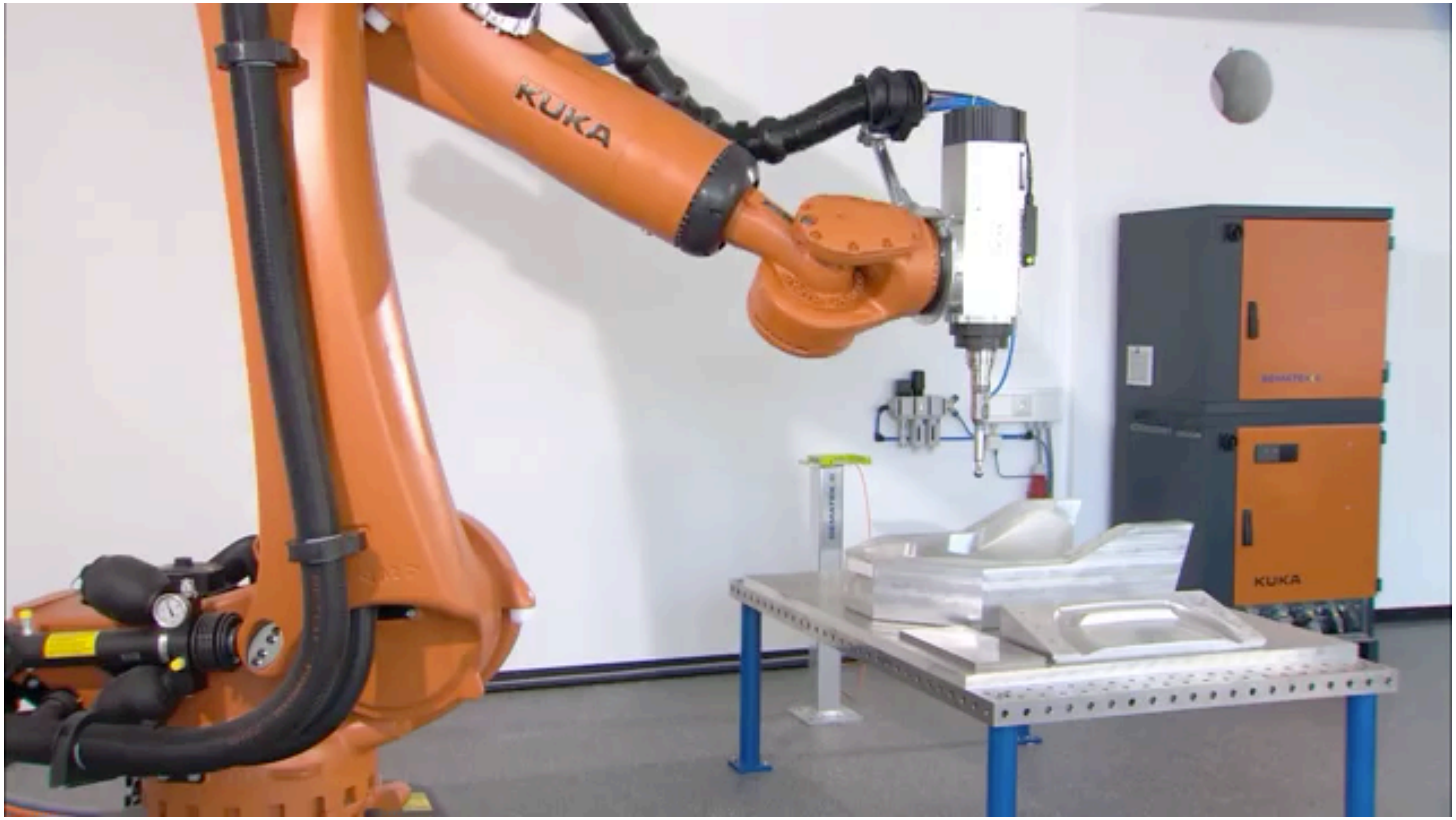




**Main properties:**

- synchronous motor
- rotation : 100 - 36.000 rpm
- power : 6 kW
- mass : 16 kg
- automated tool exchanger
- pneumatic canals for the force control (x3)

# Abrasive finishing of surfaces

video




technological processes: cold forging of surfaces and hammer peening by pneumatic machine

# Non-contact surface finishing

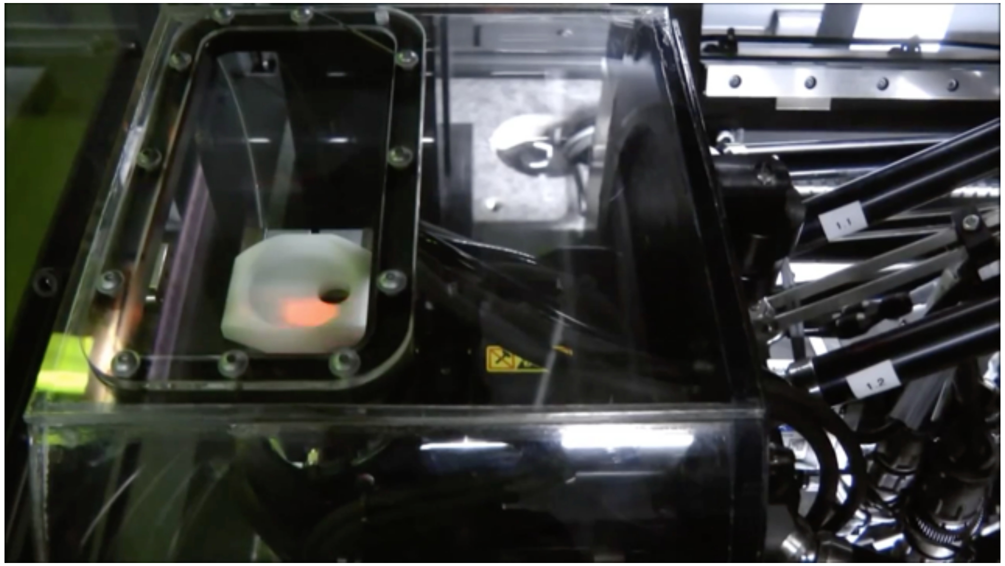
video


Fluid Jet technology

SYMPLEXITY

H2020 EU project for the
Factory of the Future (FoF)

Pulsed Laser technology



video

# In all cases ...

- for physical interaction tasks, the desired motion specification and execution should be integrated with complementary data for the desired force

  ➔ hybrid planning and control objectives

- the exchanged forces/torques at the contact(s) with the environment can be explicitly set under control or simply kept limited in an indirect way

# Evolution of control approaches
## a bit of history from the late 70's-mid '80s …

- explicit control of forces/torques only [Whitney]
  - used in quasi-static operations (assembly) in order to avoid deadlocks during part insertion
- active admittance and compliance control [Paul, Shimano, Salisbury]
  - contact forces handled through position (stiffness) or velocity (damping) control of the robot end-effector
  - robot reacts as a compressed spring (with damper) in selected/all directions
- impedance control [Hogan]
  - a desired dynamic behavior is imposed to the robot-environment interaction, e.g., a "model" with forces acting on a mass-spring-damper
  - mimics the human arm behavior moving in an unknown environment
- hybrid force-motion control [Mason]
  - decomposes the task space in complementary sets of directions where either force or motion is controlled, based on
    - a purely kinematic robot model [Raibert, Craig]
    - the actual dynamic model of the robot [Khatib]

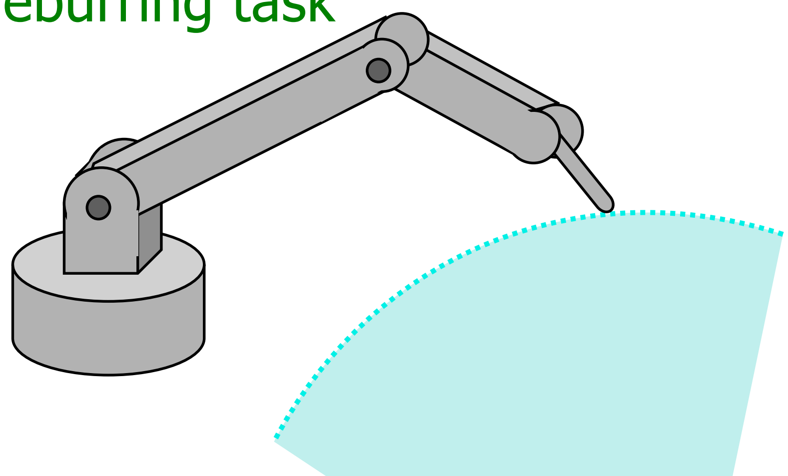
appropriate for fast and accurate motion in dynamic interaction...

# Interaction tasks of interest

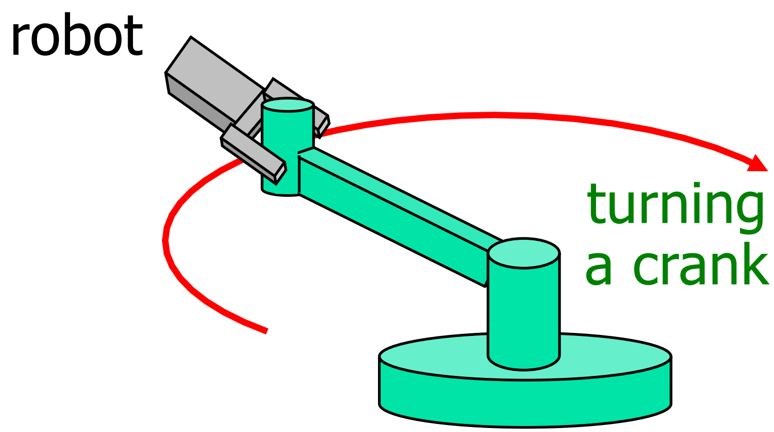
interaction tasks with the environment that require

