Robotics 2

Control in the Cartesian Space

Prof. Alessandro De Luca

Dipartimento di Ingegneria Informatica
Automatica e Gestionale Antonio Ruberti

Sapienza
Università di Roma

Regulation of robot Cartesian pose

- "PD +" type control for regulation problems
  - proportional to the Cartesian pose error, with a derivative term (on velocity) + cancellation/compensation of gravity in joint space
- robot
  - dynamics $M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = u$
  - kinematics $p = f(q) \longrightarrow \dot{p} = J(q)\dot{q}$

dimension of spaces
joint = n
Cartesian = m

- goal: asymptotic stabilization of the end-effector pose

$$p = p_d, \dot{q} = \dot{q}_d = 0 \longrightarrow \dot{p}_d = 0$$

Note: if $m = n$, then $\dot{q} = 0 \Leftrightarrow \dot{p} = 0$ up to singularities

if $m < n$, then the goal is not uniquely associated to a complete robot state: $n - m$ joint coordinates are missing…

A Cartesian regulation law

(*) $$u = J^T(q)K_P(p_d - p) - K_D\dot{q} + g(q)$$

$K_P, K_D > 0$ (symmetric)

Theorem

under the control law (*), the robot state will converge asymptotically to the set $A = \{\dot{q} = 0, q: K_P\left(p_d - f(q)\right) \in N(J^T(q))\}$

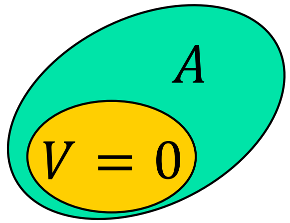
$$\supseteq \{\dot{q} = 0, q: f(q) = p_d\}$$

Proof

define $e_P = p_d - p$ (Cartesian error) and the associated Lyapunov-like candidate function

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}e_P^T K_P e_P$$

with $V = 0 \Leftrightarrow (q, \dot{q}) \in \{\dot{q} = 0, q: f(q) = p_d\} \subseteq A$



Proof (cont)

differentiating $V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2} e_P^T K_P e_P \geq 0$

$$\dot{V} = \dot{q}^T\left(M\ddot{q} + \frac{1}{2}\dot{M}\dot{q}\right) - e_P^T K_P \dot{p}$$
$$= \dot{q}^T\left(u - S\dot{q} - g + \frac{1}{2}\dot{M}\dot{q}\right) - e_P^T K_P \dot{p}$$
$$= \dot{q}^T\left(J^T K_P e_P - K_D\dot{q} + g - g\right) - e_P^T K_P J\dot{q}$$
$$= -\dot{q}^T K_D \dot{q} \leq 0$$

with $\dot{V} = 0 \Leftrightarrow \dot{q} = 0$

in this situation, the closed-loop equations become

$$M(q)\ddot{q} + g(q) = J^T(q)K_P e_P + g(q) \Rightarrow \ddot{q} = M^{-1}(q)J^T(q)K_P e_P$$

$$\Rightarrow \ddot{q} = 0 \Leftrightarrow K_P e_P \in N(J^T(q))$$

by applying LaSalle theorem, the thesis follows ◁

Corollary

for a given initial state $(q(0), \dot{q}(0))$, if the robot does not encounter any singularity of $J^T(q)$ (configurations where $\rho(J^T) < m \leq n$) during its motion, then there is asymptotic stabilization to one single state $(m = n)$ or to a set of states $(m < n)$ such that

