Robotics 2

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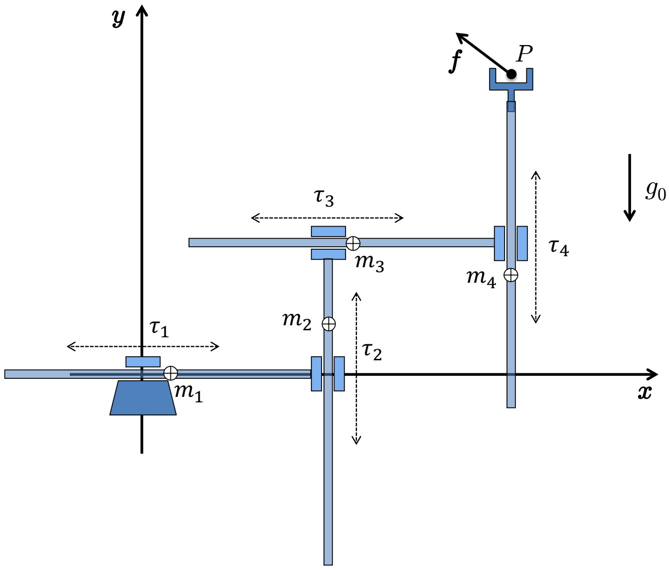

Figure 1: A 4P planar robot in the vertical plane.

Consider the 4-dof planar robot shown in Fig. 1, moving under gravity. All robot joints are prismatic and thus the joint torques $\boldsymbol{\tau} \in \mathbb{R}^4$ are in fact forces (with units expressed in [N]).

1) Derive in symbolic form the dynamic model of this robot in the joint space, including the external force $\boldsymbol{f} \in \mathbb{R}^2$ acting on the end-effector.

2) Determine the 2×2 Cartesian inertia matrix $\boldsymbol{M}_r$ at the end-effector level. Compute $\boldsymbol{M}_r$ numerically for $\boldsymbol{q} = \mathbf{0}$ and equal link masses $m_i = 1$ [kg], $i = 1, \ldots, 4$.

3) Design the simplest control law for $\boldsymbol{\tau}$ that is able to globally asymptotically stabilize the robot to a desired constant configuration $\boldsymbol{q}_d$, also in the presence of a known force $\boldsymbol{f}$ acting on the end-effector. Discuss the necessity and/or sufficiency of suitable choices for the feedback gains on position error $\boldsymbol{e} = \boldsymbol{q}_d - \boldsymbol{q}$ and on velocity error $\dot{\boldsymbol{e}} = -\dot{\boldsymbol{q}}$.

Set now $\boldsymbol{f} = \mathbf{0}$.

4) Let $\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \boldsymbol{g}(\boldsymbol{q})$ (gravity is cancelled from the picture). Find a non-zero joint torque $\boldsymbol{\tau}_0 \in \mathbb{R}^4$ that does not generate any acceleration $\ddot{\boldsymbol{r}} = (\ddot{p}_x, \ddot{p}_y) \in \mathbb{R}^2$ at the end-effector level.

5) Provide the expression of the joint torque $\boldsymbol{\tau}$ of minimum norm that realizes a desired acceleration $\ddot{\boldsymbol{r}} \in \mathbb{R}^2$ of the end-effector. Compute $\boldsymbol{\tau}$ numerically for $\boldsymbol{q} = \dot{\boldsymbol{q}} = \mathbf{0}$, equal link masses $m_i = 1$ [kg], $i = 1, \ldots, 4$, and $\ddot{\boldsymbol{r}} = (2, -3)$ [m/s^2].

6) Suppose you have no knowledge about the values of the dynamic parameters of this 4P robot. Design an adaptive control law for $\boldsymbol{\tau}$ that is able to globally asymptotically stabilize the tracking error along a smooth desired trajectory $\boldsymbol{q}_d(t)$. The controller should be of minimal dimension.

[180 minutes; open books]