- accurate following/reproduction by the robot end-effector of desired trajectories (even at high speed) defined on the surface of objects
- control of forces/torques applied at the contact with environments having low (soft) or high (rigid) stiffness

deburring task

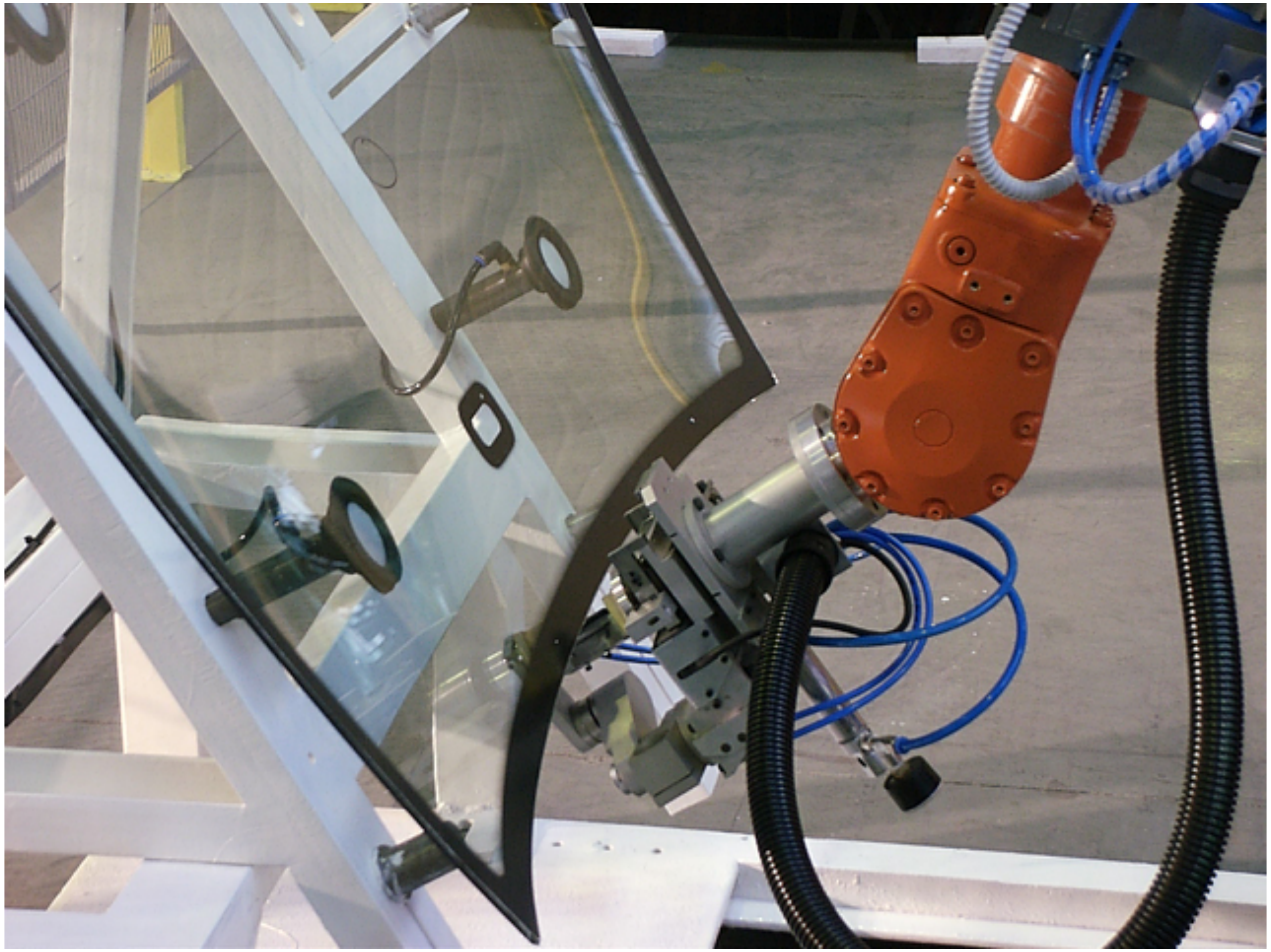


e.g., removing extra glue from the border of a car windshield




e.g., opening a door

# Robotized deburring of windshields



c/o ABB Excellence Center in Cecchina (Roma), 2002

# Impedance vs. Hybrid control

environment model ( ⬌ domain of control application)

**impedance control**

- environment = mechanical system undergoing small but finite deformations
- contact forces arise as the result of a balance of two coupled dynamic systems (robot+environment)

➔ desired dynamic characteristics are assigned to the force/motion interaction

**hybrid force/motion control**

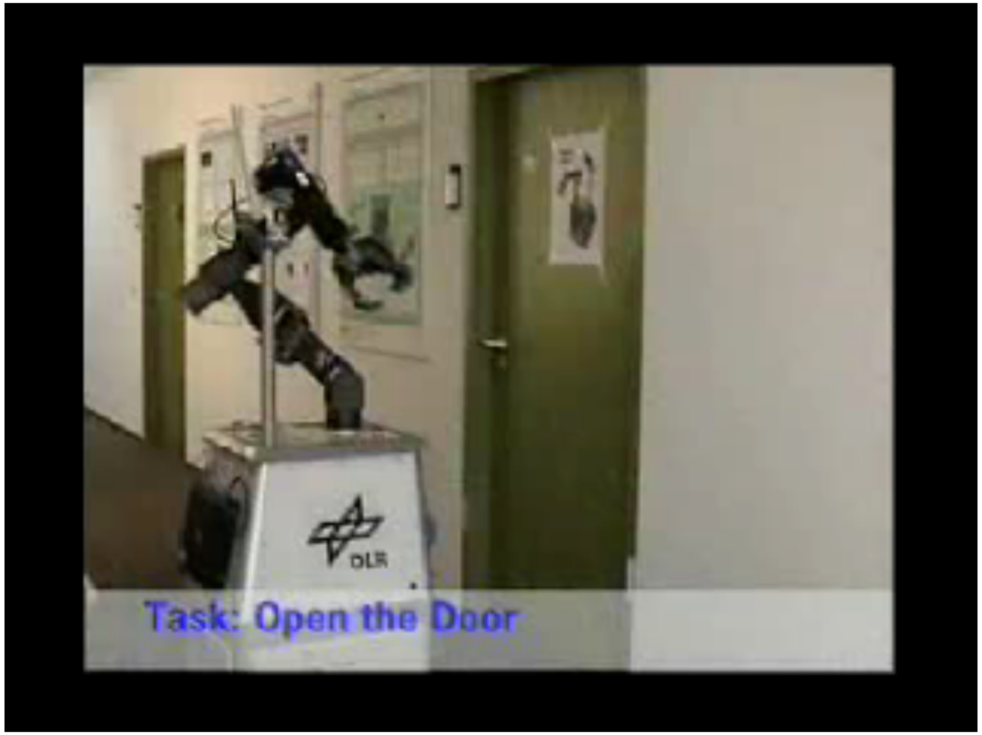
- a rigid environment reduces the degrees of freedom of the robot when in (bi-/uni-lateral) contact
- contact forces result from attempts to violate geometric constraints imposed by the environment

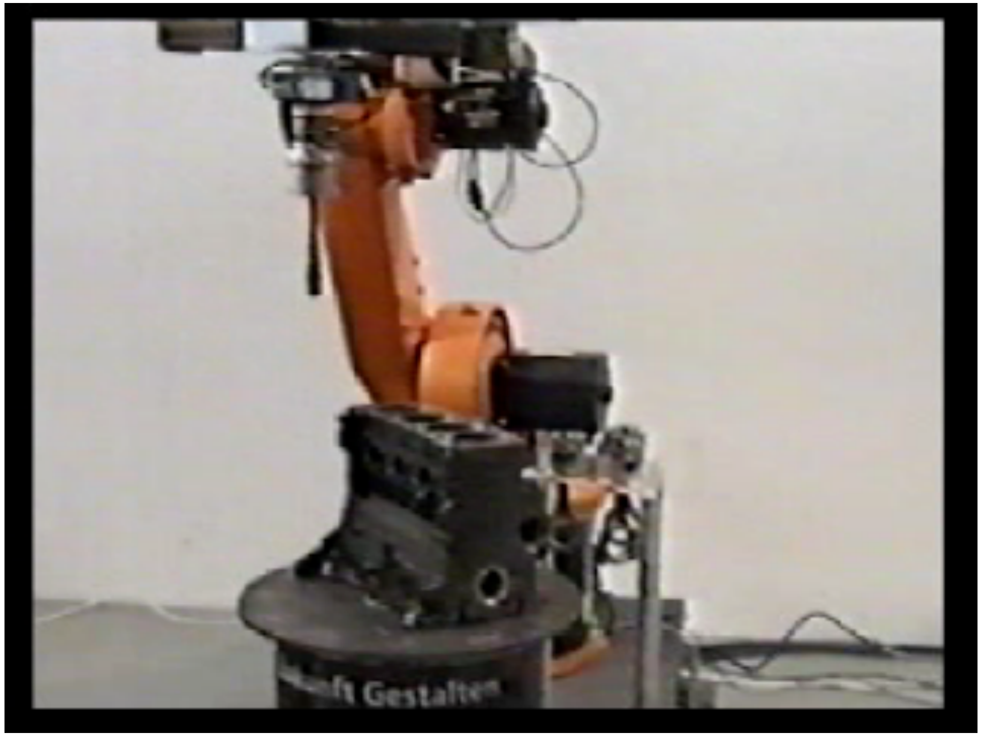
➔ task space is decomposed in sets of directions where only motion or only reaction forces are feasible

- the required level of knowledge about the environment geometry is only *apparently* different between the two control approaches
- however, measuring contact forces may not be needed in impedance control, while it always necessary in hybrid force/motion control

# Impedance vs. Hybrid control

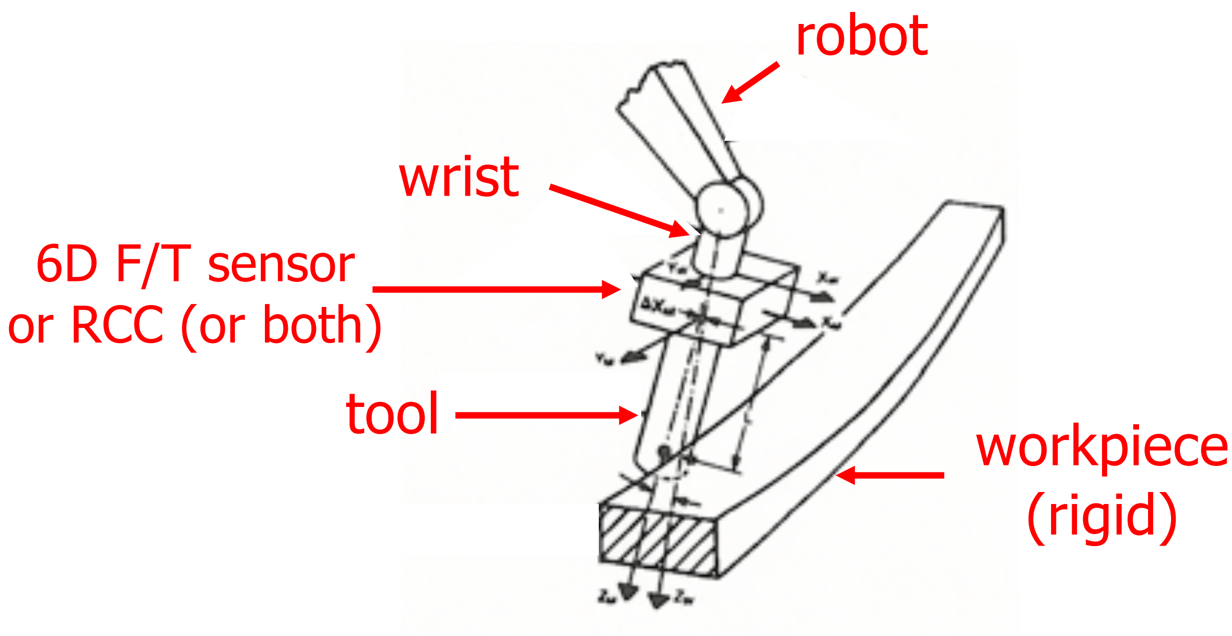
- opening a door with a mobile manipulator under impedance control
- piston insertion in a motor based on hybrid control of force-position (visual)



