Robotics 2

Trajectory Tracking Control

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Inverse dynamics control

given the robot dynamic model with N joints

$$M(q)\ddot{q} + \underbrace{n(q,\dot{q})}_{c(q,\dot{q})\,+\,g(q)\,+\,\text{friction model}} = u$$

and a twice-differentiable desired trajectory for $t \in [0, T]$

$$q_d(t) \;\; \rightarrow \;\; \dot{q}_d(t), \ddot{q}_d(t)$$

applying the feedforward torque in nominal conditions

$$\boxed{u_d = M(q_d)\ddot{q}_d + n(q_d, \dot{q}_d)}$$

yields exact reproduction of the desired motion, provided that $q(0) = q_d(0), \dot{q}(0) = \dot{q}_d(0)$ (initial matched state)

In practice …

a number of differences from the nominal condition

- initial state is "not matched" to the desired trajectory $q_d(t)$
- disturbances on the actuators, from unexpected collisions, truncation errors on data, …
- inaccurate knowledge of robot dynamic parameters $\pi \longrightarrow \hat{\pi}$ (link masses, inertias, center of mass positions)
- unknown value of the carried payload
- presence of unmodeled dynamics (complex friction phenomena, transmission elasticity, …)

⇓

require the use of feedback information

Introducing feedback

$$\hat{u}_d = \hat{M}(q_d)\ddot{q}_d + \hat{n}(q_d, \dot{q}_d)$$

with $\hat{M}$, $\hat{n}$ estimates of terms (or coefficients) in the dynamic model

note: $\hat{u}_d$ can be computed off line [e.g., by $\widehat{NE}_\alpha(q_d, \dot{q}_d, \ddot{q}_d)$]

feedback is introduced to make the control scheme more robust

different possible implementations depending on amount of computational load share

- OFF LINE (⇔ open loop)
- ON LINE (⇔ closed loop)

two-step control design:

1. compensation (feedforward) or cancellation (feedback) of nonlinearities
2. synthesis of a linear control law stabilizing the trajectory error to zero

A series of trajectory controllers

(assuming the nominal case: $\widehat{M} = M, \hat{n} = n$)

1. inverse dynamics compensation (FFW) + PD

$$\boxed{u = \hat{u}_d + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})}$$

2. inverse dynamics compensation (FFW) + variable PD

$$\boxed{u = \hat{u}_d + \widehat{M}(q_d)[K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})]}$$

local stabilization of trajectory error $e(t) = q_d(t) - q(t)$

global if additional conditions on K_P and K_D

3. feedback linearization (FBL) + [PD+FFW] = "COMPUTED TORQUE"

$$\boxed{u = \widehat{M}(q)[\ddot{q}_d + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})] + \hat{n}(q, \dot{q})}$$

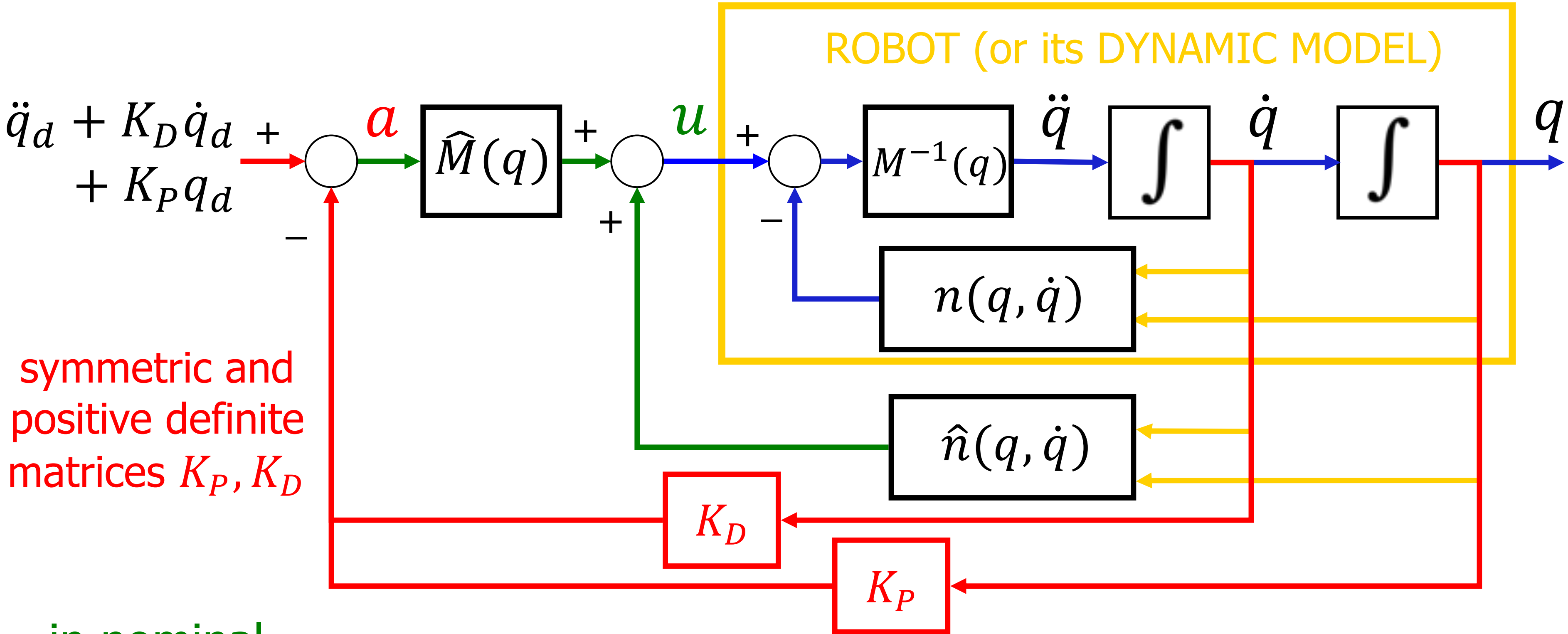
4. feedback linearization (FBL) + [PID+FFW]

$$\boxed{u = \widehat{M}(q)\left[\ddot{q}_d + K_P(q_d - q) + K_D(\dot{q}_d - \dot{q}) + K_I \int (q_d - q)dt\right] + \hat{n}(q, \dot{q})}$$

global stabilization for any $K_P > 0, K_D > 0$ (and not too large $K_I > 0$)

more robust to small uncertainties/disturbances, even if more complex to implement in real time

Feedback linearization control



symmetric and positive definite matrices K_P, K_D

in nominal conditions ($\hat{M} = M, \hat{n} = n$)

$$\boxed{\underbrace{M(q)\ddot{q} + n(q,\dot{q})}_{\text{nonlinear robot dynamics}} = u = \underbrace{M(q)a + n(q,\dot{q})}_{\text{nonlinear control law}}} \Rightarrow \boxed{\ddot{q} = a}$$

linear and decoupled system

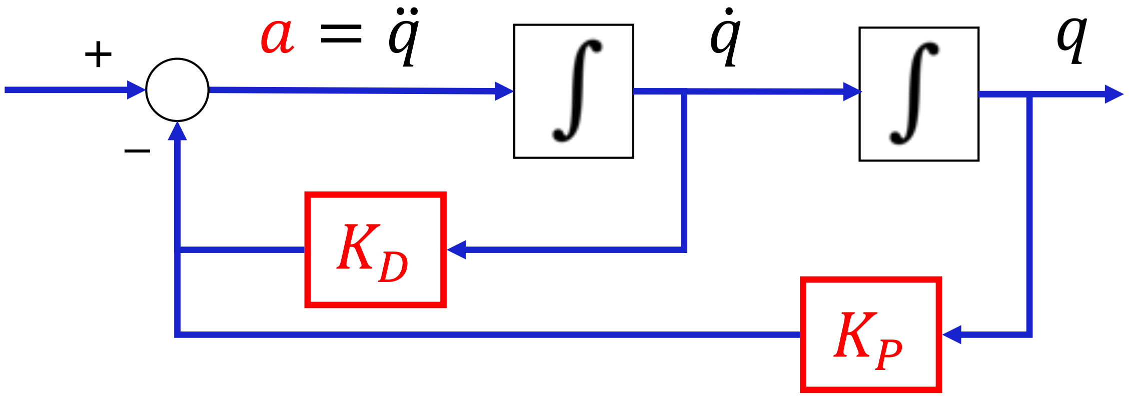
global asymptotic stabilization of tracking error

$$\boxed{a = \ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q)}$$

Interpretation in the linear domain

$\ddot{q}_d + K_D \dot{q}_d + K_P q_d$

> 0, diagonal



under feedback linearization control, the robot has a dynamic behavior that is invariant, linear and decoupled in its whole state space ($\forall (q, \dot{q})$)

a unitary mass ($m = 1$) in the joint space !!

