Robotics 2

Midterm Test – April 24, 2024

Exercise #1

Consider the 3R planar robot in the configuration $\boldsymbol{q}$ shown in Fig. 1, controlled by the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$.

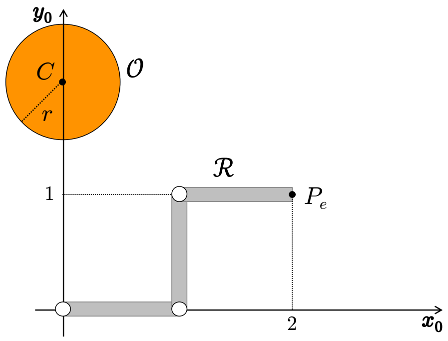

Figure 1: The 3R planar robot $\mathcal{R}$ with a single circular obstacle $\mathcal{O}$ in its workspace.

i) A desired Cartesian velocity $\boldsymbol{v}_e \in \mathbb{R}^2$ should be imposed to the end-effector point P_e of the robot $\mathcal{R}$, while maximizing the distance function $H_{\text{dist}}(\boldsymbol{q})$ between the robot body (i.e., its kinematic skeleton) and a circular obstacle $\mathcal{O}$ centered at $C = (0, 2)$ and of radius $r = 0.5$ m, as expressed by

$$H_{\text{dist}}(\boldsymbol{q}) = \min_{\boldsymbol{p}\in\mathcal{R}, \boldsymbol{o}\in\mathcal{O}} \|\boldsymbol{p}(\boldsymbol{q}) - \boldsymbol{o}\| = \|P_m(\boldsymbol{q}) - O_m\|,$$

where $P_m(\boldsymbol{q})$ and O_m are the closest points, respectively on the robot and on the obstacle, when the robot is in the given configuration $\boldsymbol{q}$. Determine the symbolic expression and the numerical value of the velocity command $\dot{\boldsymbol{q}}_{PG}$ according to the Projected Gradient (PG) method when $\boldsymbol{v}_e = \begin{pmatrix} 0 & 1 \end{pmatrix}^T$ m/s. Verify your result and compute also the resulting velocity $\boldsymbol{v}_m \in \mathbb{R}^2$ of the point P_m.

ii) Suppose now that the robot $\mathcal{R}$ has two tasks assigned, ordered by priority: the first task is to impose the same previous velocity $\boldsymbol{v}_e$ to the point P_e; the second task is to impose the velocity $\boldsymbol{v}_m$ obtained in the previous item to the point P_m. Determine the symbolic expression and the numerical value of the velocity command $\dot{\boldsymbol{q}}_{TP}$ according to the Task Priority (TP) method. Do the joint velocities $\dot{\boldsymbol{q}}_{TP}$ and $\dot{\boldsymbol{q}}_{PG}$ have different directions in the joint space or not? Explain why. What if we choose $\boldsymbol{v}_m = \alpha\,(P_m - O_m)$, for some $\alpha > 0$, as desired velocity for P_m?

Exercise #2

Figure 2 shows a 2R robot moving under gravity in the 3D space, with the associated Denavit-Hartenberg (D-H) frames. The following assumptions are made on the position of the center of mass and on the barycentric inertia of the two links, when these quantities are expressed in the frame attached to the respective link[1]:

$${}^1\boldsymbol{r}_{c1} = \begin{pmatrix} r_{c1,x} \\ r_{c1,y} \\ r_{c1,z} \end{pmatrix}, \; {}^2\boldsymbol{r}_{c2} = \begin{pmatrix} r_{c2,x} \\ 0 \\ 0 \end{pmatrix}, \; {}^1\boldsymbol{I}_{c1} = \begin{pmatrix} I_{c1,xx} & I_{c1,xy} & I_{c1,xz} \\ I_{c1,xy} & I_{c1,yy} & I_{c1,yz} \\ I_{c1,xz} & I_{c1,yz} & I_{c1,zz} \end{pmatrix}, \; {}^2\boldsymbol{I}_{c2} = \text{diag}\left\{I_{c2,xx}, I_{c2,yy}, I_{c2,zz}\right\}.$$

Note in particular that the center of mass of the first link is not on the axis of joint 1.

i) Derive the dynamic model in the form

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau},$$

and a factorization matrix $\boldsymbol{S}$ such that $\boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and matrix $\dot{\boldsymbol{M}} - 2\boldsymbol{S}$ is skew-symmetric.

[1] All given symbolic quantities are assumed to be generically non-zero.