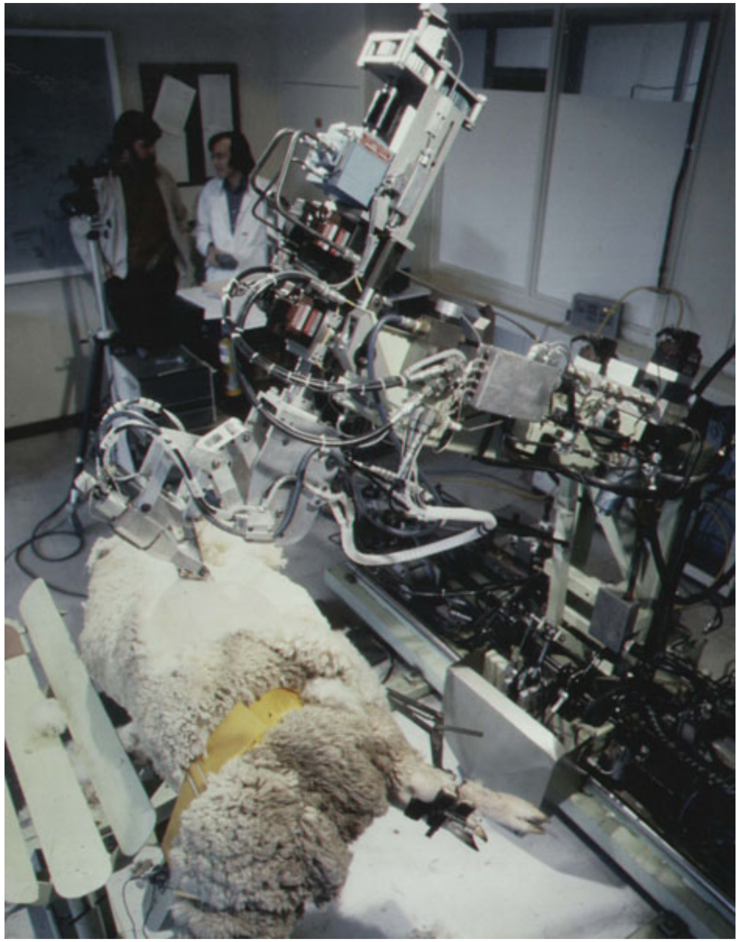
video video

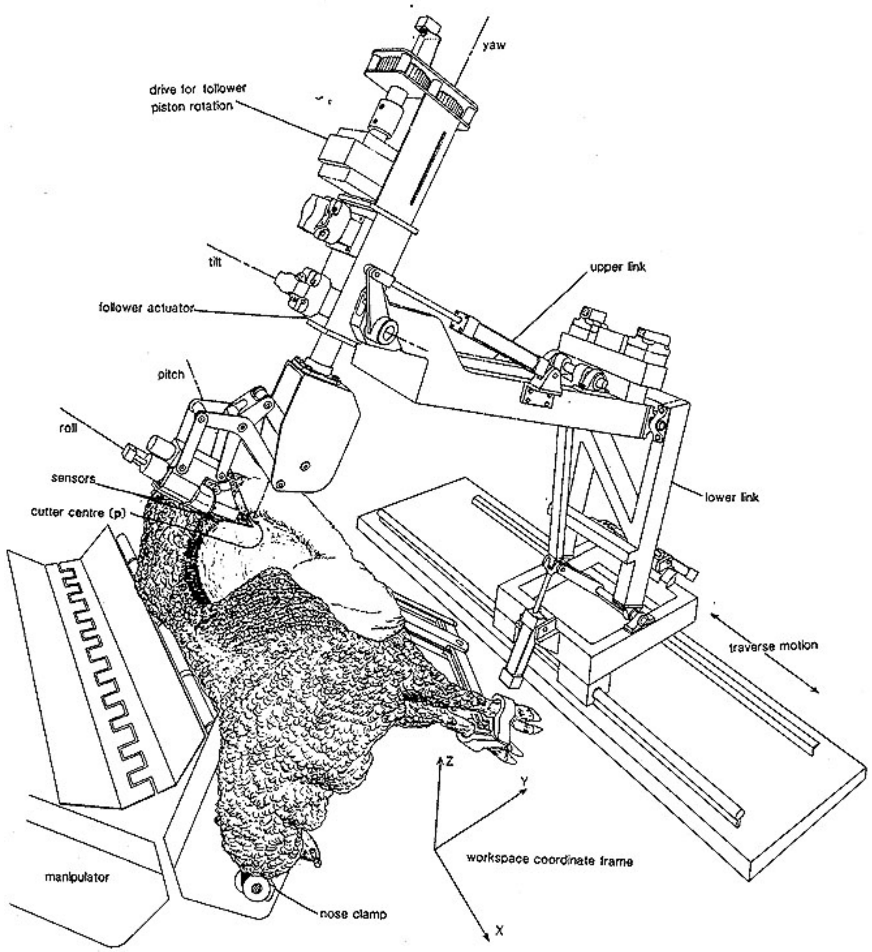
# A typical constrained situation …




the robot end-effector follows in a stable and accurate way the geometric profile of a very stiff workpiece, while applying a desired contact force

# An unusual compliant situation …

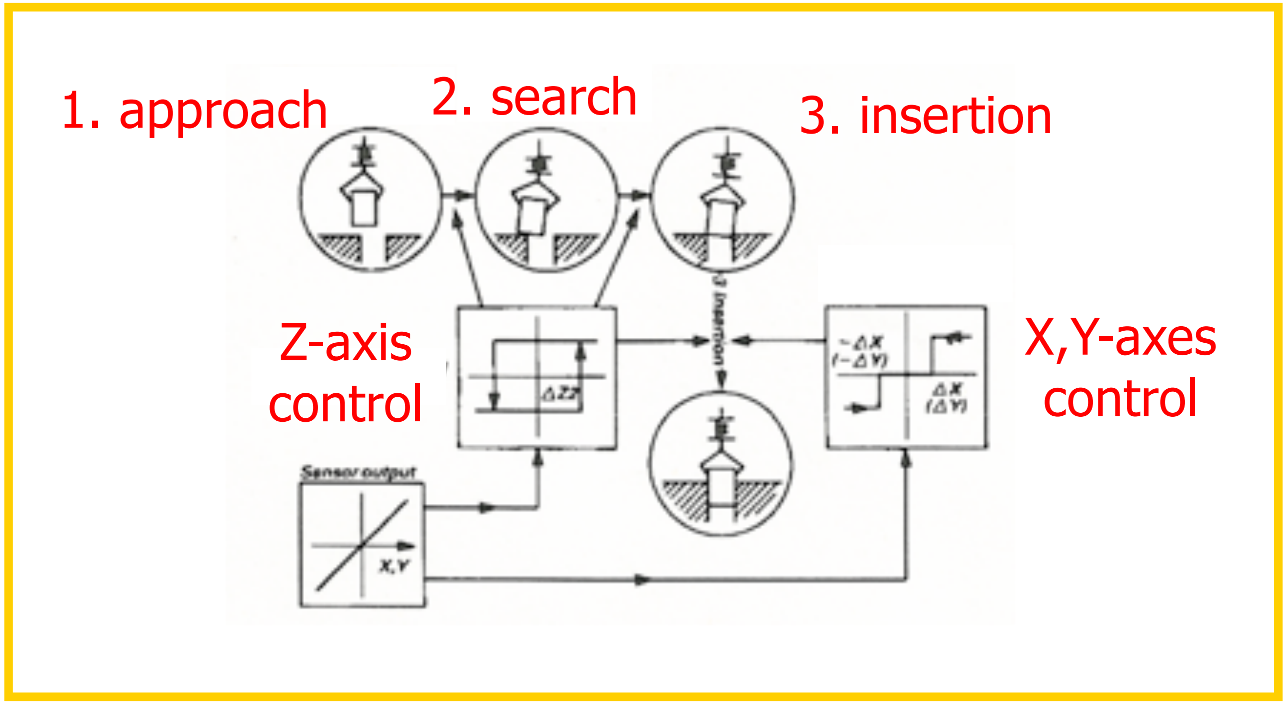





Trevelyan (AUS): Oracle robotic system in a test dated 1981

*…is the sheep happy?*

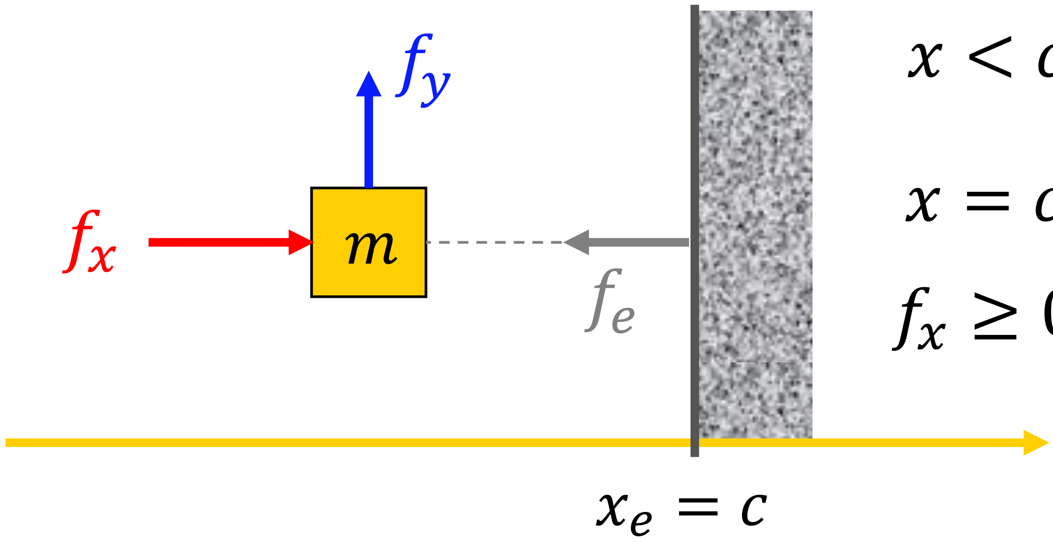
# A mixed interaction situation




processing/reasoning on force measurements leads to a sequence of fine motions
⇒ correct completion of insertion task with help of (sufficiently large) passive compliance