linearity

error transients $e_i = q_{di} - q_i \rightarrow 0$ exponentially, prescribed by K_{Pi}, K_{Di} choice

decoupling

each joint coordinate q_i evolves independently from the others, forced by a_i

$$\ddot{e} + K_D \dot{e} + K_P e = 0 \quad \Leftrightarrow \quad \ddot{e}_i + K_{Di} \dot{e}_i + K_{Pi} e_i = 0$$

Addition of an integral term: PID

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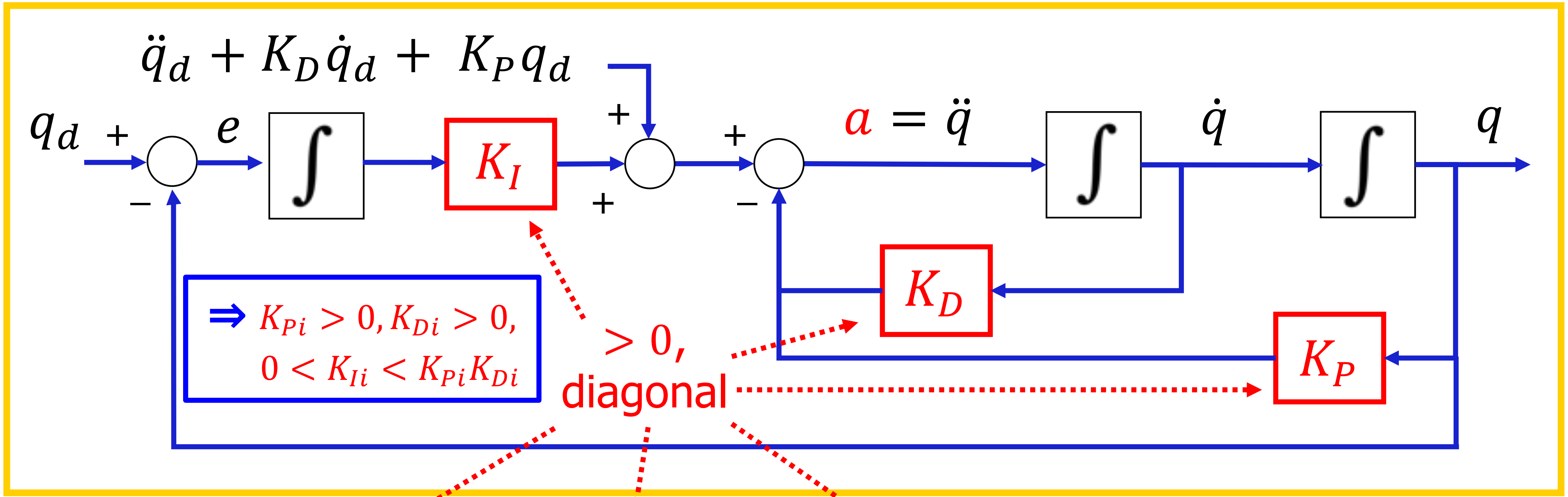


$$\ddot{q} = a = \ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P(q_d - q) + K_I \int (q_d - q)\, d\tau \qquad e = q_d - q$$

$\Rightarrow$ **(1)** $e_i = q_{di} - q_i \ (i = 1, \ldots, N)$ $\quad\Rightarrow$ **(2)** $\ddot{e}_i + K_{Di}\dot{e}_i + K_{Pi}e_i + K_{Ii}\int e_i d\tau = 0$

$\mathcal{L}[e_i(t)] \Rightarrow$ **(3)** $\left(s^2 + K_{Di}s + K_{Pi} + K_{Ii}\frac{1}{s}\right) e_i(s) = 0$

$s \times \Rightarrow$ **(4)** $(s^3 + K_{Di}s^2 + K_{Pi}s + K_{Ii})e_i(s) = 0 \ \Rightarrow$ **(5)**

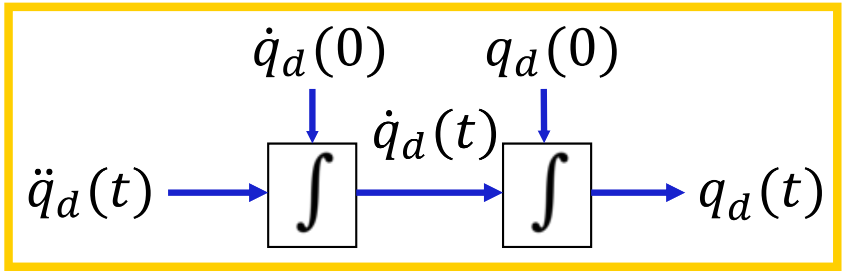
3	1	K_{Pi}
2	K_{Di}	K_{Ii}
1	$(K_{Di}K_{Pi} - K_{Ii})/K_{Di}$	
0	K_{Ii}	

$\Rightarrow$ **(6)** exponential stability conditions by Routh criterion

Remarks

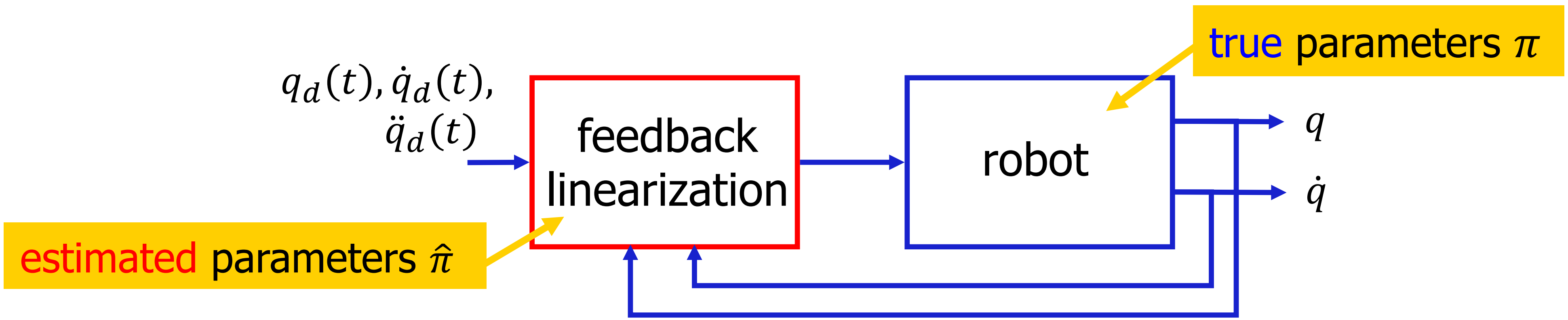
- desired joint trajectory can be generated from Cartesian data

$$\ddot{p}_d(t), \dot{p}_d(0), p_d(0)$$



$$q_d(0) = f^{-1}\big(p_d(0)\big)$$
$$\dot{q}_d(0) = J^{-1}\big(q_d(0)\big)\dot{p}_d(0)$$
$$\ddot{q}_d(t) = J^{-1}(q_d)\big[\ddot{p}_d(t) - \dot{J}(q_d)\dot{q}_d\big]$$

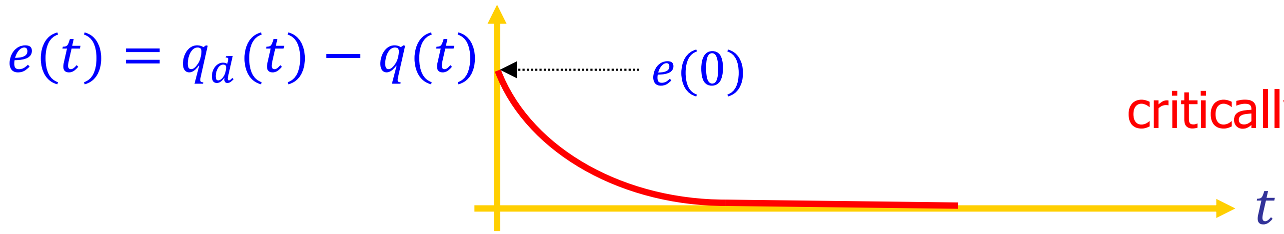
- real-time computation by Newton-Euler algo: $u_{FBL} = \widehat{NE}_{\alpha}(q, \dot{q}, a)$
- simulation of feedback linearization control



Hint: there is no use in simulating this control law in the ideal case ($\hat{\pi} = \pi$); robot behavior will be identical to the linear and decoupled case of stabilized double integrators!!

