Robotics 2

Adaptive Trajectory Control

Prof. Alessandro De Luca
Motivation and approach

- need of adaptation in robot motion control laws
  - large uncertainty on the robot dynamic parameters
  - poor knowledge of the inertial payload
- characteristics of direct adaptive control
  - the direct aim is zeroing asymptotically the trajectory error (position and velocity)
  - without necessarily identifying on line the true values of the dynamic coefficients of the robot (as opposed to indirect adaptive control)
- main tool and methodology
  - linear parameterization of robot dynamics
  - nonlinear control law of the dynamic type (the controller has its own ‘states’)

Robotics 2
Summary of robot parameters

- parameters assumed to be known
  - kinematic description based, e.g., on Denavit-Hartenberg parameters ($\{a_i, d_i, a_i, i = 1,...,N\}$ in the all revolute joints case), including link lengths (kinematic calibration)

- uncertain parameters that can be identified off line
  - masses $m_i$, positions $r_{ci}$ of CoMs, and inertia matrices $I_i$ of each link, appearing in combinations (dynamic coefficients) ⇒ $10 \times N$

- parameters that are (slowly) varying during operation
  - viscous $F_{vi}$, dry $F_{si}$, and stiction $F_{Di}$ friction at each joint ⇒ $1\div3 \times N$

- unknown and abruptly changing parameters
  - mass, CoM, inertia matrix of the payload w.r.t. the tool center point

when a payload is firmly attached to the robot E-E, only the 10 parameters of the last link are modified, influencing however most part of the robot dynamics
Goal of adaptive control

- given a twice-differentiable desired joint trajectory $q_d(t)$
  - possibly obtained by kinematic inversion + joint interpolation
  - with desired velocity $\dot{q}_d(t)$ and acceleration $\ddot{q}_d(t)$ also known
- execute this trajectory under large dynamic uncertainties
  - with a trajectory tracking error vanishing asymptotically
    \[ e = q_d - q \rightarrow 0 \quad \dot{e} = \dot{q}_d - \dot{q} \rightarrow 0 \]
  - guaranteeing global stability, no matter how far are the initial estimates of the unknown/uncertain parameters from their true values and how large is the initial trajectory error
- parameter identification is not of particular concern
- if the desired trajectory is persistently exciting, one obtains also parameter identification as a by-product
- indirect adaptive control schemes are more complex, but allow also systematic convergence of dynamic coefficients to their true values
Linear parameterization

\[ B(q)\ddot{q} + S(q, \dot{q})\dot{q} + g(q) + F_v \dot{q} + F_s \text{sgn}(q) = u \]

- there always exists a (p-dimensional) vector of dynamic coefficients, allowing to write the robot model in the linear form

\[ Y(q, \dot{q}, \ddot{q}) a = u \]

- vector contains only unknown or uncertain coefficients
- each component of \( a \) is in general a combination of the robot physical parameters (not necessarily all of them)
- the model regression matrix \( Y \) depends: linearly on \( \ddot{q} \), quadratically on \( \dot{q} \), nonlinearly (trigonometrically) on \( q \)
Trajectory controllers based on model estimates

- inverse dynamics feedforward + PD (linear) control
  \[ u = \hat{B}(q_d)\ddot{q}_d + \hat{S}(q_d, \dot{q}_d)\dot{q}_d + \hat{g}(q_d) + \hat{F}_v \dot{q}_d + K_P e + K_D \dot{e} \]

- (nonlinear) control based on feedback linearization
  \[ u = \hat{B}(q)\ddot{q} + K_P e + K_D \dot{e} + \hat{S}(q, \dot{q})\dot{q} + \hat{g}(q) + \hat{F}_v \dot{q} \]

- approximate estimates of dynamic coefficients may also lead to instability due to a partial and/or inappropriate cancelation of nonlinearities from the robot dynamics

- making these schemes adaptive is possible but not trivial
A control law easily made ‘adaptive’

- nonlinear trajectory tracking control (without cancelations) having global asymptotic stabilization properties

\[ u = \hat{B}(q)\ddot{q}_d + \hat{S}(q, \dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}_v\dot{q}_d + K_Pe + K_D\dot{e} \]

- a natural adaptive version would require ...

\[ \hat{a} = \text{designing a suitable update law} \]

  (in continuous time)

- it can be shown that velocities could be “clamped” in this way to the desired trajectory (eventually, with zero velocity error), but that a permanent residual position error may be left