# Ideally constrained contact situation

a first possible modeling choice for very stiff environments




$$x < c \quad \Rightarrow \quad f_e = 0$$

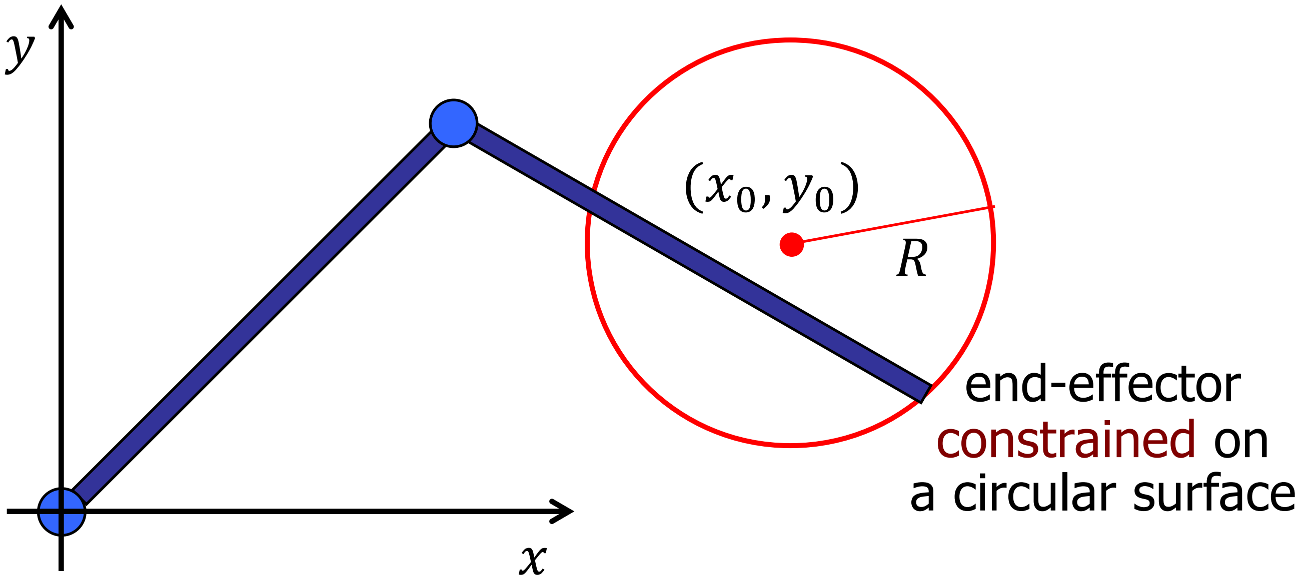
$$\left.\begin{array}{l} x = c \\ f_x \geq 0 \end{array}\right\} \quad \Rightarrow \quad \begin{array}{l} \dot{x} = 0 \\ \ddot{x} = 0 \end{array}$$

$$\begin{cases} m\ddot{x} = f_x + f_e \\ m\ddot{y} = f_y \end{cases} \qquad \longrightarrow \qquad \boxed{f_e = -f_x}$$

"ideal" = robot (sketched here as a Cartesian mass)
+ environment are both infinitely STIFF
(and without friction at the contact)

# In more complex situations

- how can we describe more complex contact situations, where the end-effector of an articulated robot (not yet reduced to a Cartesian mass via feedback linearization control) is constrained to move on an environment surface with nonlinear geometry?
- example: a planar 2R robot with end-effector moving on a circle



# Constrained robot dynamics - 1

- consider a robot in free space described by its Lagrange dynamic model and a task output function (e.g., the end-effector pose)

$$\boxed{M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u} \qquad \boxed{r = f(q)} \qquad q \in \mathbb{R}^N$$

- suppose that the task variables are subject to $M < N$ (bilateral) geometric constraints in the general form $k(r) = 0$ and define

$$h(q) = k\big(f(q)\big) = 0$$

- the constrained robot dynamics can be derived using again the Lagrange formalism, by defining an augmented Lagrangian as

$$L_a(q,\dot{q},\lambda) = L(q,\dot{q}) + \lambda^T h(q) = T(q,\dot{q}) - U(q) + \lambda^T h(q)$$

where the Lagrange multipliers $\lambda$ (a $M$-dimensional vector) can be interpreted as the generalized forces that arise at the contact when attempting to violate the constraints

# Constrained robot dynamics - 2

- applying the Euler-Lagrange equations in the extended space of generalized coordinates $q$ AND multipliers $\lambda$ yields

$$\frac{d}{dt}\left(\frac{\partial L_a}{\partial \dot{q}}\right)^T - \left(\frac{\partial L_a}{\partial q}\right)^T = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)^T - \left(\frac{\partial L}{\partial q}\right)^T - \left(\frac{\partial}{\partial q}(\lambda^T h(q))\right)^T = u$$

$$\left(\frac{\partial L_a}{\partial \lambda}\right)^T = h(q) = 0 \quad \longleftarrow$$

contact forces do NOT produce work

$$\Rightarrow \begin{cases} M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u + A^T(q)\lambda & (\star) \\ \text{subject to} \quad h(q) = 0 \end{cases}$$

where we defined the Jacobian of the constraints as the matrix

$$A(q) = \frac{\partial h(q)}{\partial q}$$

that will be assumed of full row rank ($= M$)

# Constrained robot dynamics - 3

- we can eliminate the appearance of the multipliers as follows
  - differentiate the constraints twice w.r.t. time

$$h(q) = 0 \;\Rightarrow\; \dot{h} = \frac{\partial h(q)}{\partial q}\dot{q} = A(q)\dot{q} = 0 \;\Rightarrow\; \ddot{h} = A(q)\ddot{q} + \dot{A}(q)\dot{q} = 0$$

  - substitute the joint accelerations from the dynamic model (★) (dropping dependencies)

$$AM^{-1}(u + A^T\lambda - c - g) + \dot{A}\dot{q} = 0$$

  - solve for the multipliers

invertible $M \times M$ matrix, when $A$ is full rank

$$\lambda = (AM^{-1}A^T)^{-1}\big(AM^{-1}(c + g - u) - \dot{A}\dot{q}\big)$$
$$= \left(A_M^{\#}\right)^T(c + g - u) - (AM^{-1}A^T)^{-1}\dot{A}\dot{q}$$

the inertia-weighted pseudoinverse of the constraint Jacobian $A$

constraint forces $\lambda$ are uniquely determined by the robot state $(q, \dot{q})$ and input $u$ !!

to be replaced in the dynamic model...

# Constrained robot dynamics - 4

- the final constrained dynamic model can be rewritten as

$$\boxed{M(q)\ddot{q} = \underbrace{\left[I - A^T(q)\left(A_M^{\#}(q)\right)^T\right]}_{\text{dynamically consistent projection matrix}}(u - c(q,\dot{q}) - g(q)) - M(q)A_M^{\#}(q)\dot{A}(q)\dot{q}}$$

where $A_M^{\#}(q) = M^{-1}(q)A^T(q)(A(q)M^{-1}(q)A^T(q))^{-1}$ and with

$$\lambda = \left(A_M^{\#}(q)\right)^T(c(q,\dot{q}) + g(q) - u) - \left(A(q)M^{-1}(q)A^T(q)\right)^{-1}\dot{A}(q)\dot{q}$$

- if the robot state $(q(0),\dot{q}(0))$ at time $t = 0$ satisfies the constraints, i.e.,

$$h\big(q(0)\big) = 0, \qquad A\big(q(0)\big)\dot{q}(0) = 0$$

then the robot evolution described by the above dynamics will be consistent with the constraints for all $t \geq 0$ and for any $u(t)$

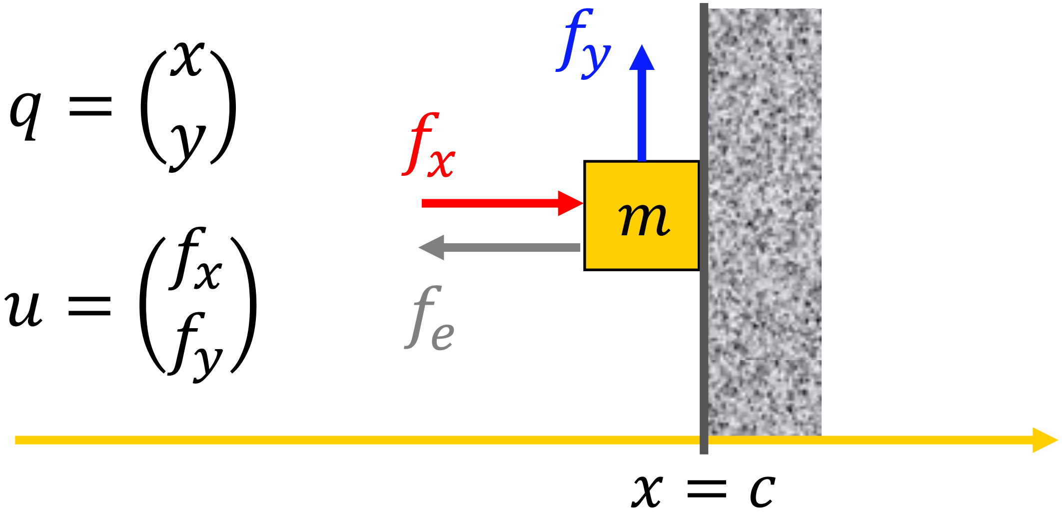
- this is a useful simulation model (constrained direct dynamics)

# Example – ideal mass

## constrained robot dynamics

$$q = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$u = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$




$$M\ddot{q} = u$$ robot dynamics in free motion

$$M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$h(q) = x - c = 0 \quad \Rightarrow \quad A(q) = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad \Rightarrow \quad A_M^{\#}(q) = \cdots = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\left(I - A^T(q)\left(A_M^{\#}(q)\right)^T\right) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$ dynamically consistent projection matrix