Further comments

- choice of the diagonal elements of K_P, K_D (and K_I)
  - shaping the error transients, with an eye also to motor saturations...



- parametric identification
  - to be done in advance, using the property of linearity in the dynamic coefficients of the robot dynamic model
- choice of the sampling time of a digital implementation
  - compromise between computational time and tracking accuracy, typically $T_c = 0.5 \div 10$ ms
- exact linearization by (state) feedback is a general technique of nonlinear control theory
  - can be used for robots with elastic joints, wheeled mobile robots, ...
  - non-robotics applications: satellites, induction motors, helicopters, ...

Another example of feedback linearization design

- dynamic model of robots with elastic joints
  - q = link position
  - θ = motor position (after reduction gears)

    } $2N$ generalized coordinates (q, θ)
  - B_m = diagonal matrix (> 0) of inertia of the (balanced) motors
  - K = diagonal matrix (> 0) of (finite) stiffness of the joints

$4N$ state variables $x = (q, \theta, \dot{q}, \dot{\theta})$

$$\begin{cases} M(q)\ddot{q} + c(q, \dot{q}) + g(q) + K(q - \theta) = 0 & (1) \\ B_m\ddot{\theta} + K(\theta - q) = u & (2) \end{cases}$$

- is there a control law that achieves exact linearization via feedback?

$$u = \alpha(q, \theta, \dot{q}, \dot{\theta}) + \beta(q, \theta, \dot{q}, \dot{\theta})\, a$$

YES and it yields $\dfrac{d^4 q_i}{dt^4} = a_i, \quad i = 1, \dots, N$

linear and decoupled system: N chains of 4 integrators (to be stabilized by linear control design)

Hint: differentiate (1) *w.r.t. time until motor acceleration* $\ddot{\theta}$ *appears; substitute this from* (2)*; choose* u *so as to cancel all nonlinearities …*

Alternative global trajectory controller

$$u = M(q)\ddot{q}_d + S(q,\dot{q})\dot{q}_d + g(q) + F_V\dot{q}_d + K_P e + K_D \dot{e}$$

SPECIAL factorization such that $\dot{M} - 2S$ is skew-symmetric

symmetric and positive definite matrices

- global asymptotic stability of $(e,\dot{e}) = (0,0)$ (trajectory tracking)
- proven by Lyapunov +Barbalat (time-varying system) +LaSalle
- does not produce a complete cancellation of nonlinearities
  - the variables $\dot{q}$ and $\ddot{q}$ that appear linearly in the model are evaluated on the desired trajectory
- does not induce a linear and decoupled behavior of the trajectory error $e(t) = q_d(t) - q(t)$ in the closed-loop system
- however, it lends itself more easily to an adaptive version
- computation: by $4\times$ standard or $1\times$ modified NE algorithm

Analysis of asymptotic stability
of the trajectory error - 1

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) + F_V\dot{q} = u$$ robot dynamics (including friction)

control law $u = M(q)\ddot{q}_d + S(q,\dot{q})\dot{q}_d + g(q) + F_V\dot{q}_d + K_P e + K_D\dot{e}$

- Lyapunov candidate and its time derivative (with $e = q_d - q$)

$$V = \frac{1}{2}\dot{e}^T M(q)\dot{e} + \frac{1}{2}e^T K_P e \geq 0 \;\Rightarrow\; \dot{V} = \frac{1}{2}\dot{e}^T \dot{M}(q)\dot{e} + \dot{e}^T \underbrace{M(q)\ddot{e}} + e^T K_P \dot{e}$$

- the closed-loop system equations yield

$$M(q)\ddot{e} = -S(q,\dot{q})\dot{e} - (K_D + F_V)\dot{e} - K_P e$$

- substituting and using the skew-symmetric property of $\dot{M} - 2S$

$$\dot{V} = -\dot{e}^T(K_D + F_V)\dot{e} \leq 0 \qquad\qquad \dot{V} = 0 \;\Leftrightarrow\; \dot{e} = 0$$

- since the system is time-varying (due to $q_d(t)$), direct application of LaSalle theorem is NOT allowed ⇒ use Barbalat lemma…

$$q = q_d(t) - e,\; \dot{q} = \dot{q}_d(t) - \dot{e} \;\Rightarrow\; V = V(\underbrace{e,\dot{e}}_{\text{error state } x},t) = V(x,t)$$

⇒ go to slide 10 in block 8

Analysis of asymptotic stability
of the trajectory error - 2

- since i) V is lower bounded and ii) $\dot{V} \le 0$, we have to check only condition iii) in order to apply Barbalat lemma

$$\ddot{V} = -2\dot{e}^T (K_D + F_V)\ddot{e}$$

... is this bounded?

- from i) + ii), V is bounded $\Rightarrow$ e and $\dot{e}$ are bounded
- assume that the desired trajectory has bounded velocity $\dot{q}_d$

$\Rightarrow$ $\dot{q}$ is bounded

- using the following two properties of dynamic model terms

$$0 < \alpha_m \le \|M^{-1}(q)\| \le \alpha_M < \infty \qquad \|S(q,\dot{q})\| \le \alpha_S \|\dot{q}\|$$

then also $\ddot{e}$ will be bounded (in norm) since

$$\ddot{e} = -\underbrace{M^{-1}(q)}_{\text{bounded in norm by } \alpha_M}[\underbrace{S(q,\dot{q})}_{\text{bounded in norm by } \alpha_S\|\dot{q}\|}\underbrace{\dot{e}}_{\text{bounded}} + K_P \underbrace{e}_{\text{bounded}} + (K_D + F_V)\underbrace{\dot{e}}_{\text{bounded}}]$$

in norm by $\alpha_S\|\dot{q}\|$ $\longleftarrow$ bounded

$$\Rightarrow \lim_{t\to\infty} \dot{V}(t) = 0$$

Analysis of asymptotic stability
of the trajectory error – end of proof

- we can conclude by proceeding as in LaSalle theorem

$$\dot{V} = 0 \quad \Leftrightarrow \quad \dot{e} = 0$$

- the closed-loop dynamics in this situation is

$$M(q)\ddot{e} = -K_P e$$

$$\Longrightarrow \quad \ddot{e} = 0 \quad \Leftrightarrow \quad e = 0$$

➡ $(e, \dot{e}) = (0, 0)$ is the largest invariant set in $\dot{V} = 0$

➡ (global) asymptotic tracking will be achieved

Regulation as a special case

- what happens to the control laws designed for trajectory tracking when q_d is constant? are there simplifications?
- feedback linearization

$$u = M(q)[K_P(q_d - q) - K_D\dot{q}] + c(q,\dot{q}) + g(q)$$

  - no special simplifications
  - however, this is a solution to the regulation problem with exponential stability (and decoupled transients at each joint!)
- alternative global controller

$$u = K_P(q_d - q) - K_D\dot{q} + g(q)$$

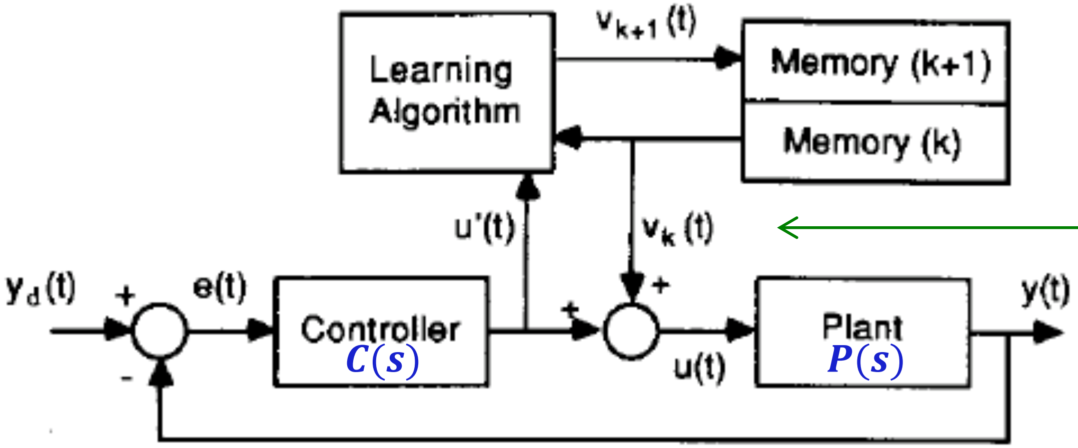
  - we recover the simpler PD + gravity cancellation control law!!

