Robotics 2

February 13, 2026

Exercise #1

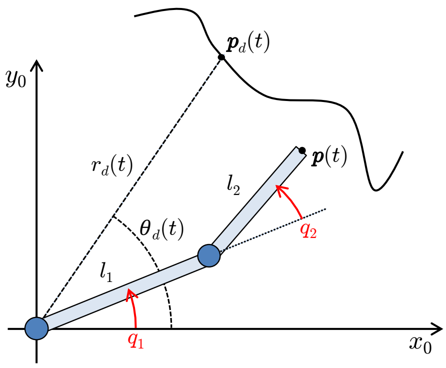

Figure 1: A 2R planar robot tracking a Cartesian trajectory

With reference to Fig. 1, the end-effector of a 2R robot, having different link lengths ($l_1 \neq l_2$) and moving on a horizontal plane, should track a smooth $\boldsymbol{p}_d(t) = (p_{xd}(t), p_{yd}(t))$. Design a control law for the joint torques $\boldsymbol{\tau} \in \mathbb{R}^2$ such that, for any initial state $(\boldsymbol{q}(0), \dot{\boldsymbol{q}}(0))$, the trajectory tracking error converges exponentially to zero in a decoupled way when expressed in polar coordinates $\boldsymbol{z} = (r, \theta)$. i.e., along the radial direction r and with respect to the pointing angle θ, with the closed-loop tracking errors $e_r = r_d - r$ and $e_\theta = \theta_d - \theta$ satisfying the dynamics

$$\ddot{e}_r + 2\lambda_r \dot{e}_r + \lambda_r^2 e_r = 0 \qquad \ddot{e}_\theta + 2\lambda_\theta \dot{e}_\theta + \lambda_\theta^2 e_\theta = 0,$$

with $\lambda_r > 0$, $\lambda_\theta > 0$. Provide the expression of all symbolic terms appearing in the chosen control law. Determine if and when the proposed control law may run into a singularity. For this, consider for instance the link lengths $l_1 = 2$ and $l_2 = 1.5$ [m], and the desired Cartesian trajectory

$$\boldsymbol{p}_d(t) = \begin{pmatrix} p_{xd}(t) \\ p_{yd}(t) \end{pmatrix} = \begin{pmatrix} 0.5\,(2 + \sin t) \\ 1.5 + 0.5 \cos t \end{pmatrix} \text{ [m]}. \tag{1}$$

Exercise #2

The end-effector of the RP robot in Fig. 2 is constrained to move on the Cartesian line $y = k$, with $k > 0$.

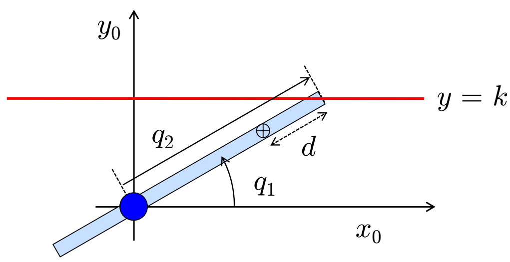

Figure 2: An RP robot moving on a horizontal plane with its end-effector constrained to a line

Derive the expression of the *reduced* robot dynamics, written in terms of pseudoacceleration and automatically satisfying the constraint. Try to provide global validity to this model. Design a control law that regulates to constant desired values v_d and λ_d, respectively the tangent velocity and the normal force to the end-effector constraint.

[180 minutes (3 hours); open books]