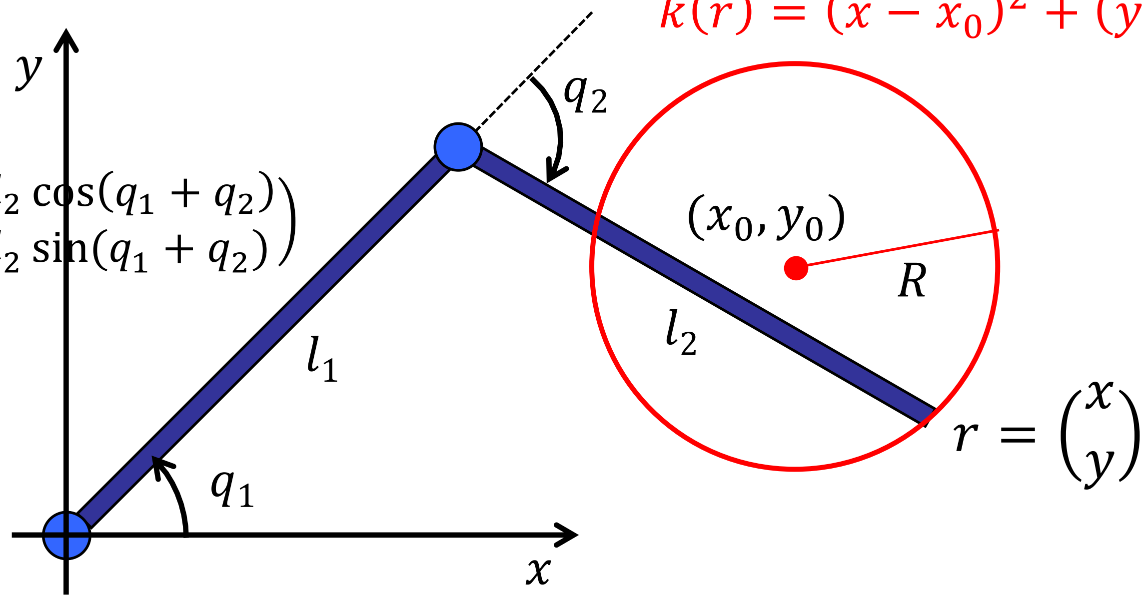
$$\lambda = -\left(A_M^{\#}(q)\right)^T u = -\begin{pmatrix} 1 & 0 \end{pmatrix} u = -f_x$$ multiplier (contact force $f_e$)

$$M \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = M\ddot{q} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} u = \begin{pmatrix} 0 \\ f_y \end{pmatrix}$$ constrained robot dynamics

# Example – planar 2R robot
## constrained robot dynamics

$$k(r) = (x - x_0)^2 + (y - y_0)^2 - R^2 = 0$$

$$r = f(q) = \begin{pmatrix} l_1 \cos q_1 + l_2 \cos(q_1 + q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \end{pmatrix}$$




$$h(q) = k(f(q)) =$$
$$(l_1 \cos q_1 + l_2 \cos(q_1 + q_2) - x_0)^2 + (l_1 \sin q_1 + l_2 \sin(q_1 + q_2) - y_0)^2 - R^2 = 0$$

$$\dot{h} = \frac{\partial k}{\partial r}\frac{\partial r}{\partial q}\dot{q} = [2(x - x_0) \quad 2(y - y_0)]\, J_r(q)\dot{q}$$
$$= [2(l_1 c_1 + l_2 c_{12} - x_0) \quad 2(l_1 s_1 + l_2 s_{12} - y_0)]\, J_r(q)\dot{q} = A(q)\dot{q}$$

# Reduced robot dynamics - 1

- by imposing $M$ constraints $h(q) = 0$ on the $N$ generalized coordinates $q$, it is also possible to reduce the description of the constrained robot dynamics to a $N - M$ dimensional configuration space
- start from constraint matrix $A(q)$ and select a matrix $D(q)$ such that

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} \text{ is a nonsingular } N{\times}N \text{ matrix} \quad \Rightarrow \quad \begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = \begin{pmatrix} E(q) & F(q) \end{pmatrix}$$

- define the $(N - M)$-dimensional vector of pseudo-velocities $v$ as the linear combination (at a given $q$) of the robot generalized velocities

$$v = D(q)\dot{q} \quad \Rightarrow \quad \dot{v} = D(q)\ddot{q} + \dot{D}(q)\dot{q}$$

- inverse relationships (from "pseudo" to "generalized" velocities and accelerations) are given by

$$\dot{q} = F(q)v \qquad \ddot{q} = F(q)\dot{v} - \big(E(q)\dot{A}(q) + F(q)\dot{D}(q)\big)F(q)v$$

⇔ properties of block products in inverse matrices have been used for eliminating the appearance of $\dot{F}$ (often $F$ is only known numerically)

# Reduced robot dynamics – 2

whiteboard …

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = \begin{pmatrix} E(q) & F(q) \end{pmatrix}$$

a number of properties from this definition…

two matrix inverse products

three useful identities!

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} \begin{pmatrix} E(q) & F(q) \end{pmatrix} = \begin{pmatrix} A(q)E(q) & A(q)F(q) \\ D(q)E(q) & D(q)F(q) \end{pmatrix} = \begin{pmatrix} I_{M\times M} & 0 \\ 0 & I_{(N-M)\times(N-M)} \end{pmatrix}$$

$$\begin{pmatrix} E(q) & F(q) \end{pmatrix} \begin{pmatrix} A(q) \\ D(q) \end{pmatrix} = E(q)A(q) + F(q)D(q) = I_{N\times N}$$

➡ differentiating w.r.t. time $\quad \dot{E}A + E\dot{A} + \dot{F}D + F\dot{D} = 0 \quad \triangleleft$

from pseudo-velocity $v = D(q)\dot{q}$ since $F$ is a right inverse of the full row rank matrix $D$ $(DF = I)$ ➡

$$\dot{q} = F(q)v = D^T(q)\left(D(q)D^T(q)\right)^{-1}v$$

(in fact $D\dot{q} = DFv = v$)

➡ differentiating w.r.t. time $\dot{q} = F(q)v$

$$\ddot{q} = F\dot{v} + \dot{F}v = F\dot{v} + (\dot{F}D)\dot{q} \overset{(\triangleleft)}{=} F\dot{v} - \left(\dot{E}\cancel{A} + E\dot{A} + F\dot{D}\right)Fv$$

$$= F(q)\dot{v} - \left(E(q)\dot{A}(q) + F(q)\dot{D}(q)\right)F(q)v$$

# Reduced robot dynamics - 3

- consider again the dynamic model (★), dropping dependencies

$$M\ddot{q} + c + g = u + A^T\lambda$$

- since $AE = I$, multiplying on the left by $E^T$ isolates the multipliers

$$E^T(M\ddot{q} + c + g - u) = \lambda$$

- since $AF = 0$, multiplying on the left by $F^T$ eliminates the multipliers

$$F^T M\ddot{q} = F^T(u - c - g)$$

- substituting in the latter the generalized accelerations and velocities with the pseudo-accelerations and pseudo-velocities leads finally to

invertible
$(N-M)\times(N-M)$
positive definite matrix

$$\boxed{(F^T M F)\dot{v} = F^T\big(u - c - g + M\big(E\dot{A} + F\dot{D}\big)Fv\big)}$$

which is the reduced $(N-M)$-dimensional dynamic model

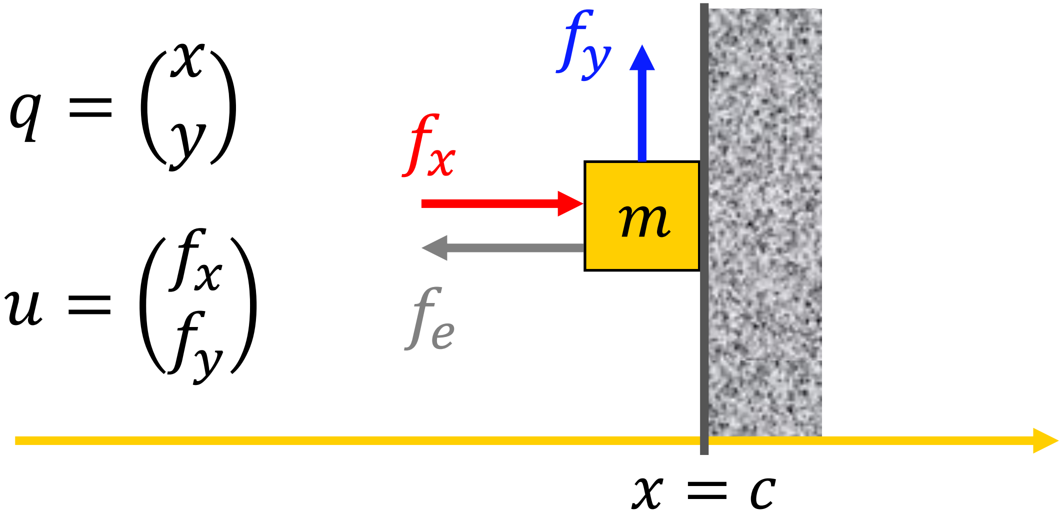
- similarly, the expression of the multipliers becomes

$$\lambda = E^T\big(MF\dot{v} - M\big(E\dot{A} + F\dot{D}\big)Fv + c + g - u\big) \qquad (\S)$$

# Example – ideal mass

## reduced robot dynamics

$$q = \begin{pmatrix} x \\ y \end{pmatrix} \qquad u = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$




$$M\ddot{q} = u$$ robot dynamics in free motion

$$M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$h(q) = x - c = 0 \;\Rightarrow\; A = (1 \quad 0) \;\Rightarrow\; \begin{pmatrix} A \\ D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} E & F \end{pmatrix}$$

$$v = D\dot{q} = \dot{y}$$ pseudo-velocity

$$\lambda = E^T(MF\dot{v} - u)$$

$$= (1 \quad 0)\left(\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\ddot{y} - \begin{pmatrix} f_x \\ f_y \end{pmatrix}\right) = -(1 \quad 0)\begin{pmatrix} f_x \\ f_y \end{pmatrix} = -f_x$$