Trajectory execution without a model

- is it possible to accurately reproduce a desired smooth joint-space reference trajectory with reduced or no information on the robot dynamic model?
- this is feasible (and possibly simple) in case of repetitive motion tasks over a finite interval of time
  - trials are performed iteratively, storing the trajectory error information of the current execution [k-th iteration] and processing it off line before the next trial [$(k+1)$-iteration] starts
  - the robot should be reinitialized in the same initial state at the beginning of each trial (typically, with $\dot{q} = 0$)
  - the control law is made of a non-model based part (often, a decentralized PD law) + a time-varying feedforward which is updated before every trial
- this scheme is called iterative trajectory learning

Scheme of iterative trajectory learning

- control design can be illustrated on a SISO linear system in the Laplace domain



"plug-in" signal: $v_1(t) \equiv 0$ at first $(k = 1)$ iteration

$$W(s) = \frac{y(s)}{y_d(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

closed-loop system without learning ($C(s)$ is, e.g., a PD control law)

$$u_k(s) = u'_k(s) + v_k(s) = C(s)e_k(s) + v_k(s)$$

control law at iteration k

$$y_k(s) = W(s)y_d(s) + \frac{P(s)}{1 + P(s)C(s)} v_k(s)$$

system output at iteration k

Background math on feedback loops

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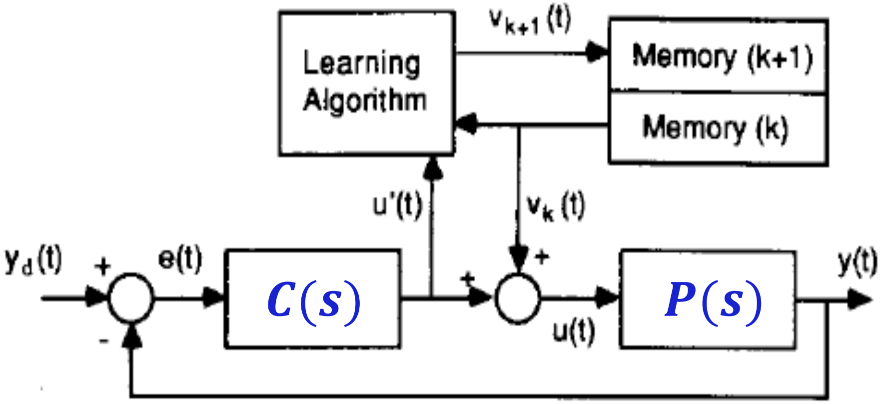
- algebraic manipulations on block diagram signals in the Laplace domain: $x(s) = \mathcal{L}[x(t)]$, $x = \{y_d, y, u', v, e\} \Rightarrow \{y_d, y_k, u_k', v_k, e_k\}$, with transfer functions



$$\begin{aligned} y(s) &= P(s)u(s) = P(s)\big(v(s) + u'(s)\big) \\ &= P(s)v(s) + P(s)C(s)e(s) \\ &= P(s)v(s) + P(s)C(s)(y_d(s) - y(s)) \end{aligned}$$

$$\Rightarrow \; (1 + P(s)C(s))\, y(s) = \\ = P(s)v(s) + P(s)C(s)y_d(s)$$

$$\Rightarrow \; y(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}\, y_d(s) + \frac{P(s)}{1 + P(s)C(s)}\, v(s) = W(s)y_d(s) + W_v(s)v(s)$$

- feedback control law at iteration k

$$u_k'(s) = C(s)\big(y_d(s) - y_k(s)\big) = C(s)y_d(s) - P(s)C(s)\big(v_k(s) + u_k'(s)\big)$$

$$\Rightarrow \; u_k'(s) = \frac{C(s)}{1 + P(s)C(s)}\, y_d(s) - \frac{P(s)C(s)}{1 + P(s)C(s)}\, v_k(s) = W_c(s)y_d(s) - W(s)v_k(s)$$

- error at iteration k

$$e_k(s) = y_d(s) - y_k(s) = y_d(s) - \big(W(s)y_d(s) + W_v(s)v_k(s)\big) = \big(1 - W(s)\big)y_d(s) - W_v(s)v_k(s)$$

$$W_e(s) = 1/(1 + P(s)C(s))$$

Learning update law

- the update of the feedforward term is designed as

$$\boxed{v_{k+1}(s) = \alpha(s)u'_k(s) + \beta(s)v_k(s)}$$

with α and β suitable filters (also non-causal, of the FIR type)

recursive expression of feedforward term

$$v_{k+1}(s) = \frac{\alpha(s)C(s)}{1+P(s)C(s)}y_d(s) + (\beta(s) - \alpha(s)W(s))v_k(s)$$

recursive expression of error $e = y_d - y$

$$e_{k+1}(s) = \frac{1-\beta(s)}{1+P(s)C(s)}y_d(s) + \big(\beta(s) - \alpha(s)W(s)\big)e_k(s)$$

- if a contraction condition can be enforced

$$\boxed{|\beta(s) - \alpha(s)W(s)| < 1}$$

(for all $s = j\omega$ frequencies such that ...)

then convergence is obtained for $k \to \infty$

$$v_\infty(s) = \frac{y_d(s)}{P(s)} \frac{\alpha(s)W(s)}{1-\beta(s)+\alpha(s)W(s)} \qquad e_\infty(s) = \frac{y_d(s)}{1+P(s)C(s)} \frac{1-\beta(s)}{1-\beta(s)+\alpha(s)W(s)}$$

Proof of recursive updates

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- recursive expression for the feedworward v_k

$$v_{k+1}(s) = \alpha(s)u'_k(s) + \beta(s)v_k(s) = \alpha(s)C(s)e_k(s) + \beta(s)v_k(s)$$
$$= \alpha(s)C(s)[W_e(s)y_d(s) - W_v(s)v_k(s)] + \beta(s)v_k(s)$$
$$= \frac{\alpha(s)C(s)}{1+P(s)C(s)}y_d(s) + (\beta(s) - \alpha(s)W(s))\, v_k(s)$$

- recursive expression for the error e_k

$$e_k(s) = y_d(s) - y_k(s) = y_d(s) - P(s)(v_k(s) + u'_k(s))$$
$$\Rightarrow \quad v_k(s) = \frac{1}{P(s)}\big(y_d(s) - e_k(s)\big) - u'_k(s)$$

$$y_{k+1}(s) = P(s)\big(v_{k+1}(s) + u'_{k+1}(s)\big) = P(s)\big(\alpha(s)u'_k(s) + \beta(s)v_k(s) + u'_{k+1}(s)\big)$$
$$= P(s)\left(\alpha(s)C(s)e_k(s) + \beta(s)\frac{1}{P(s)}\big(y_d(s) - e_k(s)\big) - \beta(s)C(s)e_k(s) + C(s)e_{k+1}(s)\right)$$

$$e_{k+1}(s) = y_d(s) - y_{k+1}(s)$$
$$= (1-\beta(s))\, y_d(s) - \big[\big(\alpha(s) - \beta(s)\big)P(s)C(s) - \beta(s)\big]e_k(s) - P(s)C(s)e_{k+1}(s)$$

$$\Rightarrow \quad e_{k+1}(s) = \frac{1-\beta(s)}{1+P(s)C(s)}y_d(s) + \big(\beta(s) - \alpha(s)W(s)\big)\, e_k(s)$$

Proof of convergence

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from recursive expressions

$$v_{k+1}(s) = \frac{\alpha(s)C(s)}{1+P(s)C(s)} y_d(s) + (\beta(s) - \alpha(s)W(s))\, v_k(s)$$

$$e_{k+1}(s) = \frac{1-\beta(s)}{1+P(s)C(s)} y_d(s) + \big(\beta(s) - \alpha(s)W(s)\big)\, e_k(s)$$

compute variations from k to $k+1$ (repetitive term in trajectory y_d vanishes!)