$$e_P = 0, \dot{q} = 0$$

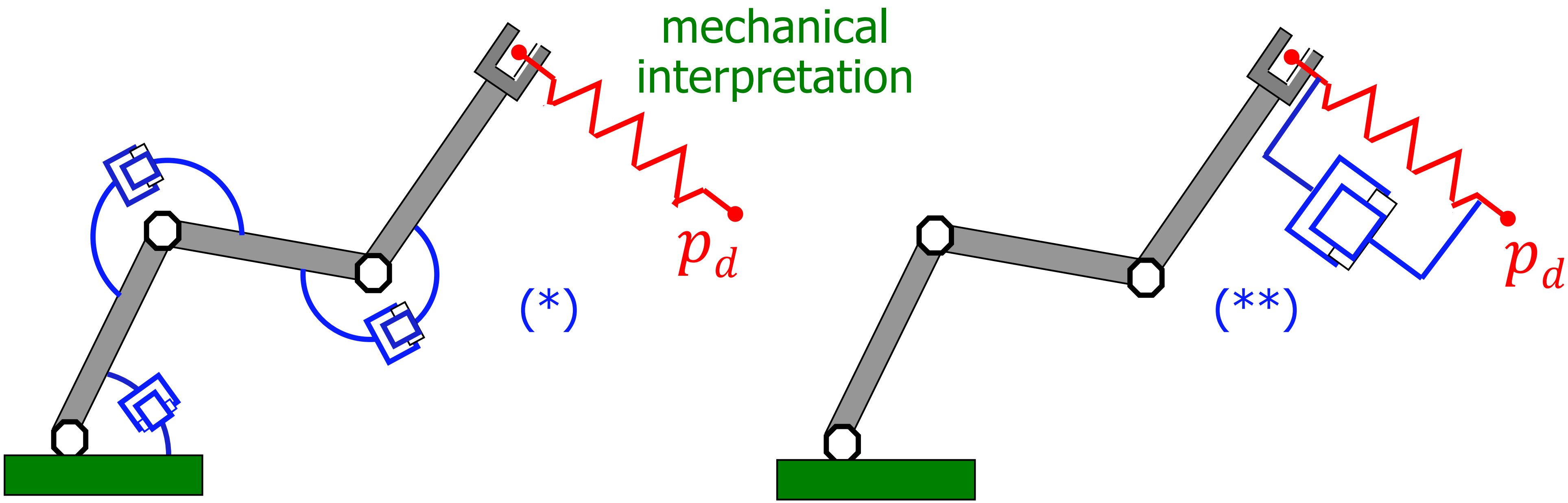
Note: singular configurations q of $J^T(q)$ coincide with those of $J(q)$

A possible variant for regulation

"all Cartesian" PD control + gravity cancellation in joint space

$$(**)\quad u = J^T(q)[K_P(p_d - p) - K_D\dot{p}] + g(q)$$

$K_P, K_D > 0$ (symmetric)



J^T transforms the "virtual" elastic, for (*), or visco-elastic, for (**), force/torque acting on the end-effector into control torques at the joints

Feedback linearization in Cartesian space

robot $M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u$

output $y = p, \quad p = f(q)$ Cartesian position/orientation

assume: $m = n$

algorithm: differentiate the output(s) as many times as needed up to the appearance of (at least one of) the input torque(s), then verify if it is possible to solve for the input = "inversion"

$$y = f(q)$$
$$\dot{y} = J(q)\dot{q}$$
$$\ddot{y} = J(q)\ddot{q} + \dot{J}(q)\dot{q}$$
$$= J(q)M^{-1}(q)[u - c(q,\dot{q}) - g(q)] + \dot{J}(q)\dot{q}$$

from the dynamic model ($\ddot{q}$)

uniform "relative degree" $\rho = 2$ for all outputs

Theorem

for a non-redundant robot, it is possible to exactly linearize and decouple the dynamic behavior at the Cartesian level if and only if

$$\det J(q) \neq 0$$

Feedback linearization in Cartesian space
(in the right coordinates!)

control law

$$u = M(q)J^{-1}(q)a + c(q,\dot{q}) + g(q) - M(q)J^{-1}(q)\dot{J}(q)\dot{q}$$

$$= \beta(q)a + \alpha(q,\dot{q})$$

$$\Rightarrow \quad \ddot{y} = \ddot{p} = J(q)M^{-1}(q)[u - c(q,\dot{q}) - g(q)] + \dot{J}(q)\dot{q} = a$$