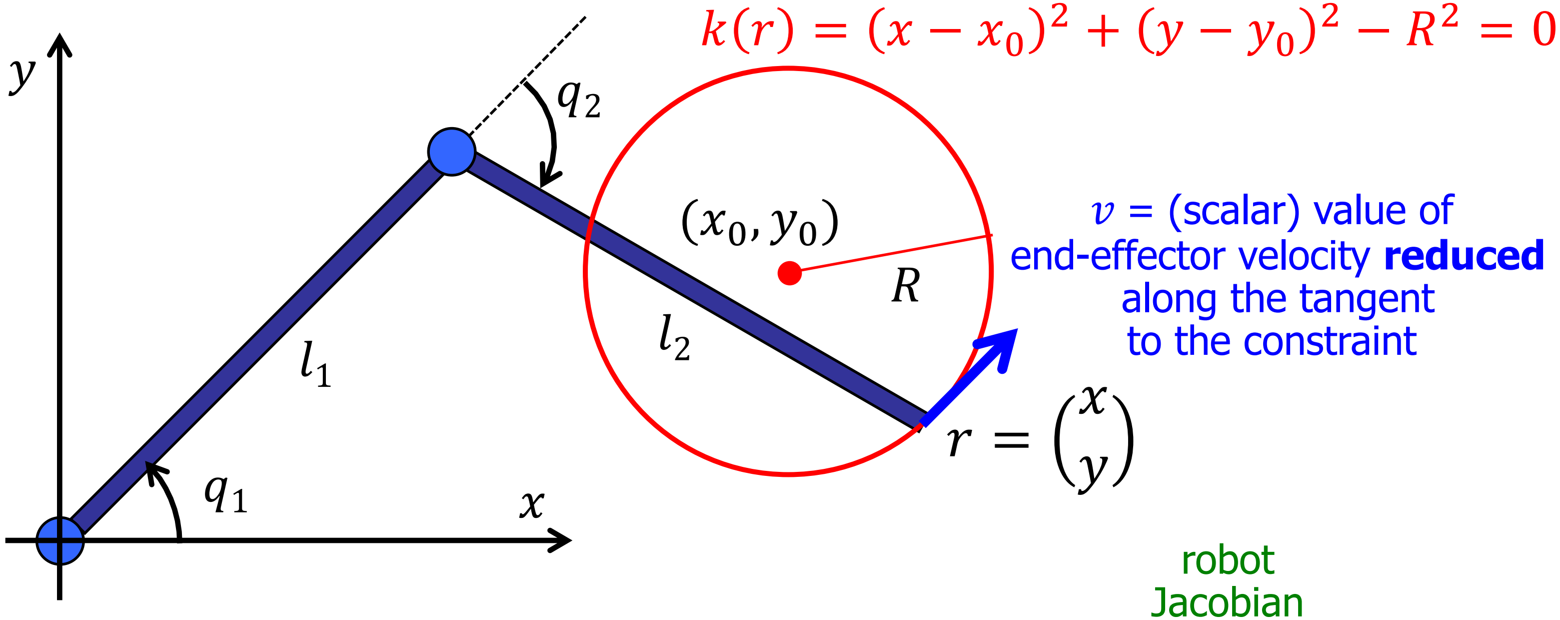
multiplier (contact force $f_e$)

$$(F^T M F)\dot{v} = (0 \quad 1)\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\dot{v} = m\ddot{y} = f_y = F^T u$$

reduced robot dynamics

# Example – planar 2R robot

## reduced robot dynamics




$v$ = (scalar) value of end-effector velocity **reduced** along the tangent to the constraint

robot Jacobian

$$A(q) = \begin{bmatrix} 2(x-x_0) & 2(y-y_0) \end{bmatrix} J_r(q)$$
$$= \begin{bmatrix} 2(l_1c_1 + l_2c_{12} - x_0) & 2(l_1s_1 + l_2s_{12} - y_0) \end{bmatrix} J_r(q)$$

a feasible selection of matrix $D(q)$

out of robot singularities

$$D(q) = \begin{bmatrix} -\frac{1}{2}(y-y_0) & \frac{1}{2}(x-x_0) \end{bmatrix} J_r(q) \quad \Rightarrow \quad \det\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} = R^2 \cdot \det J_r(q) \neq 0$$

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = \begin{pmatrix} E(q) & F(q) \end{pmatrix} \quad \Rightarrow \quad \boxed{v} = D(q)\dot{q} \quad \Rightarrow \quad \dot{q} = F(q)v = J_r^{-1}(q) \begin{pmatrix} -2(y-y_0)/R^2 \\ 2(x-x_0)/R^2 \end{pmatrix} v$$

a scalar

# Control based on reduced robot dynamics

- the reduced $N - M$ dynamic expressions are more compact but also more complex and less used for simulation purposes than the $N$-dimensional constrained dynamics
- however, they are useful for control design (reduced inverse dynamics)
- in fact, it is straightforward to verify that the feedback linearizing control law

$$u = \left(c + g - M\left(E\dot{A} + F\dot{D}\right)Fv\right) + MFu_1 - A^T u_2$$

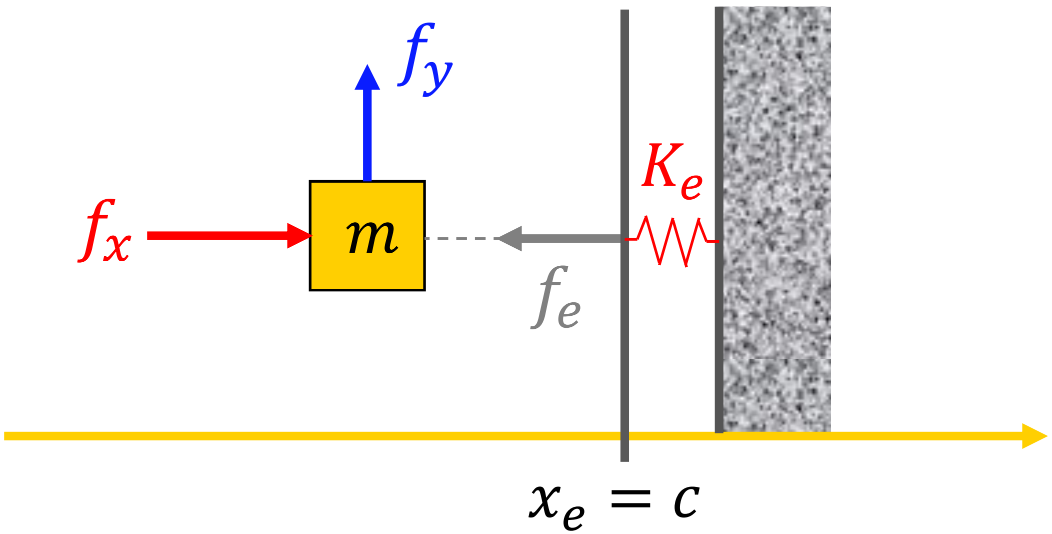
applied to the reduced robot dynamics and to the expression ( § ) of the multipliers leads to the closed-loop system

$$\dot{v} = u_1 \qquad\qquad \lambda = u_2$$

Note: these are exactly in the form of the ideal mass example of slide #24, with $v = \dot{y}$, $u_1 = f_y/m$, $\lambda = f_e$, $u_2 = -f_x$ (being $N = 2$, $M = 1$, $N - M = 1$)

# Compliant contact situation

a second possible modeling choice for softer environments




compliance/impedance control (in all directions) is here a good choice that allows to handle
- uncertain position
- uncertain orientation

of the wall

$$\begin{cases} m\ddot{x} = f_x + f_e \\ m\ddot{y} = f_y \end{cases}$$

$$\begin{cases} x < c \quad \Rightarrow \quad f_e = 0 \\ x \geq c \quad \Rightarrow \quad f_e = K_e(x - c) \end{cases}$$

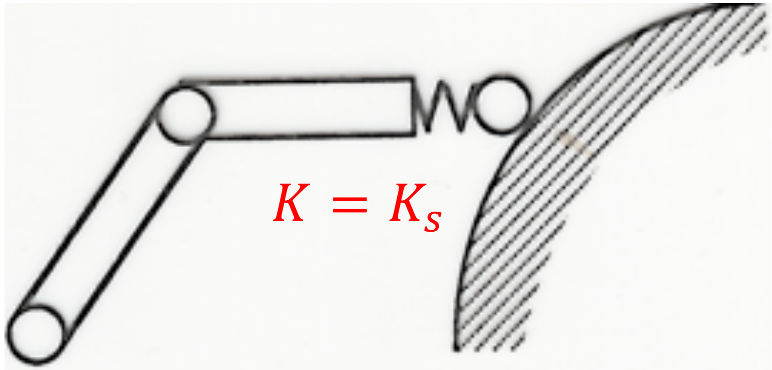
with $K_e > 0$ being the stiffness of the environment

# Robot-environment contact types

## modeled by a single elastic constant

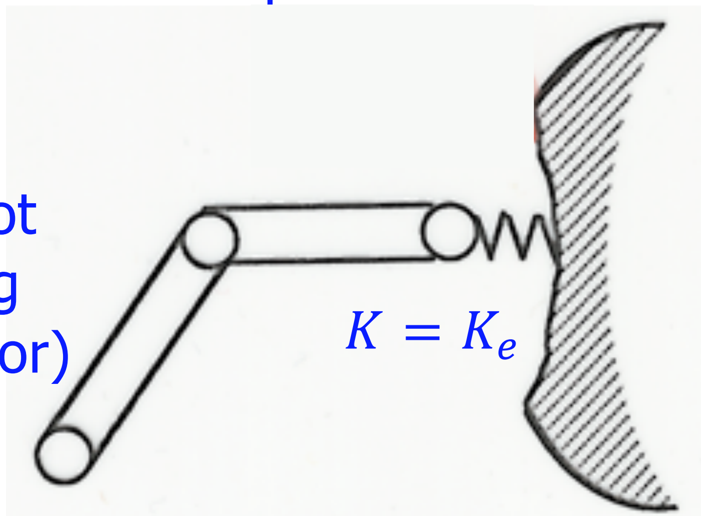
compliant
force sensor

rigid environment

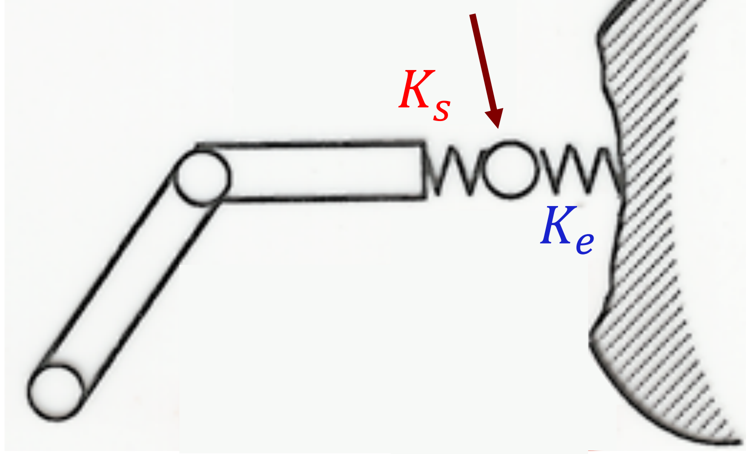



compliant environment

rigid robot
(including
force sensor)




negligible intermediate
mass

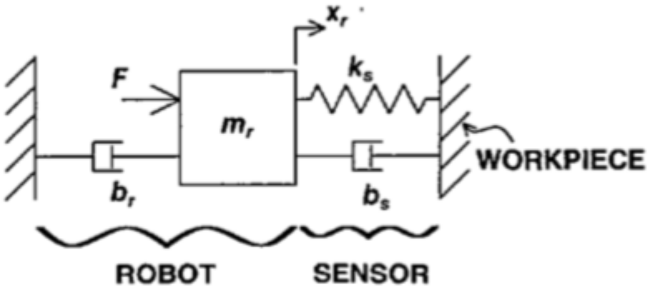



$$\frac{1}{K} = \frac{1}{K_s} + \frac{1}{K_e} \quad \Rightarrow \quad K = \frac{K_s K_e}{K_s + K_e}$$

series of springs =
sum of compliances
(inverse of stiffnesses)

