Robotics 2

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Exercise 1

Figure 1 shows a 2P planar robot having the two prismatic joint axes skewed by a fixed angle $\alpha = \pi/3$ rad.

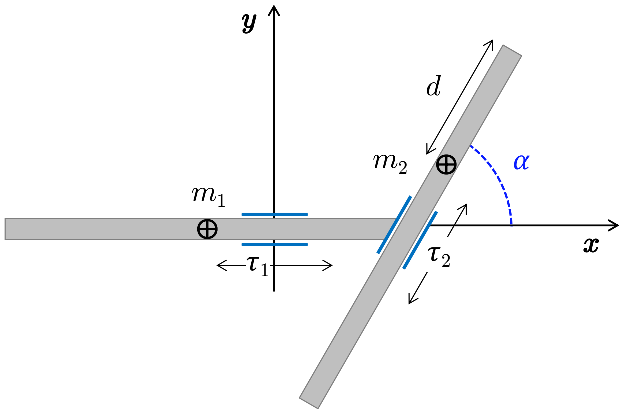

Figure 1: A 2P planar robot with skewed joint axes.

- Let the plane of motion be horizontal and the joint torques $\boldsymbol{\tau} \in \mathbb{R}^2$ be bounded componentwise as $|\tau_i| \leq T_i$, $i = 1, 2$, so that

$$-\boldsymbol{T} \leq \boldsymbol{\tau} \leq \boldsymbol{T}, \qquad \boldsymbol{T} = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \in \mathbb{R}^2. \tag{1}$$

  As a result of (1), is it true that the joint acceleration vector $\ddot{\boldsymbol{q}}$ is bounded by

$$-\boldsymbol{A} \leq \ddot{\boldsymbol{q}} \leq \boldsymbol{A}, \qquad \text{with } \boldsymbol{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \boldsymbol{M}^{-1}\boldsymbol{T},$$

  where $\boldsymbol{M}$ is the constant, symmetric, and positive definite 2×2 inertia matrix of the robot? Motivate your answer.

- Suppose next that the robot is subject also to gravity, i.e., the plane of motion is vertical. Let the mass of the two links be $m_1 = 5$ and $m_2 = 3$ [kg], while the distance of the center of mass of the second link from the tip is $d = 0.2$ m. Determine the region of all feasible accelerations $\ddot{\boldsymbol{q}} \in \mathbb{R}^2$ under the bound (1), for $T_1 = T_2 = 30$ N.

Exercise 2

Consider the robot of Exercise 1 moving in a vertical plane. By means of a revolute joint, add at the tip end of the second link also a third link of length $l_3 = 0.5$ m, uniformly distributed mass $m_3 = 2$ kg, and barycentric inertia (normal to the motion plane) $I_3 = m_3 l_3^2/12$. The resulting structure is a (skewed) PPR planar robot. For the third joint variable, let $q_3 = 0$ correspond to the third link being horizontal. Neglect all dissipative effects.

- Derive the dynamic model of this robot and find a minimal linear parametrization with dynamic coefficients a_i, $i = 1, \ldots, p$, with associated $3 \times p$ regressor matrix $\boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$.

- Design a control law based on feedback linearization that achieves exponential tracking of the desired trajectory

$$\boldsymbol{q}_d(t) = \begin{pmatrix} 1 - \cos 4t \\ 0 \\ \pi(t-1)^2/2 + \pi/4 \end{pmatrix} \quad \text{[m,m,rad]},$$

  with diagonal PD gains $\boldsymbol{K}_P = \text{diag}\{25, 25, 25\}$ and $\boldsymbol{K}_D = \text{diag}\{10, 10, 10\}$. If the robot is at rest at $t = 0$ in the configuration $\boldsymbol{q}(0) = (1, 1, \pi/4)$ [m,m,rad], compute the value of the initial control input $\boldsymbol{\tau}(0) \in \mathbb{R}^3$.

Exercise 3

With reference to Fig. 2, consider an assembly task in which a sphere is fully inserted in a cylindric hole with reduced clearance. A 6R robot can move the sphere within the hole using an end-effector equipped with a suction cup that firmly holds the sphere, without interfering with the lateral sides of the hole. Neglecting contact friction, define a suitable task frame and write the natural and artificial constraints for this task. Draw a block diagram of a hybrid force-velocity controller for the task, indicting the number and type of variables that are motion controlled and force controlled.

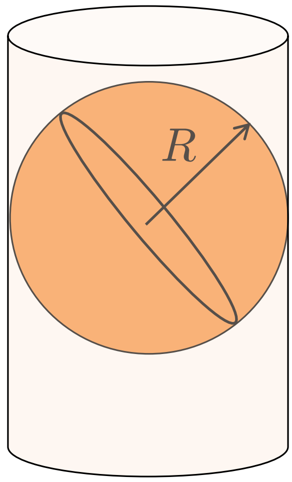

Figure 2: A sphere-in-hole task.

Exercise 4

Consider again the (skewed) PPR robot of Exercise 2 and suppose that the control architecture allows to command directly the joint velocities $\dot{\boldsymbol{q}} \in \mathbb{R}^3$. If a motion task is specified only for the robot end-effector velocity as $\boldsymbol{v}_d = (1, 0)$ [m/s], determine the explicit expression of the joint velocity command $\dot{\boldsymbol{q}}_d$ that executes the task while minimizing instantaneously the kinetic energy $T = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}$ of the robot.

[180 minutes, open books]