$p, \dot{p}$ are the so-called "**linearizing**" coordinates

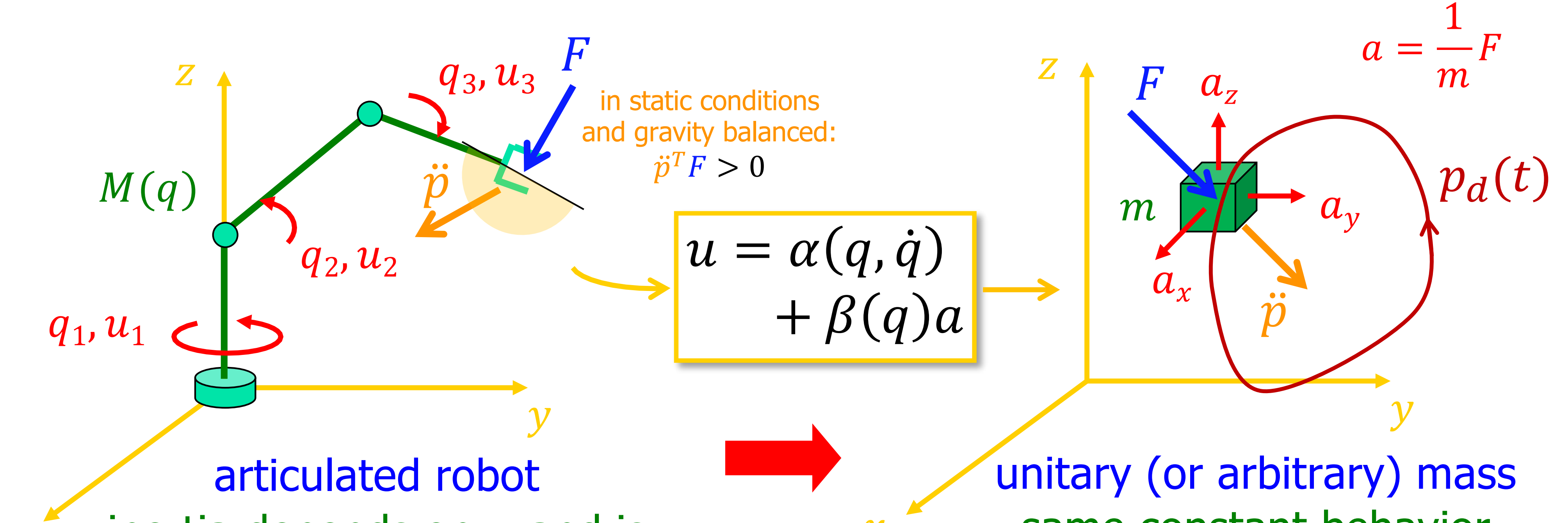
closed-loop equations (in the joint space)

$$M^{-1} * \quad M\ddot{q} + c + g = MJ^{-1}\left[a - \dot{J}\dot{q}\right] + c + g$$

$$\Rightarrow \quad \ddot{q} = J^{-1}(q)a - J^{-1}(q)\dot{J}(q)\dot{q}$$

purely kinematic equations (but still **nonlinear** and **coupled**!!)

Physical interpretation



$$\ddot{p}^T F > 0$$

$$u = \alpha(q,\dot{q}) + \beta(q)a$$

$$a = \frac{1}{m}F$$

articulated robot
inertia depends on q and is variable in different Cartesian directions

unitary (or arbitrary) mass
same constant behavior in all Cartesian directions

when a control force F is applied at the end-effector

- the uncontrolled robot will accelerate with $\ddot{p}$ in a different direction
- the mass accelerates in the same direction of the applied force F

Alternative derivation
in purely Cartesian terms

the previous exact linearizing and decoupling law can be rewritten in Cartesian terms using a control force/torque F

$$u = M(q)J^{-1}(q)a + c(q,\dot{q}) - M(q)J^{-1}(q)\dot{J}(q)\dot{q} + g(q)$$

joint torque u is moved to the Cartesian space as $F = J^{-T}(q)u$ (for $m = n$)

$$F = [J^{-T}MJ^{-1}]a \longrightarrow \text{Cartesian inertia} = [JM^{-1}J^{T}]^{-1} = M_p(p)$$