# Force control
## 1-dof robot-environment linear dynamic models

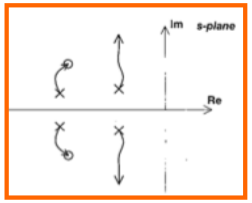
- with a force sensor (having stiffness $k_s$ and damping $b_s$) measuring the contact force $F_c$
- stability analysis of a proportional control loop for regulation of the contact force (to a desired constant value $F_d$) can be made using the root-locus method (for a varying $k_f$)
- by including/excluding work-piece compliance and/or robot (transmission) compliance

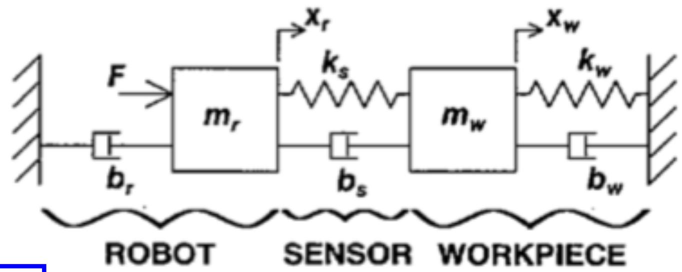



$$\boxed{F_c = k_s x_r}$$

stable
for all $k_f$

+ work-piece compliance

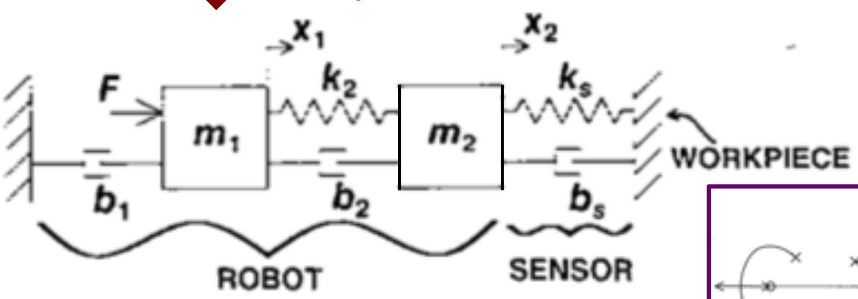


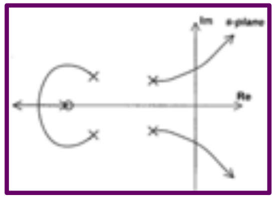

$$\boxed{F = k_f(F_d - F_c), \quad k_f > 0}$$

$$\boxed{F_c = k_s(x_r - x_w)}$$

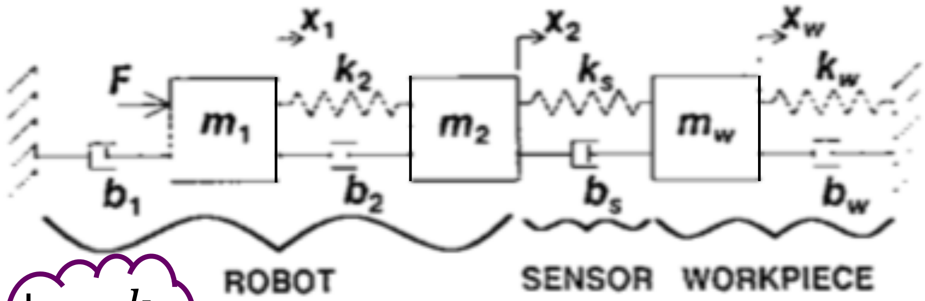
+ robot compliance




$$\boxed{F_c = k_s x_2}$$



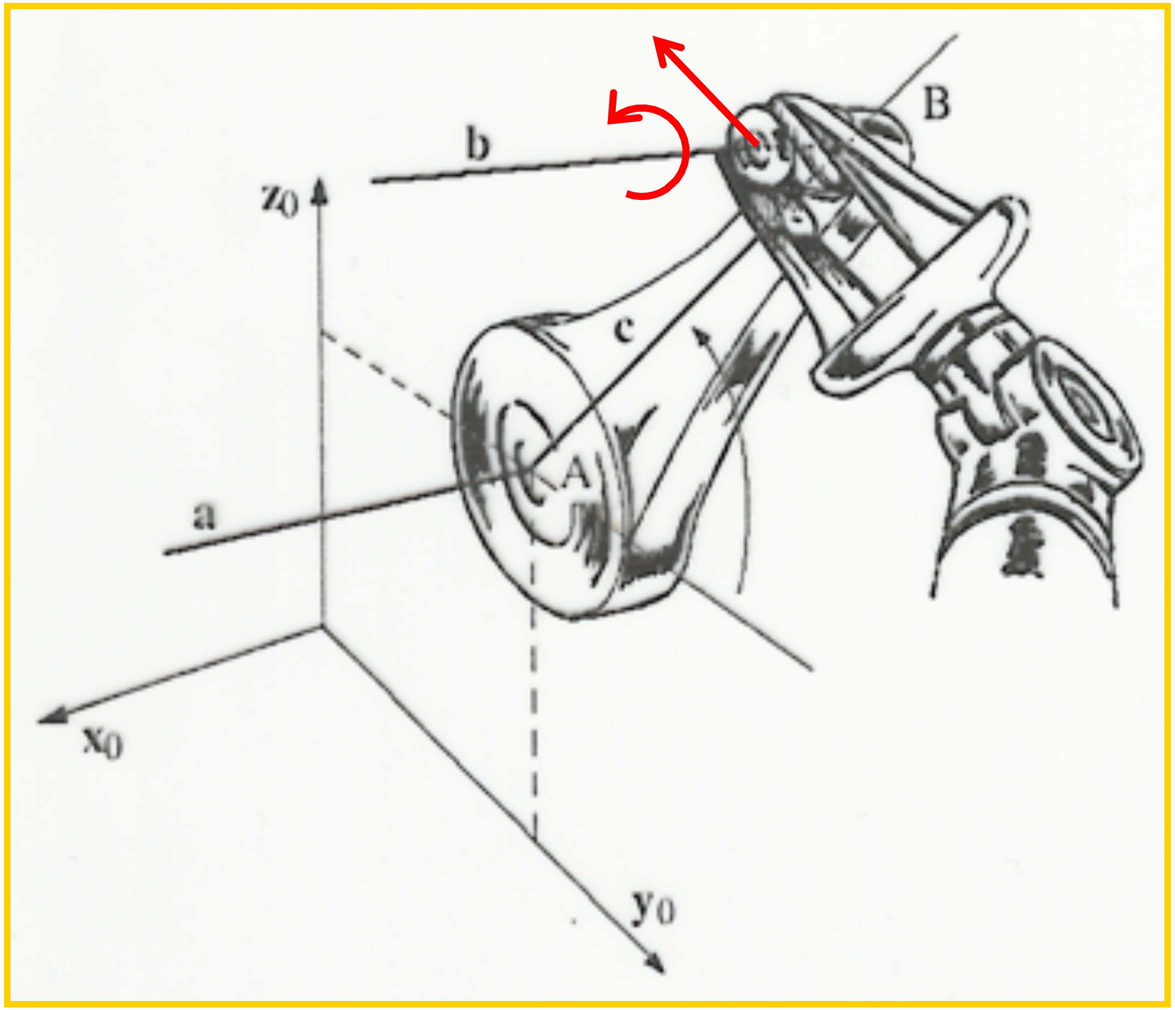

large $k_f$
$\Downarrow$
unstable




$$\boxed{F_c = k_2(x_2 - x_w)}$$

(see the paper by S. Eppinger, W.P. Seering: IEEE Control Systems Mag., 1987; available as extra material on the course web)