$$\Delta v_{k+1}(s) = v_{k+1}(s) - v_k(s) = (\beta(s) - \alpha(s)W(s))\, \Delta v_k(s)$$

$$\Delta e_{k+1}(s) = e_{k+1}(s) - e_k(s) = \big(\beta(s) - \alpha(s)W(s)\big)\, \Delta e_k(s)$$

by contraction mapping condition $|\beta(s) - \alpha(s)W(s)| < 1 \Rightarrow \{v_k\} \to v_\infty,\ \{e_k\} \to e_\infty$

$$v_\infty(s) = \frac{\alpha(s)C(s)}{1+P(s)C(s)} y_d(s) + (\beta(s) - \alpha(s)W(s))\, v_\infty(s)$$

$$e_\infty(s) = \frac{1-\beta(s)}{1+P(s)C(s)} y_d(s) + \big(\beta(s) - \alpha(s)W(s)\big)\, e_\infty(s)$$

$$\Rightarrow \quad v_\infty(s) = \frac{y_d(s)}{P(s)} \frac{\alpha(s)W(s)}{1-\beta(s)+\alpha(s)W(s)} \qquad e_\infty(s) = \frac{y_d(s)}{1+P(s)C(s)} \frac{1-\beta(s)}{1-\beta(s)+\alpha(s)W(s)}$$

Comments on convergence

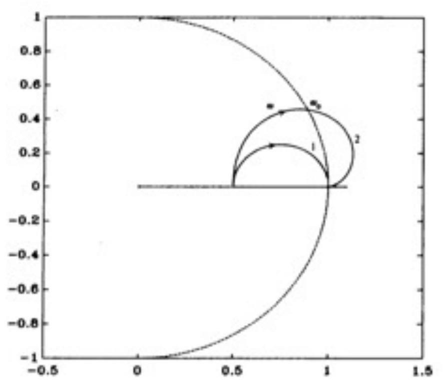
- if the choice $\beta = 1$ allows to satisfy the contraction condition, then convergence to zero tracking error is obtained

$$e_\infty(s) = 0$$

  and the inverse dynamics command has been learned

$$v_\infty(s) = \frac{y_d(s)}{P(s)}$$

- in particular, for $\alpha(s) = 1/W(s)$ convergence would be in 1 iteration only!
- the use of filter $\beta(s) \neq 1$ allows to obtain convergence (but with residual tracking error) even in presence of unmodeled high-frequency dynamics
- the two filters can be designed from very poor information on system dynamics, using classic tools (e.g., Nyquist plots)



Application to robots

- for N-dof robots modeled as

$$[B_m + M(q)]\ddot{q} + [F_V + S(q,\dot{q})]\dot{q} + g(q) = u$$

we choose as (initial = pre-learning) control law

$$u = u' = K_P(q_d - q) + K_D(\dot{q}_d - \dot{q}) + \hat{g}(q)$$

and design the learning filters (at each joint) using the linear approximation

$$W_i(s) = \frac{q_i(s)}{q_{di}(s)} = \frac{K_{Di}s + K_{Pi}}{\hat{B}_{mi}s^2 + \left(\hat{F}_{Vi} + K_{Di}\right)s + K_{Pi}} \qquad i = 1, \cdots, N$$

- initialization of feedforward uses the best estimates

$$v_1 = \left[\hat{B}_m + \widehat{M}(q_d)\right]\ddot{q}_d + \left[\hat{F}_V + \hat{S}(q_d, \dot{q}_d)\right]\dot{q}_d + \hat{g}(q_d)$$

or simply $v_1 = 0$ (in the worst case) at first trial $k = 1$

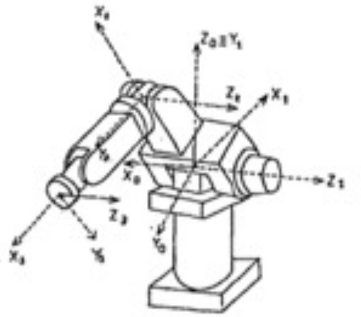
Experimental set-up

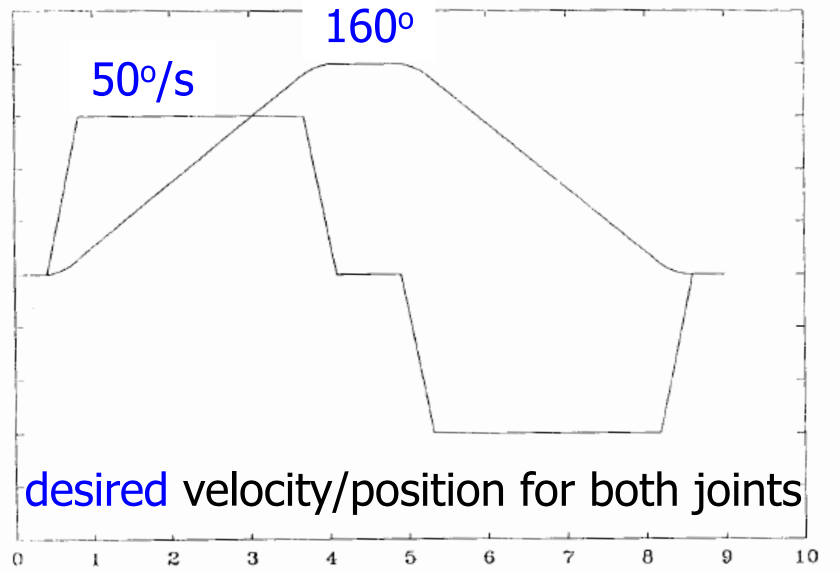
- joints 2 and 3 of 6R MIMO CRF robot prototype @DIS

≈ 90% gravity balanced through springs

high level of dry friction

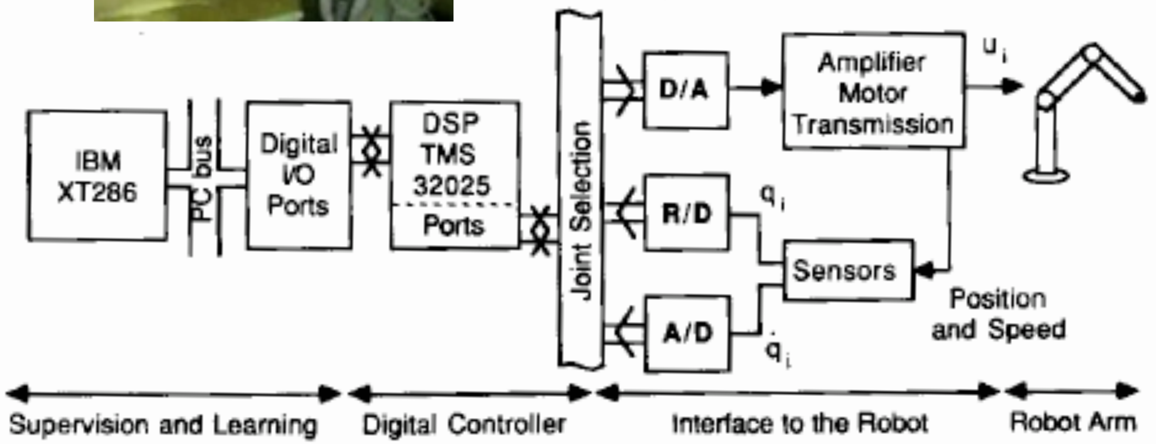
Harmonic Drives transmissions with ratio 160:1





desired velocity/position for both joints

DSP T_c = 400μs
D/A = 12 bit
R/D = 16 bit/2π
A/D = 11 bit/(rad/s)