- idea: on-line modification of a reference velocity

\[ \dot{q}_d \rightarrow \dot{q}_r = \dot{q}_d + \Lambda(q_d - q) \quad \Lambda > 0 \]

  typically \( \Lambda = K_D^{-1}K_P \) (all matrices will be chosen diagonal)
Intuitive interpretation of $\dot{q}_r$

- elementary case
  - a mass ‘lagging behind’ its mobile reference on a linear rail

  ![Diagram showing a mass controlled by $u$ and a mobile reference.
  $\dot{q}_r = \dot{q}_d + \Delta e$
  $s = \dot{q}_r - \dot{q} > \dot{q}_d - \dot{q} = \dot{e}$
  $u = K_D s = K_D (\dot{q}_r - \dot{q}) = K_D (\dot{q}_d + \Delta e - \dot{q}) = K_D \dot{e} + K_D \Delta e$

  - a mass ‘leading in front’ of its mobile reference
    - in a symmetric way, a ‘reduced’ velocity error will appear ($s < \dot{e}$)
Adaptive control law design

- substituting $\dot{q}_r = \dot{q}_d + \Delta e$, $\ddot{q}_r = \ddot{q}_d + \Delta \dot{e}$ in the previous nonlinear controller for trajectory tracking

$$u = \hat{B}(q)\dddot{q}_r + \hat{S}(q, \dot{q})\ddot{q}_r + \ddot{g}(q) + \hat{F}_v\dot{q}_r + K_P e + K_D \dot{e}$$

$$= Y(q, \dot{q}, \ddot{q}_r, \dot{\dddot{q}_r}) \hat{a} + K_P e + K_D \dot{e}$$

dynamic parameterization of the control law using current estimates
PD stabilization (diagonal matrices, $>0$)
(note here the 4 arguments in $Y$!)

- **update law** for the estimates of the dynamic coefficients
($\hat{a}$ becomes the $p$-dimensional state of the dynamic controller)

$$\dot{\hat{a}} = \Gamma Y^T(q, \dot{q}, \ddot{q}_r, \dot{\dddot{q}_r})(\ddot{q}_r - \dot{q})$$

$\Gamma > 0$ (diagonal)

estimation gains (rate of variations of estimates)
Asymptotic stability of trajectory error

Theorem

The introduced adaptive controller makes the tracking error for the desired trajectory globally asymptotically stable:

\[ e = q_d - q \rightarrow 0, \quad \dot{e} = \dot{q}_d - \dot{q} \rightarrow 0 \]

Proof

- A Lyapunov candidate for the closed-loop system (robot + dynamic controller) is given by

\[ V = \frac{1}{2} s^T B(q)s + \frac{1}{2} e^T R e + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} \geq 0 \]

\[ s = \dot{q}_r - \dot{q} \quad (= \dot{e} + \Lambda e) \quad R > 0 \quad \tilde{a} = a - \hat{a} \]

'Modified' velocity error \hspace{1cm} Constant matrix \hspace{1cm} Error in parametric estimation

\[ \Rightarrow \hat{a} = a \quad \& \quad q = q_d, \quad s = 0 \quad (\Rightarrow \dot{q} = \dot{q}_d) \]
Proof (cont)

- the time derivative of $V$ is

$$\dot{V} = \frac{1}{2} s^T \dot{B}(q) s + s^T \dot{B}(q) \dot{s} + e^T R \dot{e} - \dot{a}^T \Gamma^{-1} \dot{a}$$

since $\dot{a} = -\dot{a}$ ($\dot{a} = 0$)

- the closed-loop dynamics is given by

$$B(q) \ddot{q} + S(q, \dot{q}) \dot{q} + F_v \dot{q} + g(q) =$$

$$= \hat{B}(q) \ddot{q}_r + \hat{S}(q, \dot{q}) \dot{q}_r + \hat{F}_v \dot{q}_r + \hat{g}(q) + K_P e + K_D \dot{e}$$

subtracting the two sides from $B(q) \ddot{q}_r + S(q, \dot{q}) \dot{q}_r + F_v \dot{q}_r + g(q)$ leads to

$$B(q) \dot{\dot{s}} + (S(q, \dot{q}) + F_v) s$$

$$= \hat{B}(q) \ddot{q}_r + \hat{S}(q, \dot{q}) \dot{q}_r + \hat{F}_v \dot{q}_r + \hat{g}(q) - K_P e - K_D \dot{e}$$

with $\tilde{B} = B - \hat{B}$, $\tilde{S} = S - \hat{S}$, $\tilde{F}_v = F_v - \hat{F}_v$, $\tilde{g} = g - \hat{g}$
Proof (cont)

