Robotics II

January 11, 2017

Exercise 1

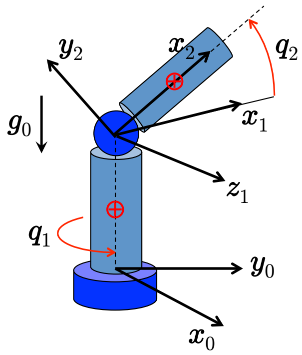

Figure 1: A 2R polar robot with associated link frames.

The 2R polar robot shown in Fig. 1 moves in the presence of gravity and has links of cylindric form and uniformly distributed mass. Its dynamic model is

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau},$$

where

$$\boldsymbol{B}(\boldsymbol{q}) = \begin{pmatrix} a_1 + a_2 \sin^2 q_2 & 0 \\ 0 & a_3 \end{pmatrix}, \quad \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{pmatrix} 2a_2 \sin q_2 \cos q_2 \, \dot{q}_1 \dot{q}_2 \\ -a_2 \sin q_2 \cos q_2 \, \dot{q}_1^2 \end{pmatrix}, \quad \boldsymbol{g}(\boldsymbol{q}) = \begin{pmatrix} 0 \\ a_4 \cos q_2 \end{pmatrix}.$$

with $a_1 = I_{1y} + I_{2y} + m_2 d_2^2$, $a_2 = I_{2x} - I_{2y} - m_2 d_2^2$, $a_3 = I_{2z} + m_2 d_2^2$, and $a_4 = m_2 g_0 d_2$.

- Give a physical interpretation of the inertia matrix elements that confirms their correctness.
- Write down all expressions of feedback control laws for $\boldsymbol{\tau}$ that you are aware of, which guarantee regulation to a desired (generic) constant configuration $\boldsymbol{q}_d$. Specify for each law the design conditions for success and the type of convergence/stability achieved.

Exercise 2

In inverse dynamics problems for serial manipulators, the most efficient implementations are based on a numerical Newton-Euler (NE) algorithm that contains a forward recursive (FR) part, which computes from the base to the tip all relevant differential kinematic terms associated to the links, and a backward recursive (BR) part, which computes from the tip to the base the exchanged forces/torques between links. Suppose now that we compute the (linear/angular) acceleration vector $\ddot{\boldsymbol{p}} \in \mathbb{R}^6$ of the end-effector by

$$\ddot{\boldsymbol{p}} = NEFR(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}},$$

where $NEFR$ denotes compactly the FR part only of the NE algorithm.

- How can the $NEFR$ algorithm be used to evaluate numerically and separately the Jacobian matrix $\boldsymbol{J}$ and the product term $\dot{\boldsymbol{J}}\dot{\boldsymbol{q}}$? How many times is the algorithm called in total?
- With the same algorithm, can we evaluate also the matrix $\dot{\boldsymbol{J}}$ alone? If so, how?

Exercise 3

Consider a planar 3R robot with unitary link lengths. Taking into account robot redundancy, a kinematic control scheme is active at the velocity level so as to track a desired end-effector position trajectory, while trying to locally maximize the minimum Cartesian distance of the robot body from obstacles.

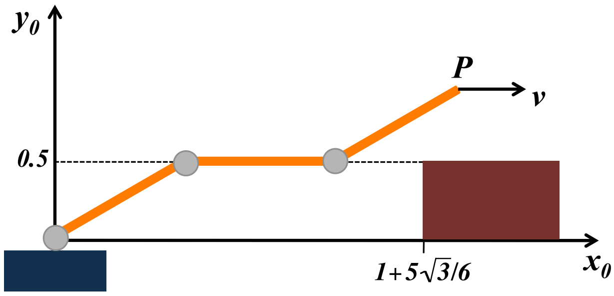

Figure 2: A planar 3R robot moving its end effector in the presence of an obstacle.

- In the shown configuration $\boldsymbol{q} = (30°, -30°, 30°)$ and with a single obstacle placed as in Fig. 2, the robot end effector is assigned a unitary velocity $\boldsymbol{v}$ in the positive $\boldsymbol{x}_0$ direction. Specify one particular kinematic control scheme achieving at best both tasks, and provide the associated numerical value of the command vector $\dot{\boldsymbol{q}} \in \mathbb{R}^3$.
- Compare with a minimum velocity norm solution that neglects the presence of the obstacle.

[180 minutes; open books but no computer or smartphone]