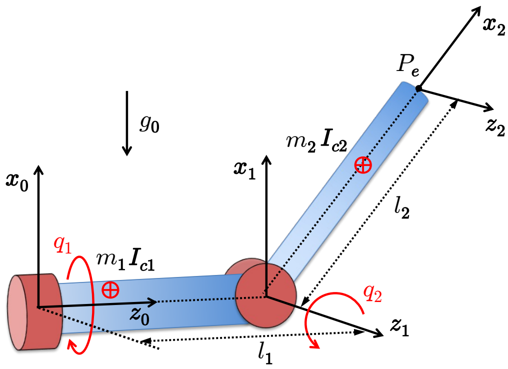

Figure 2: A 2R spatial robot with its D-H frames.

ii) Provide a minimal linear parametrization of the model

$$\boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\,\boldsymbol{a} = \boldsymbol{\tau},$$

giving the symbolic expression of the dynamic coefficients $\boldsymbol{a} \in \mathbb{R}^p$ and of the $2 \times p$ regressor matrix $\boldsymbol{Y}$. Assume that the acceleration of gravity g_0 is known.

iii) What is the expression of the torque $\boldsymbol{\tau}_d(t) = (\tau_{d1}(t), \tau_{d2}(t))$ needed to execute the desired motion $\boldsymbol{q}_d(t) = (q_{d1}(t), q_{d2}(t)) = (2\,t, \pi/4)$, with $t \in [0, \pi]$?

iv) When $\boldsymbol{\tau} = \boldsymbol{0}$, find as many as possible (unforced) equilibrium states $\boldsymbol{x}_e = (\boldsymbol{q}_e, \boldsymbol{0})$ of the robot —specify also the assumptions under which such equilibrium configurations $\boldsymbol{q}_e$ exist.

v) Choose mechanical parameters for the links such that the gravity term $\boldsymbol{g}(\boldsymbol{q})$ vanishes for all $\boldsymbol{q}$.

vi) Under the conditions found in item *v)*, with the robot at rest and being $\boldsymbol{\tau} = \boldsymbol{0}$, a force $\boldsymbol{F}_e \in \mathbb{R}^2$ is applied to the tip of the second link along a direction lying in the horizontal plane parallel to the plane $(\boldsymbol{y}_0, \boldsymbol{z}_0)$. Will the resulting tip acceleration $\ddot{\boldsymbol{p}}_e$ be in the same direction of the applied force $\boldsymbol{F}_e$ or not? Elaborate your answer.

Exercise #3

With reference to the standard recursive Newton-Euler algorithm for inverse dynamics, consider the calling instruction

$$\boldsymbol{u} = NE_0(\boldsymbol{q}, \boldsymbol{0}, \boldsymbol{0})$$

for a 6R robot in a nonsingular configuration $\boldsymbol{q}$. What will be the output $\boldsymbol{u}$ when the robot end-effector is *i)* in free space, or *ii)* subject to a known active wrench (force and moment) $\boldsymbol{F}_e \in \mathbb{R}^6$?

Exercise #4

Consider a rigid robot manipulator with n revolute joints, without external contact forces/moments and dissipative effects. The components of the motor torque $\boldsymbol{\tau} \in \mathbb{R}^n$ are bounded as

$$|\tau_i| \leq T_i, \qquad \text{with } \; T_i \geq 5 \cdot \max_{\boldsymbol{q}} |g_i(\boldsymbol{q})|\,, \qquad i = 1, \ldots, n,$$

namely with bounds that are large enough to guarantee that the robot always sustains at least its own weight under gravity (here, with a conservative margin factor of 5). Let the robot state at time $t = t_0$ be $\boldsymbol{x}(t_0) = (\boldsymbol{q}(t_0), \dot{\boldsymbol{q}}(t_0))$, with a total robot energy $E(t_0) = E_0$.

i) If the robot is in a generic state $\boldsymbol{x}(t_0) = (\boldsymbol{q}_0, \dot{\boldsymbol{q}}_0)$ with $\dot{\boldsymbol{q}}_0 \neq \boldsymbol{0}$, choose the torque $\boldsymbol{\tau}_0 = \boldsymbol{\tau}(t_0)$ to be delivered by the motors so that the total energy E will instantaneously decrease as much as possible.

ii) Suppose now that the robot is in a state $\boldsymbol{x}(t_0) = (\boldsymbol{q}_0, \boldsymbol{0})$, namely **at rest** and in a configuration $\boldsymbol{q}_0$ such that $\boldsymbol{g}(\boldsymbol{q}_0) \neq \boldsymbol{0}$. Choose again the motor torque $\boldsymbol{\tau}_0$ so that the total energy E will decrease. *Hint:* If this is not possible, choose $\boldsymbol{\tau}_0$ so that $\dot{E}$ will instantaneously decrease.

[210 minutes (3.5 hours); open books]