Robotics II

January 24, 2024

Exercise 1

The dynamic model of a rigid robot with n generalized coordinates $\boldsymbol{q}$ and associated input torques $\boldsymbol{\tau} \in \mathbb{R}^n$ is written in the usual Lagrangian form as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{F}\dot{\boldsymbol{q}} = \boldsymbol{\tau}, \tag{1}$$

with inertial, Coriolis/centrifugal, gravity, and viscous friction terms (with diagonal matrix $\boldsymbol{F} > 0$). The Coriolis and centrifugal quadratic terms in the velocity $\dot{\boldsymbol{q}}$ can be suitably factorized as $\boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}}$.

Derive the explicit expressions of a nonlinear state-space representation of the n second-order differential equations (1) in the general form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\,\boldsymbol{u}, \qquad \text{with} \quad \boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix} \in \mathbb{R}^{2n}, \qquad \boldsymbol{u} = \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{p} = \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \in \mathbb{R}^n$ is the generalized momentum of the robot.

Exercise 2

Consider the RP robot in Fig. 1 moving in a vertical plane and with the generalized coordinates $\boldsymbol{q} = (q_1, q_2)$ defined therein. The center of mass of the first link is on the axis of joint 1. The robot is commanded at the joint level by the input $\boldsymbol{\tau} = (\tau_1, \tau_2)$, with a torque τ_1 and a force τ_2. Viscous friction is acting on the joints.

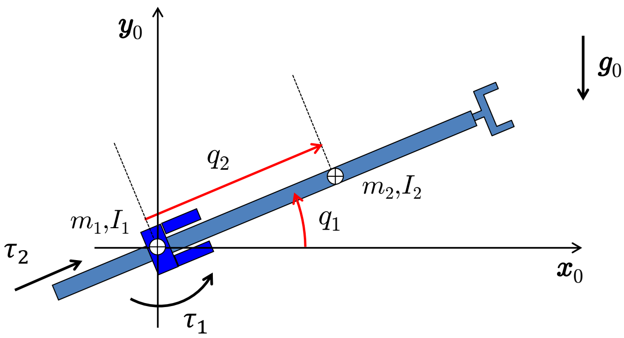

Figure 1: A RP planar robot and its relevant kinematic and dynamic parameters.

- Write down for this robot the $(2n =)\,4$ first-order scalar equations of the state-space representation (2).
- Assume the following dynamic data: $m_1 = 10$, $m_2 = 5$ [kg]; $I_1 = 0.1$, $I_2 = 0.5$ [kg $\cdot$ m^2]. Neglect the viscous friction terms. With the robot in the configuration $\boldsymbol{q} = (\pi/4, 1)$ [rad, m], having a joint velocity $\dot{\boldsymbol{q}} = (2, -0.5)$ [rad/s, m/s], and with an input command $\boldsymbol{\tau} = (1, 0)$ [Nm, N], provide the numerical values of the last two components of $\dot{\boldsymbol{x}}$, namely of $\dot{p}_1$ and $\dot{p}_2$.

Exercise 3

A number of questions and statements are reported on the attached Questionnaire Sheet. Fill in your answers and/or comments on the same or on a different sheet, providing a short motivation/explanation for each item.

[180 minutes, open books]

Robotics II - Questionnaire Sheet

January 24, 2024

Name: ______________________________

Answer to the questions or comment the statements, providing also a *short* motivation/explanation for each of the following 4 items.

1. Write down the calibration equation for a planar 2R robot in which the only inaccurate values are the link lengths l_1 and l_2. Describe briefly how to set up a kinematic calibration procedure in this case.

2. At a given $\boldsymbol{q} \in \mathbb{R}^n$, we have to choose the velocity command $\dot{\boldsymbol{q}} \in \mathbb{R}^n$ that minimizes the objective function $H = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{A}^{-1}\dot{\boldsymbol{q}}$, with $\boldsymbol{A} > 0$, while satisfying the task $\boldsymbol{J}\dot{\boldsymbol{q}} = \dot{\boldsymbol{x}} \in \mathbb{R}^m$, with $m < n$ and $\text{rank}\{\boldsymbol{J}\} = m$. Two velocity commands have been proposed as

$$\dot{\boldsymbol{q}}' = \boldsymbol{A}^{-1}\boldsymbol{J}^T\left(\boldsymbol{J}\boldsymbol{A}^{-1}\boldsymbol{J}^T\right)^{-1}\dot{\boldsymbol{x}} \qquad \text{and} \qquad \dot{\boldsymbol{q}}'' = \boldsymbol{J}^{\#}\dot{\boldsymbol{x}} - \left(\boldsymbol{I} - \boldsymbol{J}^{\#}\boldsymbol{J}\right)\nabla_{\dot{\boldsymbol{q}}}H, \quad \text{with} \;\; \nabla_{\dot{\boldsymbol{q}}}H = \boldsymbol{A}^{-1}\dot{\boldsymbol{q}}.$$

   Which command is better? Why?

3. Consider an assembly task, in which a peg having an equilateral triangle as section is to be inserted at a slow but constant speed V in a similar hole with reduced clearance. Neglecting contact friction, define a suitable task frame and write the natural and artificial constraints for this task.

4. A mass m is in free linear motion in a vertical plane. At time $t = 0$, the mass is in the position $y(0) = 0$ and has a vertical (upward) velocity $\dot{y}(0) = v_0 > 0$. At a given time $t = \delta > 0$, will the position $y(\delta)$ be positive, zero, or negative? At which time $t = \bar{\delta} > 0$, if any, will the mass invert the sign of its velocity?