DC motors with current amplifiers

resolvers and tachometers

De Luca, Paesano, Ulivi: IEEE Trans Ind Elect, 1992

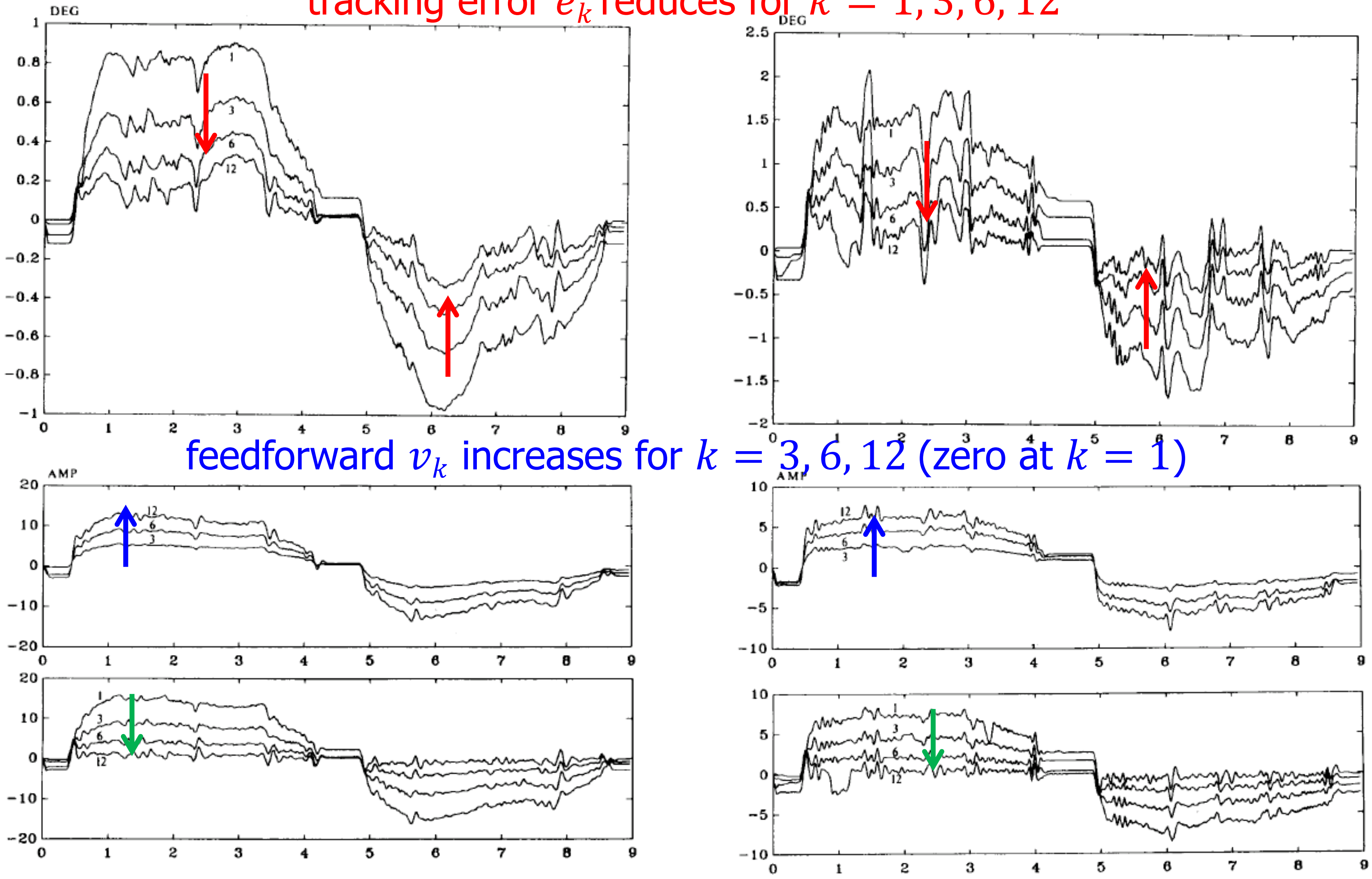
Experimental results

tracking error e_k reduces for $k = 1, 3, 6, 12$

feedforward v_k increases for $k = 3, 6, 12$ (zero at $k = 1$)

joint 2

joint 3



feedback u'_k decreases for $k = 1, 3, 6, 12$

On-line learning control

- re-visitation of the learning idea so as to acquire the missing dynamic information in model-based trajectory control
- on-line learning approach
  - the robot improves tracking performance already while executing the task in feedback mode
- uses only position measurements from encoders
  - no need of joint torque sensors
- machine learning techniques used for
  - data collection and organization
  - regressor construction for estimating model perturbations
- fast convergence
  - starting with a reasonably good robot model
- extensions to underactuated robots or with flexible components

Control with approximate FBL

- dynamic model, its nominal part and (unstructured) uncertainty

$$M(q)\ddot{q} + n(q,\dot{q}) = \tau \qquad M = \widehat{M} + \Delta M \qquad n = \hat{n} + \Delta n$$

- model-based (approximate) feedback linearization

$$\tau_{FBL} = \widehat{M}(q)a + \hat{n}(q,\dot{q})$$

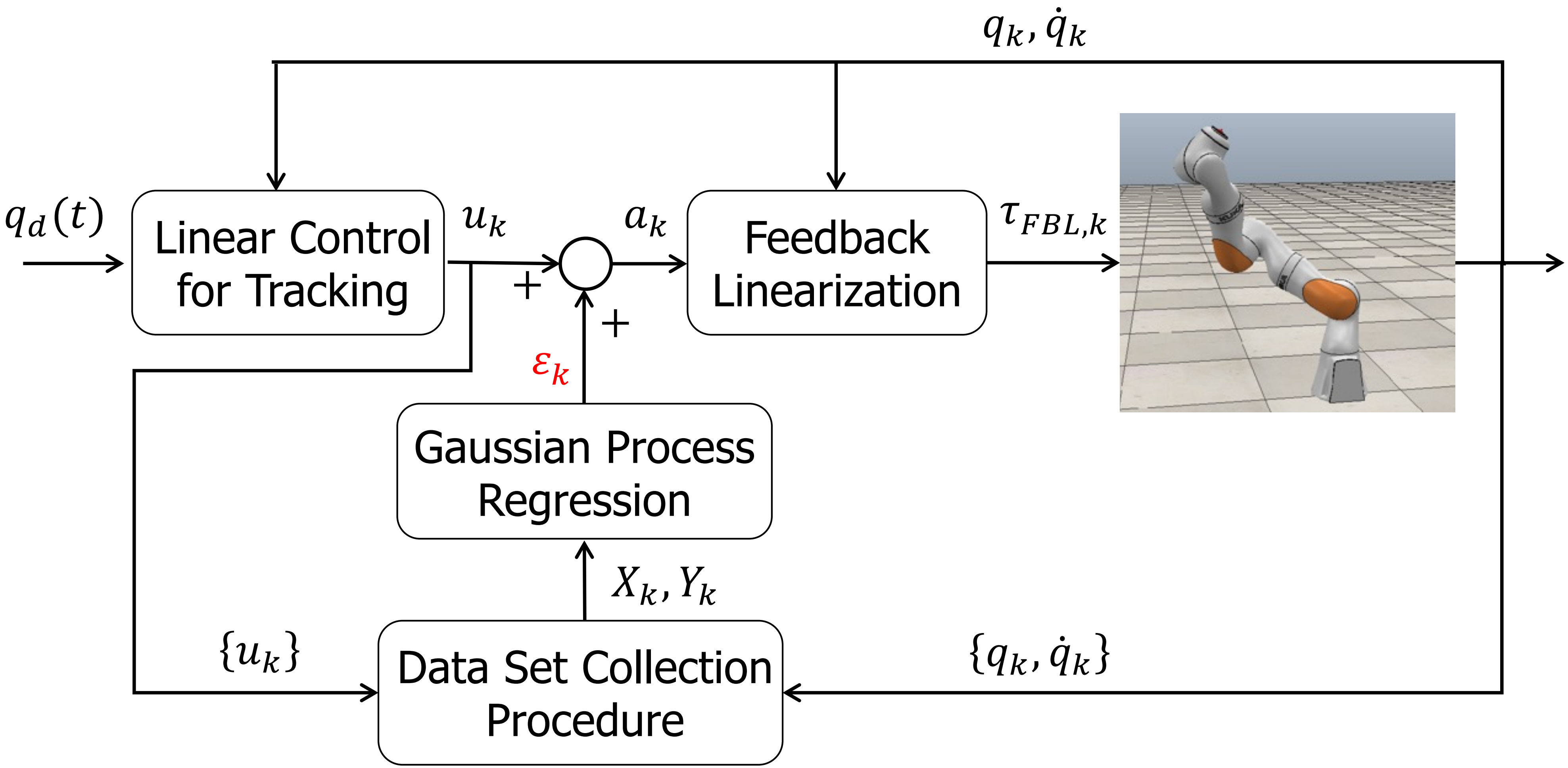
- resulting closed-loop dynamics with perturbation

$$\ddot{q} = a + \delta(q,\dot{q},a) \quad \longleftarrow \quad \delta = \left(M^{-1}\widehat{M} - I\right)a + M^{-1}(\hat{n} - n)$$

- control law for tracking $q_d(t)$ is completed by using (at $t = t_k$) a linear design (PD with feedforward) and a learning regressor ε_k

$$\begin{aligned} a = a_k &= u_k + \varepsilon_k \\ &= \ddot{q}_{d,k} + K_P(q_{d,k} - q_k) + K_D(\dot{q}_{d,k} - \dot{q}_k) + \varepsilon_k \end{aligned}$$