- from the property of linearity in the dynamic coefficients, it follows

\[ B(q) \dot{s} + (S(q, \dot{q}) + F_v) s = Y(q, \dot{q}, \ddot{q}, \dddot{q}) \dddot{a} - K_P e - K_D \dot{e} \]

- substituting in \( V \), together with \( \dot{\dddot{a}} = \Gamma Y^T s \), and using the skew-symmetry of matrix \( \dot{B} - 2S \) we obtain

\[
\dot{V} = \frac{1}{2} s^T [\dot{B}(q) - 2S(q, \dot{q})] s - s^T F_v s + s^T Y \dddot{a} \\
- s^T (K_P e + K_D \dot{e}) + e^T R \dot{e} - \dddot{a}^T Y^T s \\
= -s^T F_v s - s^T (K_P e + K_D \dot{e}) + e^T R \dot{e}
\]

- replacing \( s = \dot{e} + \Lambda e \) and being \( F_v = F_v^T \geq 0 \) diagonal

\[
\dot{V} = -e^T (\Lambda^T F_v \Lambda + \Lambda^T K_P) e \\
- e^T (2\Lambda^T F_v + \Lambda^T K_D + K_P - R) \dot{e} \\
- \dot{e}^T (F_v + K_D) \dot{e}
\]
Proof (end)

- defining now (all matrices are **diagonal**)!

\[
\Lambda = K_D^{-1} K_p \quad R = 2K_p(I + K_D^{-1}F_v)
\]

leads to

\[
\dot{V} = -e^T K_P K_D^{-1}(F_v + K_D)K_D^{-1}K_P e
- \dot{e}^T(F_v + K_D)\dot{e} \leq 0
\]

and thus

\[
\dot{V} = 0 \quad \Leftrightarrow \quad e = \dot{e} = 0
\]

... the thesis follows from Barbalat lemma + LaSalle theorem

the set of states of convergence has **zero trajectory error** and a **constant value** for \(\hat{\alpha}\), not necessarily the true one (\(\tilde{\alpha} \neq 0\))
Remarks

- if the desired trajectory $q_d(t)$ is **persistently exciting**, then also the estimates converge to their true values

- **condition** of persistent excitation
  - for **linear** systems: # of frequency components in the desired trajectory should be at least twice as large as # of unknown coefficients
  - for **nonlinear** systems: the condition can be checked only a posteriori (a certain motion integral should be permanently lower bounded)

- in case of known absence of (viscous) friction ($F_v \equiv 0$), the same proof applies (a bit easier in the final part)

- the adaptive controller **does not require** neither the inverse of the inertia matrix (true or estimated) nor the actual robot acceleration (only the desired acceleration)

- the **non-adaptive version** (using accurate estimates) is a static controller based on the property of **passivity** of robots
Case study: Single-link under gravity

model \[ I \ddot{\theta} + mgd \sin \theta + f_v \dot{\theta} = u \] (with friction)

linear parameterization
\[ Y(\theta, \dot{\theta}, \ddot{\theta}) a = \begin{bmatrix} \ddot{\theta} \\ \sin \theta \\ \dot{\theta} \end{bmatrix} \begin{bmatrix} I \\ m gd \\ f_v \end{bmatrix} = u \]

adaptive controller

\[ u = \hat{I} \ddot{\theta} + mgd \sin \theta + \hat{f}_v \dot{\theta}_r + k_P e + k_D \dot{e} \]
\[ \ddot{e} = \dot{\theta}_d - \dot{\theta} \]
\[ \dot{\theta}_r = \dot{\theta}_d + \frac{k_P}{k_D} e \]
\[ \gamma_i > 0 \]
Simulation data

- **real** dynamic coefficients
  \[ I = 7.5, \quad mgd = 6, \quad f_v = 1 \]

- **initial** estimates
  \[ \hat{I} = 5, \quad \hat{mgd} = 5, \quad \hat{f}_v = 2 \]

- **control parameters**
  \[ k_P = 25, \quad k_D = 10, \quad \gamma_i = 5, \quad i = 1, \ldots, 3 \]

- **test trajectories** (starting with \( \theta(0) = 0, \dot{\theta}(0) = 0 \))
  - **first**
    \[ \theta_d(t) = -\sin t \]
  - **second**
    \[ \ddot{\theta}_d(t) = \text{(periodic) bang-bang acceleration profile with} \]
    \[ A = 1 \text{ rad/s}^2, \omega = 1 \text{ rad/s} \]

same test trajectories used also for robust control
Results
first trajectory

note the nonlinear system dynamics
(no sinusoidal regime at steady state!)