# Tasks requiring hybrid control



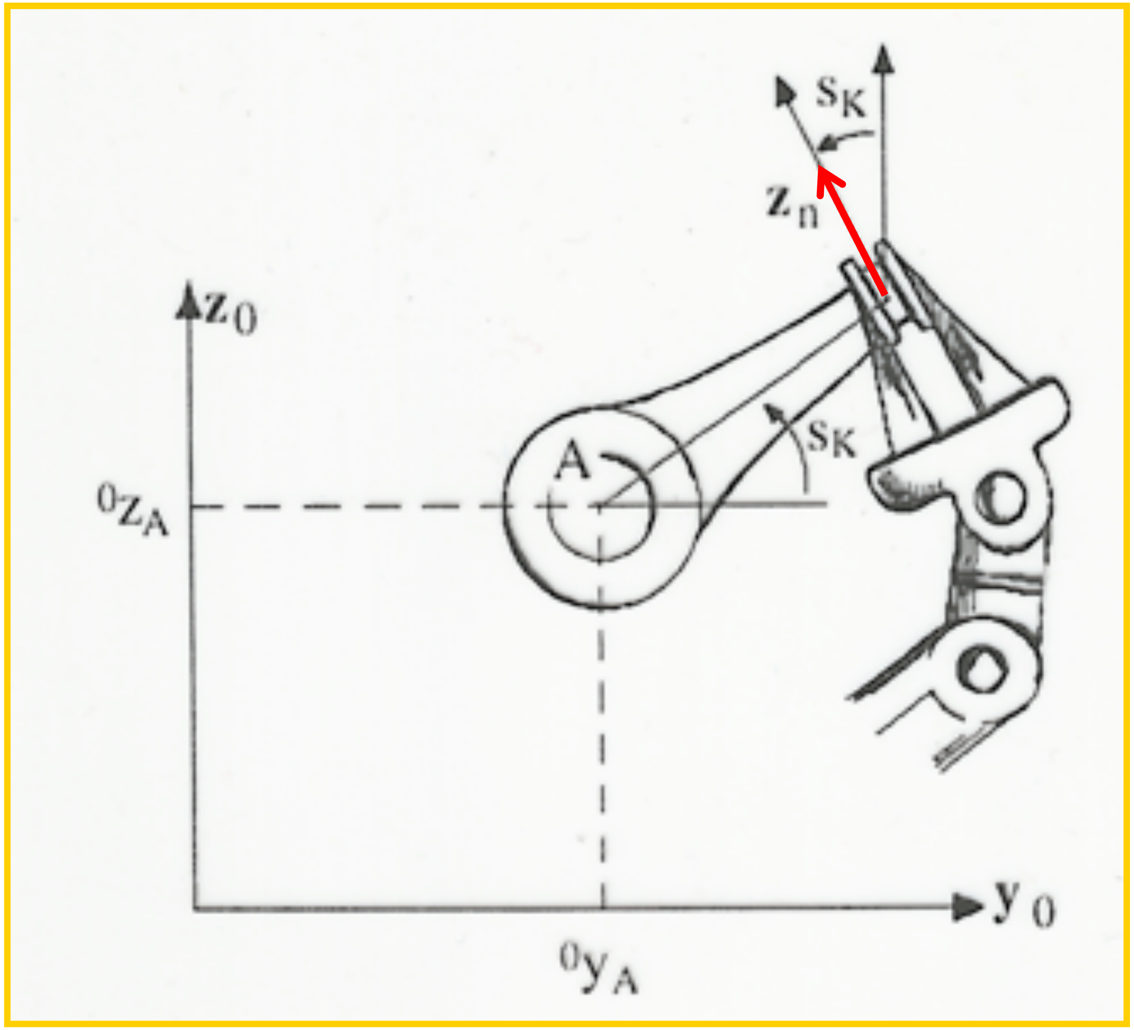

**two** generalized **directions** of instantaneous free motion at the contact: *tangential velocity & angular velocity* around handle axis

↕

**four directions** of generalized reaction forces at the contact

the robot should turn a crank having a free-spinning handle

# Tasks requiring hybrid control



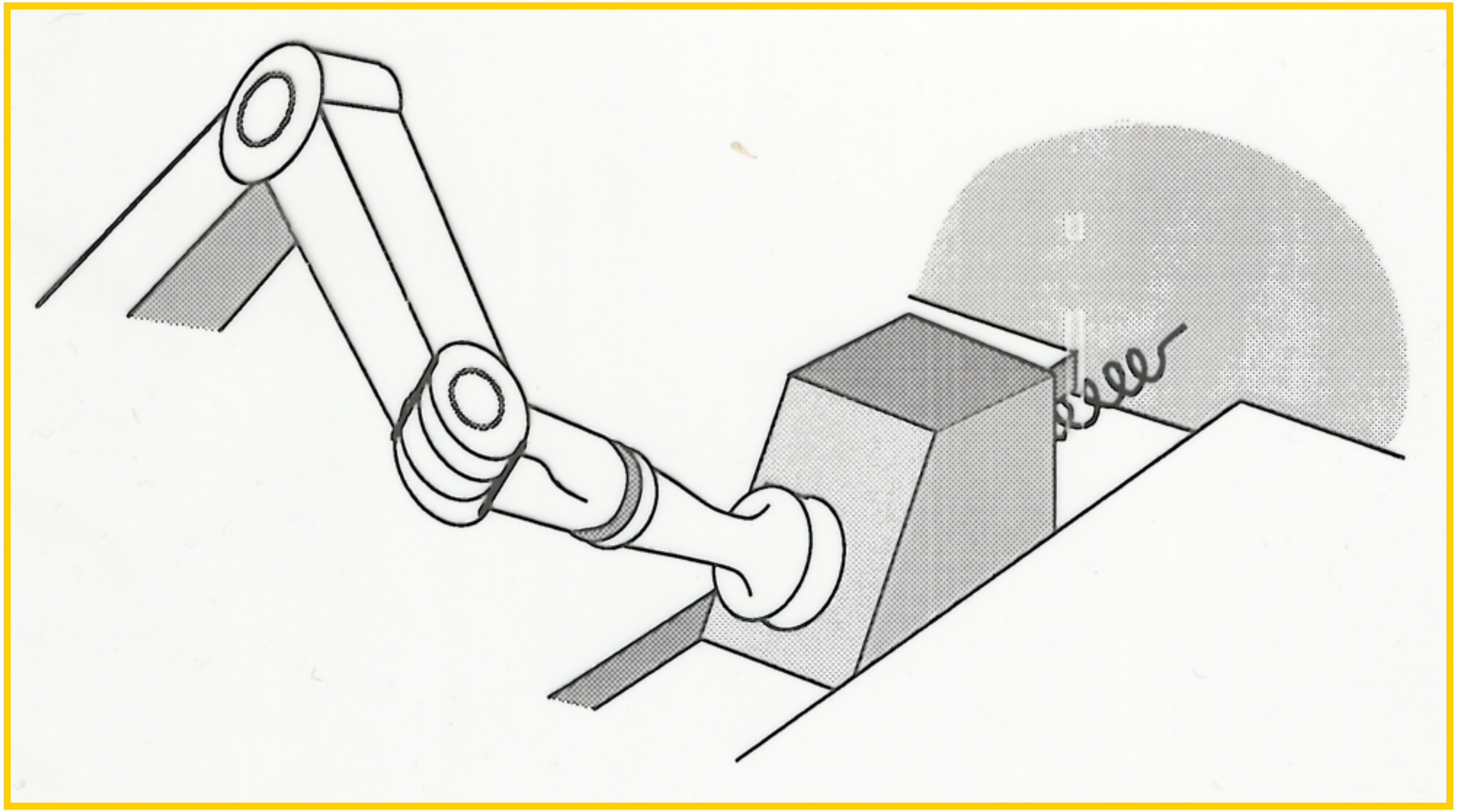

**one** direction **only** of instantaneous free motion at the contact: *tangential velocity*

↕

**five directions** of generalized reaction forces at the contact

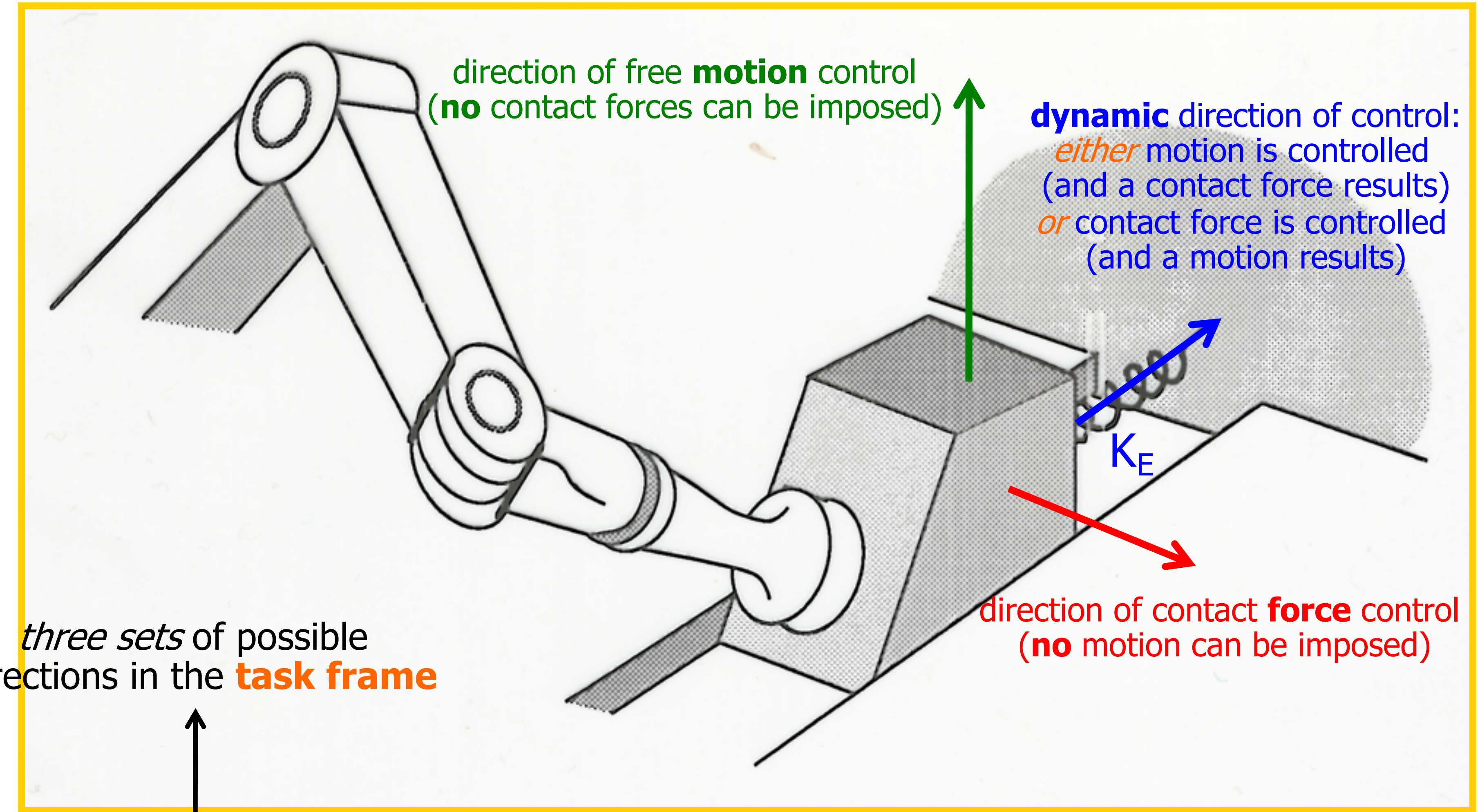
the robot should turn a crank having a fixed handle

# Tasks requiring hybrid control



the robot should push a mass
elastically coupled to a wall and constrained in a guide

# Tasks requiring hybrid control




*three sets* of possible directions in the **task frame**

*generalized* hybrid modeling and control for dynamic environments

A. De Luca, C. Manes: IEEE Trans. Robotics and Automation, vol. 10, no. 4, 1994