On-line learning scheme



On-line regressor

- Gaussian Process (GP) regression to estimate the perturbation δ
  - from input-output observations that are noisy, with $\omega \sim \mathcal{N}(0, \Sigma_\omega)$, the generated data points at the k-th control step are

$$X_k = (q_k, \dot{q}_k, u_k) \qquad\qquad Y_k = \ddot{q}_k - u_k$$

  - assuming the ensemble of n_d observations with a joint Gaussian distribution

$$\begin{pmatrix} Y_{1:n_d-1} \\ Y_{n_d} \end{pmatrix} \sim \mathcal{N}\left(0, \begin{pmatrix} K & k \\ k^T & \kappa(X_{n_d}, X_{n_d}) \end{pmatrix}\right)$$

a Kernel to be chosen

  - the predictive distribution that approximates $\delta(\hat{X})$ for a generic query $\hat{X}$ is

$$\varepsilon(\hat{X}) \sim \mathcal{N}\left(\mu(\hat{X}), \sigma^2(\hat{X})\right)$$

with

$$\mu(\hat{X}) = k^T(\hat{X})(K + \Sigma_\omega)^{-1} Y$$

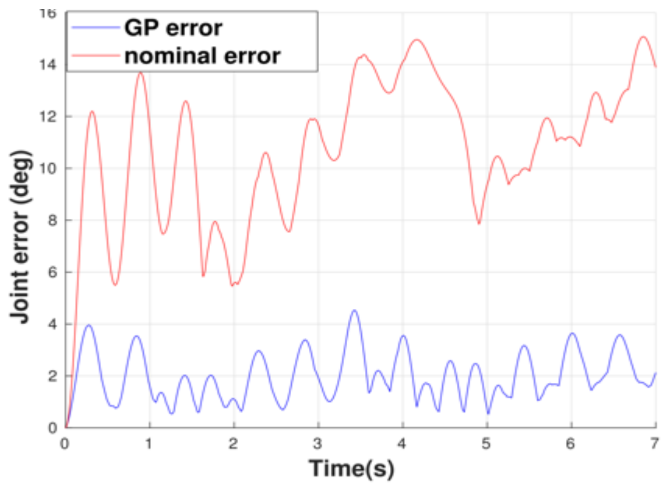
$$\sigma^2(\hat{X}) = \kappa(\hat{X}, \hat{X}) - k^T(\hat{X})(K + \Sigma_\omega)^{-1} k(\hat{X})$$

$$\Rightarrow \varepsilon_k = \varepsilon(X_k)$$

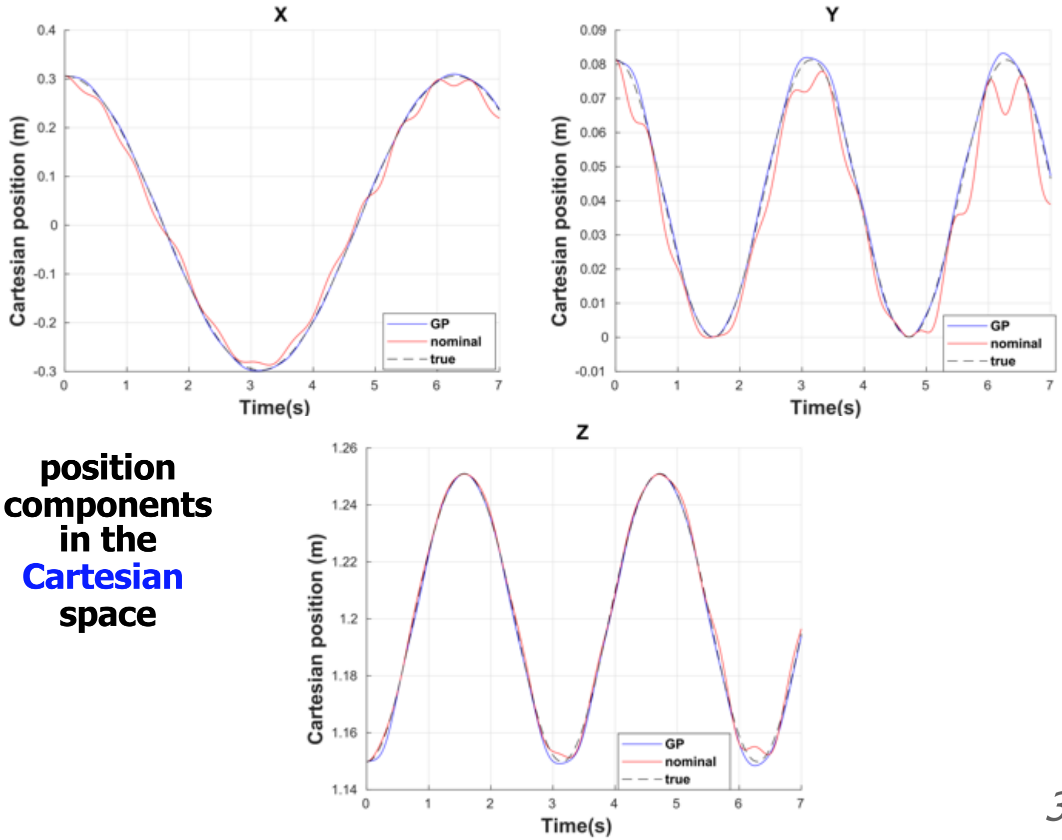
Simulation results

- Kuka LWR iiwa, 7-dof robot
- model perturbations: dynamic parameters with $\pm$ 20% variation, uncompensated joint friction
- 7 separate GPs (one for each joint), each with 21 inputs at every $t = t_k$
- sinusoidal trajectories for each joint

norm of the joint errors

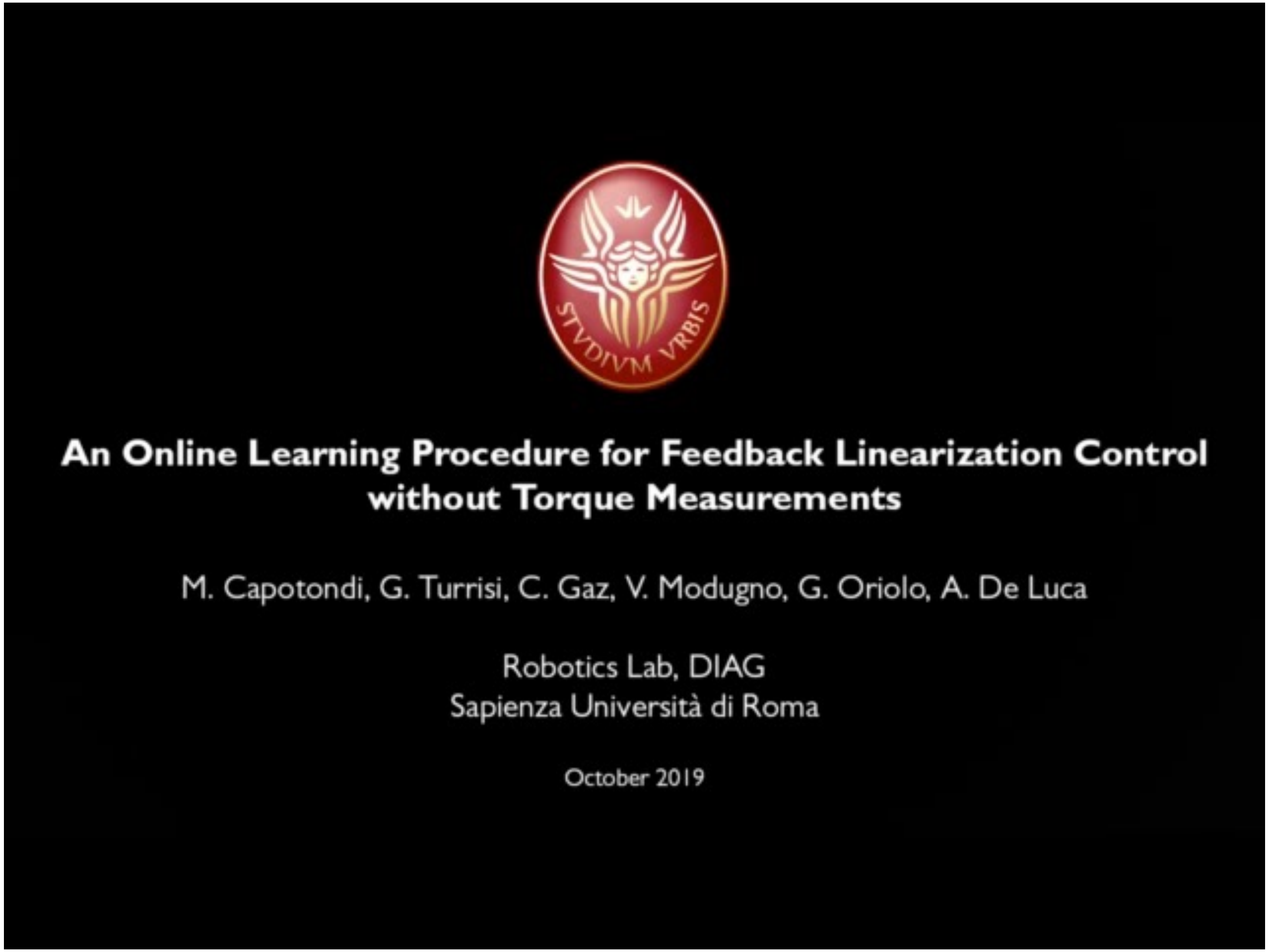


… at the first and only iteration!