position and velocity errors

control torque

\[ \theta_d(t) = - \sin t \]
Results
second trajectory

note the torque discontinuities (due to those of the desired acceleration)

position and velocity errors

\[ \ddot{\theta}_d(t) = \text{(periodic) bang-bang acceleration profile} \]
Estimates of dynamic coefficients

only the estimate of the viscous friction coefficient converges to the true value

all three estimates of dynamic coefficients converge to their true values
Homework: Comau Smart 6.10R robot

- 6R robot with spherical wrist and symmetric structure (no offsets)
- freeze the last three joints in the shown configuration (3R robot)
- assume that each link center of mass is on the kinematic axis of the link
- assume a diagonal inertia matrix for each link
- derive the dynamic model
- determine a linear parameterization (viz., using the least # of coefficients)
- write explicitly the expression of an adaptive trajectory tracking law
- simulate with initially wrong estimates (what happens if they are instead correct?)

\[ \begin{align*}
\theta_1 &= \text{arbitrary} \\
\theta_2 &= 90^\circ \\
\theta_3 &= -90^\circ 
\end{align*} \]
A special case: Adaptive regulation

- adaptation in case $q_d$ is constant
- no special simplifications for the presented adaptive control law (designed for the general tracking case...)

$$u = \hat{B}(q)\ddot{q}_r + \hat{S}(q, \dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_v\dot{q}_r + K_P e + K_D \dot{e}$$

$$\dot{\hat{a}} = \Gamma Y^T(q, \dot{q}, \ddot{q}_r, \dddot{q}_r) (\dddot{q}_r - \dddot{q})$$

since $\ddot{q}_r = \Lambda (q_d - q)$ and $\dddot{q}_r = -\Lambda \dot{q}$ do not vanish!

- a different case would be the availability of an adaptive version of the trajectory tracking controller

$$u = \hat{B}(q)\ddot{q}_d + \hat{S}(q, \dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}_v\dot{q}_d + K_P e + K_D \dot{e}$$

since, when $q_d$ collapses to a constant, only the adaptation of the gravity term would be left over (which is what one would naturally expect...)
An efficient adaptive regulator

- use a linear parameterization of the gravity term only
  \[ g(q) = G(q)a_g \]
  with a \( p_g \)-dimensional vector \( a_g \)

- an adaptive regulator yielding global asymptotic stability of the equilibrium state \((q_d,0)\) is provided by
  \[
  u = G(q)\hat{a}_g + K_P(q_d - q) - K_D \dot{q} \\
  \hat{a}_g = \gamma G^T(q) \left( \frac{2e}{1 + 2\|e\|^2} - \beta \dot{q} \right) \quad \gamma > 0 
  \]
  where \( e = q_d - q, \ K_P > 0, \ K_D > 0 \) (symmetric), and \( \beta > 0 \) is chosen sufficiently large

(see paper by P. Tomei, IEEE TRA, 1991; available as extra material on the course web)
An adaptive regulator
Sketch of asymptotic stability analysis

- use the function
  \[ V = \frac{\beta}{2} (q^T B(q) \dot{q} + e^T K_p e) - \frac{2q^T B(q) e}{1 + 2\|e\|^2} + \frac{1}{2} (\dot{a}_g - a_g)^T (\dot{a}_g - a_g) \geq 0 \]

- a sufficient condition for \( V \) to be a Lyapunov candidate is that
  \[ \beta > \frac{2B_M}{\sqrt{B_m K_{P,m}}} \]

- a sufficient condition that guarantees also
  \[ \dot{V} = \ldots \leq -a\|e\|^2 - b\|\dot{q}\|^2 \leq 0 \quad a > 0, b > 0 \]
  is that
  \[ \beta > \max\left( \frac{2B_M}{\sqrt{B_m K_{P,m}}}, \frac{1}{K_{D,m}} \left( \frac{K_{D,m}^2}{2K_{P,m}} + 4B_M + \frac{\alpha_S}{\sqrt{2}} \right) \right) \]

Note: for any symmetric, positive definite matrix \( A \)

\[ A_M = \lambda_{\max}(A) = \sqrt{\lambda_{\max}(A^T A)} = \|A\| \quad \text{and thus, e.g.} \quad \frac{1}{2} q^T B(q) \dot{q} \geq \frac{1}{2} B_m \|\dot{q}\|^2 \]

\[ A_m = \lambda_{\min}(A) \]