$$+[J^{-T}c - J^{-T}MJ^{-1}\dot{J}\dot{q}] \rightarrow \text{Cartesian Coriolis/centrifugal terms}$$

$$+[J^{-T}g] \longrightarrow \text{Cartesian gravity}$$

$$= M_p a + c_p + g_p$$

➡ this is the feedback linearization law applied to the **Cartesian dynamic model** of the robot

$$M_p(p)\ddot{p} + c_p(p,\dot{p}) + g_p(p) = F$$

➡ $\ddot{p} = a$

Remarks - 1

- the design of a Cartesian trajectory tracking control is completed by stabilizing the tracking error in the m independent chains of double integrators, i.e., by setting

$$a_i = \ddot{p}_{di} + K_{Di}(\dot{p}_{di} - \dot{p}_i) + K_{Pi}(p_{di} - p_i)$$

scalars

$K_{Pi} > 0, K_{Di} > 0$

$i = 1, \dots, m$

- the transient behavior of the Cartesian error along a desired trajectory is exponentially stable (with arbitrary eigenvalues assigned by choosing the diagonal gains of K_P, K_D)
- in redundant robots ($m < n$), by replacing $MJ^{-1} = (JM^{-1})^{-1}$ in the control law with some (weighted) pseudoinverse $(JM^{-1})^{\#}_{W}$, one still obtains input-output decoupling and linearization, but not exact linearization of the whole state dynamics
  - there is an additional internal dynamics left of dimension $n - m$

More on the redundant case …

- suppose $m < n$, but with a Jacobian J of full rank m
- let the control law (with null-space torque term u_0) be defined as

$$u = \left(J(q)M^{-1}(q)\right)^{\#}_{W}\left(a - \dot{J}(q)\dot{q} + J(q)M^{-1}(q)\left(c(q,\dot{q}) + g(q)\right)\right)$$
$$+ \left(I - \left(J(q)M^{-1}(q)\right)^{\#}_{W} J(q)M^{-1}(q)\right) u_0$$

where $(JM^{-1})^{\#}_{W} = W^{-1}M^{-1}J^{T}(JM^{-1}W^{-1}M^{-1}J^{T})^{-1}$

- three standard choices for $W > 0$

$$W = I \;\Longrightarrow\; (JM^{-1})^{\#} = M^{-1}J^{T}(JM^{-2}J^{T})^{-1}$$
$$W = M^{-1} \;\Longrightarrow\; (JM^{-1})^{\#}_{M^{-1}} = J^{T}(JM^{-1}J^{T})^{-1}$$
$$W = M^{-2} \;\Longrightarrow\; (JM^{-1})^{\#}_{M^{-2}} = M\,J^{T}(J\,J^{T})^{-1} = MJ^{\#}$$

each associated control torque optimizes a different criterion (see the slides on redundant robots)

- all give the same $\ddot{p} = a$, with u_0 available for null-space control

Remarks - 2

- the Cartesian pose/velocity can either be directly measured by external sensors (cameras) or computed through the direct and differential kinematics of the robot arm
- when applied to the case p_d = constant (regulation task), the control law becomes

$$u = M(q)J^{-1}(q)[K_P e_P - K_D J(q)\dot{q}] + c(q,\dot{q}) + g(q) - M(q)J^{-1}(q)\dot{J}(q)\dot{q}$$

which is computationally more expensive than a control law designed directly for regulation, such as the previous laws (*) or (**), but keeps the additional property of obtaining an exponentially stable transient error

Conclusions

- most of the control laws presented in the joint space (i.e., driven by a joint error) can be translated with relative ease to the Cartesian space, e.g.
  - regulation with constant gravity compensation
  - adaptive regulation
  - robust control for trajectory tracking
  - adaptive control for trajectory tracking
- the main issues are related to
  - kinematic singularities, both for the Jacobian transpose and the Jacobian inverse control laws: suitable modifications are needed to obtain singularity robustness
  - kinematic redundancy ($m < n$): use of a stabilizing null-space torque control is needed for the extra $n - m$ generalized coordinates (locally, $n - m$ joint variables)