position components in the Cartesian space

Simulation results



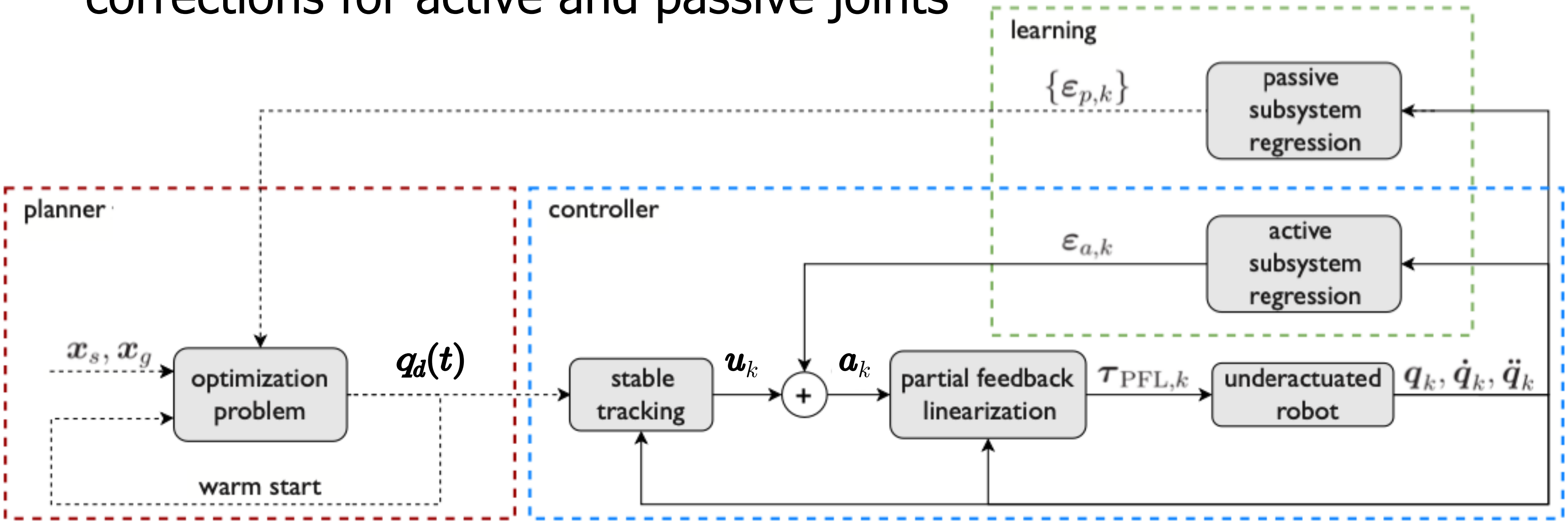
video
(slowed down)

Proc. of Machine Learning Research, vol. 100 (2020)

Extension to underactuated robots

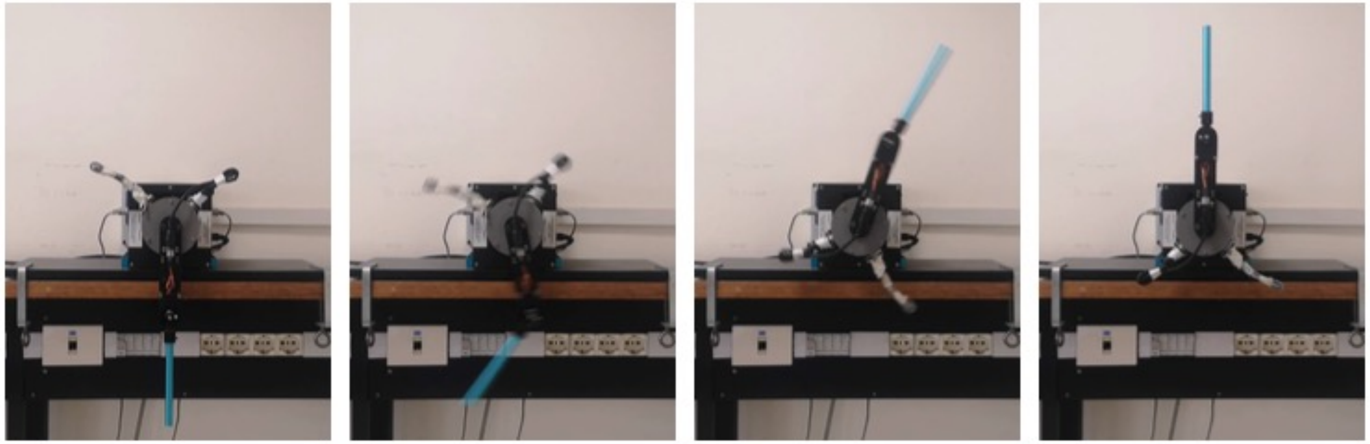
$$\begin{pmatrix} M_{aa}(q) & M_{ap}(q) \\ M_{ap}^T(q) & M_{pp}(q) \end{pmatrix} \begin{pmatrix} \ddot{q}_a \\ \ddot{q}_p \end{pmatrix} + \begin{pmatrix} n_a(q,\dot{q}) \\ n_p(q,\dot{q}) \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix}$$

- planner optimizes motion of passive joints (at every iteration)
- controller for active joints with partial feedback linearization
- two regressors (on/off-line) for learning the required acceleration corrections for active and passive joints

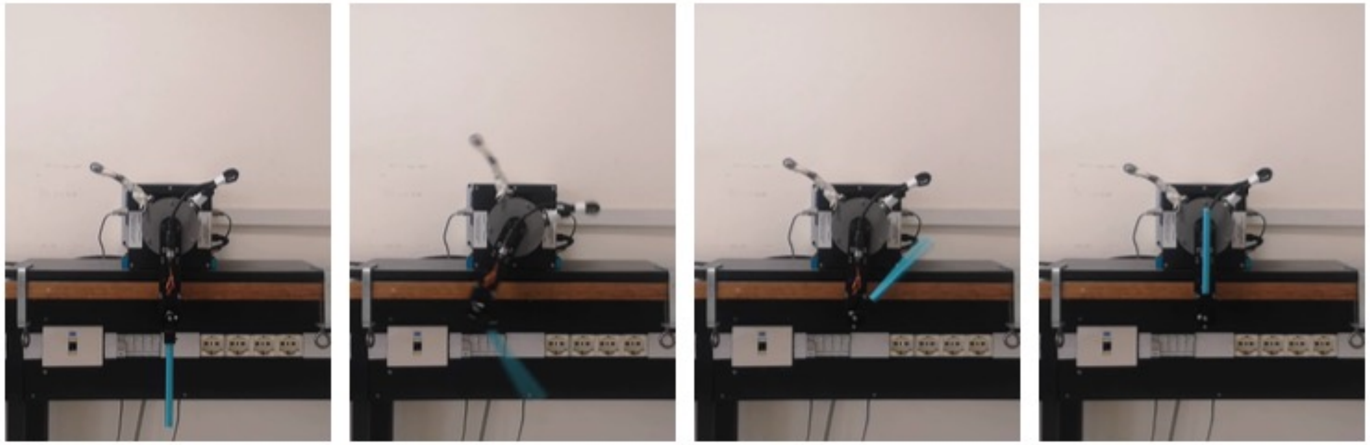


Experiments on the Pendubot

- Pendubot, 2-dof robot with passive second joint
- swing-up maneuvers from down-down to a new equilibrium state



⇒ up-up



⇒ down-up

Experimental results

IEEE Robotics and Automation Letters, vol. 7(1), 2022 video

convergence in 2 iterations!

latest video with more simulations & experiments on YouTube https://youtu.be/1aKG__8gfvk