Robotics 2

Remote Exam – October 23, 2020

Exercise #1

The PR robot in Fig. 1 moves on a horizontal plane. Its positive definite inertia matrix has the form

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{pmatrix} a & b\cos q_2 \\ b\cos q_2 & c \end{pmatrix} > 0. \tag{1}$$

Using only the symbolic coefficients a, b and c in (1), provide the expression of the regressor matrix $\boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r)$ in the *adaptive* control law that guarantees global asymptotic tracking of a smooth joint trajectory $\boldsymbol{q}_d(t)$, without a priori information on the values of the dynamic coefficients.

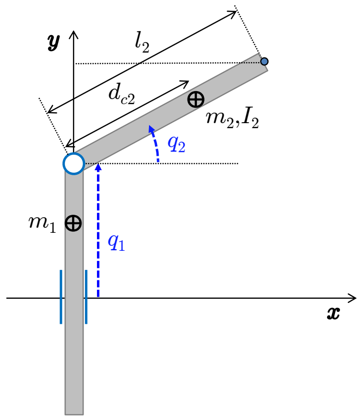

Figure 1: The planar PR robot with definition of joint variables q_1 and q_2 and relevant parameters.

Exercise #2

For the same PR robot in Fig. 1, suppose that a desired twice-differentiable trajectory $y_d(t) \in \mathbb{R}$ has been assigned to the coordinate y of the end-effector position. With the robot in the configuration $\bar{\boldsymbol{q}} = \begin{pmatrix} 1 & \pi/2 \end{pmatrix}^T$ and at rest, provide the three input vectors of joint force/torque $\boldsymbol{\tau}_A$, $\boldsymbol{\tau}_B$, and $\boldsymbol{\tau}_C$ (all $\in \mathbb{R}^2$) that execute the desired task and instantaneously minimize, respectively,

$$H_A = \frac{1}{2}\left\|\boldsymbol{\tau}\right\|^2, \qquad H_B = \frac{1}{2}\left\|\boldsymbol{\tau}\right\|^2_{\boldsymbol{M}^{-1}(\bar{\boldsymbol{q}})}, \qquad \text{or} \qquad H_C = \frac{1}{2}\left\|\boldsymbol{\tau}\right\|^2_{\boldsymbol{M}^{-2}(\bar{\boldsymbol{q}})}.$$

Which of the three solutions has the largest component at the first joint in absolute value?

Exercise #3

Assume now that the robot in Fig. 1 is moving instead in a vertical plane. Derive the expression of the gravity term $\boldsymbol{g}(\boldsymbol{q})$ in the dynamic model as a function of the robot dynamic parameters m_1, m_2, d_{c2} and I_2. In order to regulate the robot configuration at a desired constant value $\boldsymbol{q}_d$, the following control law is being applied:

$$\boldsymbol{\tau} = \boldsymbol{K}_P(\boldsymbol{q}_d - \boldsymbol{q}) - \boldsymbol{K}_D\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d). \tag{2}$$

Provide explicit sufficient conditions (depending also on the robot dynamic parameters) ensuring that the law (2) globally asymptotically stabilizes the closed-loop equilibrium state $(\boldsymbol{q}, \dot{\boldsymbol{q}}) = (\boldsymbol{q}_d, \boldsymbol{0})$.

Exercise #4

Consider the task of rolling a sphere of radius R on a rigid plane, while the contact point C follows a desired trajectory on the planar surface. The sphere will be held by a robot through a gimbal fork that allows free rotation of the sphere around the instantaneous direction of the axis $\boldsymbol{r}$ —see Fig. 2. In turn, the fork can be rotated around the axis $\boldsymbol{t}$, which remains however always vertical. Under the action of a sufficiently large force in the normal direction to the plane, friction at the contact will let the sphere only roll when in motion, without slipping. With this in mind, define a suitable task frame and specify the natural and artificial constraints associated to this hybrid force-velocity control problem. How many generalized velocity directions $\boldsymbol{V} = \begin{pmatrix} \boldsymbol{v}^T & \boldsymbol{\omega}^T \end{pmatrix}^T \in \mathbb{R}^6$ can be independently assigned by the control law?

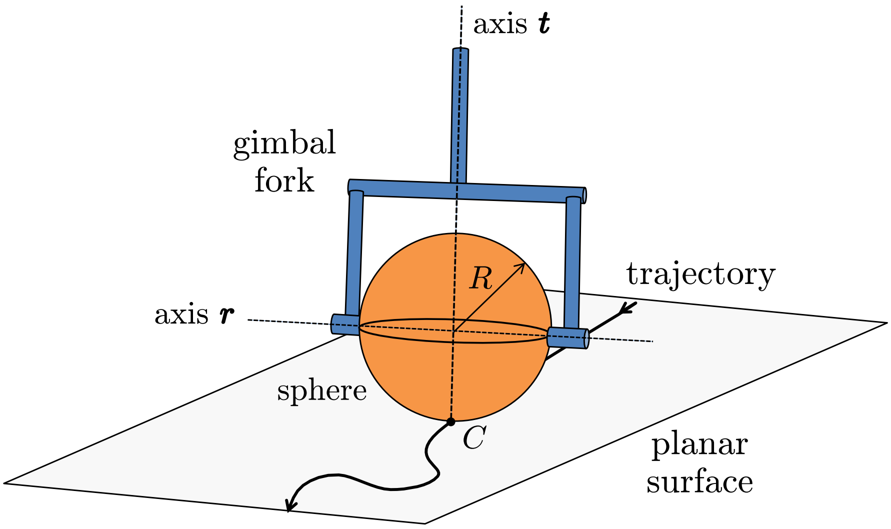

Figure 2: A sphere rolling on a plane and following a desired trajectory of the contact point C.

[180 minutes (3 hours); open books]